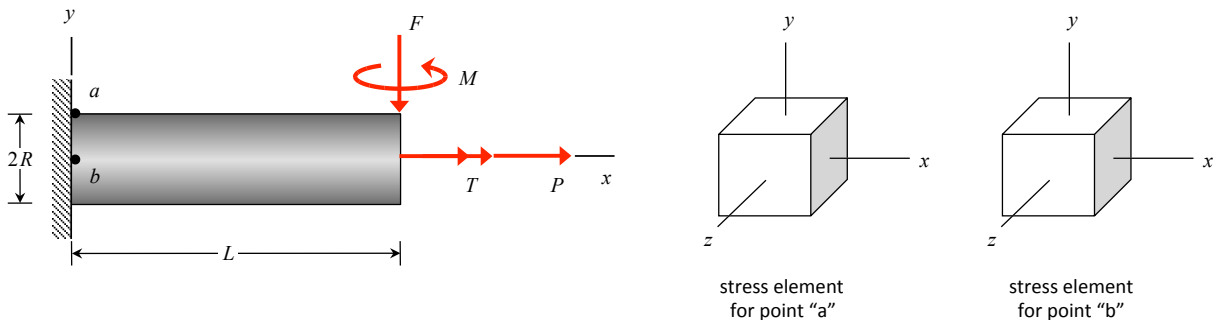


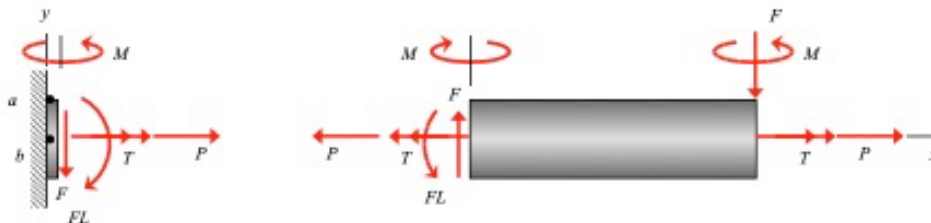
A circular cross-sectioned shaft having an outer radius of R and length L is acted upon by forces F and P , and an axial torque T at its right end. The material of the shaft (aluminum alloy 2014-T6) is ductile and has a yield strength of σ_Y (see the tables in Chapter 2 for the numerical values of the yield strength of this material).

- Determine the stress components at points “a” and “b” in the shaft, where “a” is on the top surface of the shaft along the y -axis, and “b” is on the front surface of the shaft along the z -axis. Show these components on the stress elements provided below. Leave your answers in terms of the variables defined here in the problem statement.
- Compute the *numerical* values for the principal components of stress and the absolute maximum shear stress at point “a”.
- Compute the *numerical* value for the yielding factors of safety at point “a” using the maximum shear stress (MSS) theory.
- Compute the *numerical* value for the yielding factors of safety at point “a” using the maximum distortional energy (MDE) theory.

For your numerical answers, use the following: $P = 2 \text{ kN}$, $F = 500 \text{ N}$, $T = 600 \text{ N-m}$, $R = 20 \text{ mm}$ and $L = 1.5 \text{ m}$.

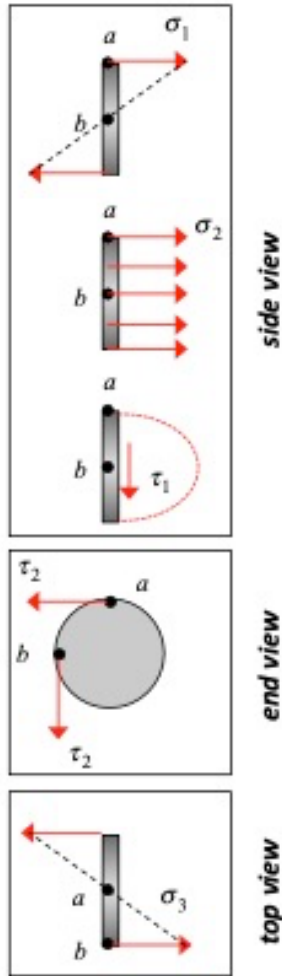


Acting on the cut surface at the wall, we have a shear force F , a bending moment $M_z = -FL$, an axial force P and a torque T .



Only the following page, we will use a table to list the components of normal and shear stress at the cut surface at points “a” and “b”. These components are then shown on the stress elements above.

For location "a":



$$I = \frac{\pi}{4} R^4$$

$$A = \pi R^2$$

$$I_p = \frac{\pi}{2} R^4$$

load	location "a"	location "b"
M_z	$\sigma_{1a} = \frac{(FL)R}{I}$	$\sigma_{1b} = 0$
P	$\sigma_{2a} = \frac{P}{A}$	$\sigma_{2b} = \frac{P}{A}$
F	$\tau_{1a} = 0$	$\tau_{1b} = \frac{4}{3} \frac{F}{A}$
T	$\tau_{2a} = \frac{TR}{I_p}$	$\tau_{2b} = \frac{TR}{I_p}$
M	$\sigma_{3a} = 0$	$\sigma_{3b} = \frac{MR}{I}$

$$\sigma_x = \sigma_{1a} + \sigma_{2a} = \frac{FLR}{I} + \frac{P}{A} = \frac{FLR}{\pi R^4 / 4} + \frac{P}{\pi R^2} = \frac{F}{\pi R^2} \left(4 \frac{L}{R} + \frac{P}{F} \right)$$

$$\tau_{xz} = \frac{TR}{I_p} = \frac{2T}{\pi R^3}$$

$$\sigma_{ave} = \frac{\sigma_x}{2} = \frac{F}{2\pi R^2} \left(4 \frac{L}{R} + \frac{P}{F} \right)$$

$$R = \sqrt{\left(\frac{\sigma_x}{2} \right)^2 + \tau_{xz}^2} = \frac{F}{2\pi R^2} \sqrt{\left(4 \frac{L}{R} + \frac{P}{F} \right)^2 + 16 \left(\frac{T}{RF} \right)^2}$$

$$\begin{aligned} \sigma_{P1} = \sigma_{ave} + R &= \frac{4FL}{\pi R^3} + \frac{P}{\pi R^2} + \frac{F}{2\pi R^2} \sqrt{\left(4 \frac{L}{R} + \frac{P}{F} \right)^2 + 16 \left(\frac{T}{RF} \right)^2} \\ &= \frac{F}{2\pi R^2} \left[8 \frac{L}{R} + 2 \frac{P}{F} + \sqrt{\left(4 \frac{L}{R} + \frac{P}{F} \right)^2 + 16 \left(\frac{T}{RF} \right)^2} \right] \end{aligned}$$

$$\sigma_{P2} = \sigma_{ave} - R = \frac{F}{2\pi R^2} \left[8\frac{L}{R} + 2\frac{P}{F} - \sqrt{\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2} \right]$$

Since $R > \sigma_{ave}$, we know that σ_{P1} and σ_{P2} have opposite signs; therefore, $|\tau_{max}|_{abs} = R$.

Maximum shear stress (MSS) theory

For the MSS theory, we write that failure occurs when: $|\tau_{max}|_{abs} = \frac{\sigma_Y}{2}$. Therefore, the factor of safety for MSS is:

$$FS = \frac{\sigma_Y / 2}{|\tau_{max}|_{abs}} = \frac{\sigma_Y}{2R} = \frac{\pi R^2 \sigma_Y}{F} \left[\left(4\frac{L}{R} + \frac{P}{F}\right)^2 + 16\left(\frac{T}{RF}\right)^2 \right]^{-1}$$

Maximum distortional energy (MDE) theory

For the MSS theory, we write that failure occurs when: $\sigma_Y = \sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}$. Therefore, the factor of safety for MDE is:

$$FS = \frac{\sigma_Y}{\sigma_M} = \frac{\sigma_Y}{\sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}}$$