ME 323: Mechanics of Materials **Homework Set H29**

Summer 2024 **Assigned/Due: July 22/July 24**

Consider the structure shown on the next page with loading applied at end H.

a) Determine the internal resultants at location B on section OC.
b) Show the stress distribution due to each internal resultant on the supplied figures on the attached worksheet.
c) Complete the table in the attached worksheet quantifying the stresses at points “a” and “b”.
d) Show the stress components on the stress elements at “a” and “b” on the attached worksheet.
e) Determine the principal components of stress and maximum absolute shear stress at point “b”.
f) *BONUS POINTS* (2 points): Explain in words or equations why the maximum in-plane shear stress is equal to the maximum absolute shear stress at point “b”, regardless of the loading on the structure at end H.

Please include the attached worksheet with your homework.

From FBD on the following page:

$$\Sigma F_x = P - F_x = 0 \Rightarrow F_x = P$$

$$\Sigma F_y = F_y = 0 \Rightarrow F_y = 0$$

$$\Sigma F_z = 2P - F_z = 0 \Rightarrow F_z = 2P$$

$$\Sigma \vec{M}_B = \vec{r}_{H/B} \times (P\hat{i} - P\hat{j} + 2P\hat{k}) - T\hat{j} + \vec{M}$$

$$\bar{0} = (2L\hat{i} + 2L\hat{j}) \times (P\hat{i} - P\hat{j} + 2P\hat{k}) - T\hat{j} + \vec{M}$$

$$= 4PL\hat{i} - (T+4PL)\hat{j} - 4PL\hat{k} + (-T_x\hat{i} + M_y\hat{j} + M_z\hat{k})$$

$$\begin{cases} \hat{i}: 0 = 4PL - T_x \Rightarrow T_x = 4PL \\ \hat{j}: 0 = -(T+4PL) + M_y \Rightarrow M_y = 4PL + T \\ \hat{k}: 0 = -4PL + M_z \Rightarrow M_z = 4PL \end{cases}$$

$$R = \frac{d}{2}\ ;\ A = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}\ ;\ I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64}\ ;\ I_p = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

(c) For point “b”:

$$\sigma_x = \sigma_1 - \sigma_3 = \frac{F_x}{A} + \frac{M_z R}{I} = \frac{P}{\pi d^2/4} + \frac{(4PL)\frac{d}{2}}{\pi d^4/64} = \frac{4P}{\pi d^2}\left[1 + 32\left(\frac{L}{d}\right)\right]$$

$$\tau_{xz} = \tau_2 + \tau_3 = \frac{4}{3}\frac{F_z}{A} + \frac{T_x R}{I_p} = \frac{4}{3}\frac{2P}{\pi d^2/4} + \frac{(4PL)d/2}{\pi d^4/32}$$

$$= \frac{32}{3\pi}\frac{P}{d^2}\left[1 + 6\left(\frac{L}{d}\right)\right]$$

$$\therefore \quad \sigma_{ave} = \frac{\sigma_x}{2} = \frac{2P}{\pi d^2}\left[1 + 32\left(\frac{L}{d}\right)\right]$$

$$R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xz}^2} = \frac{P}{\pi d^2}\sqrt{4^2\left[1+32\left(\frac{L}{d}\right)\right]^2 + \left(\frac{32}{3}\right)^2\left[1+6\left(\frac{L}{d}\right)\right]^2}$$

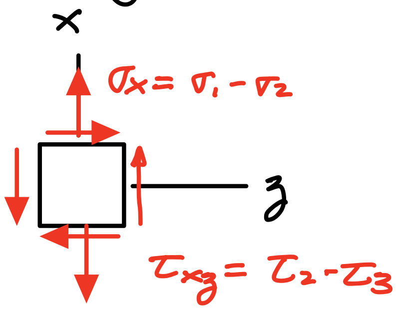
$$> \sigma_{ave}$$

$$\left.\begin{aligned} \sigma_{p1} &= \sigma_{ave} + R \\ \sigma_{p2} &= \sigma_{ave} - R \end{aligned}\right\}$$ Since $\sigma_{ave} < R$, σ_{p1} and σ_{p2} have opposite signs

$$\therefore \quad |\tau|_{max,abs} = R$$

f) As seen above, σ_{p1} and σ_{p2} have opposite signs, regardless of the value of P.

A more general answer would be that the stress element for "b" is given by:

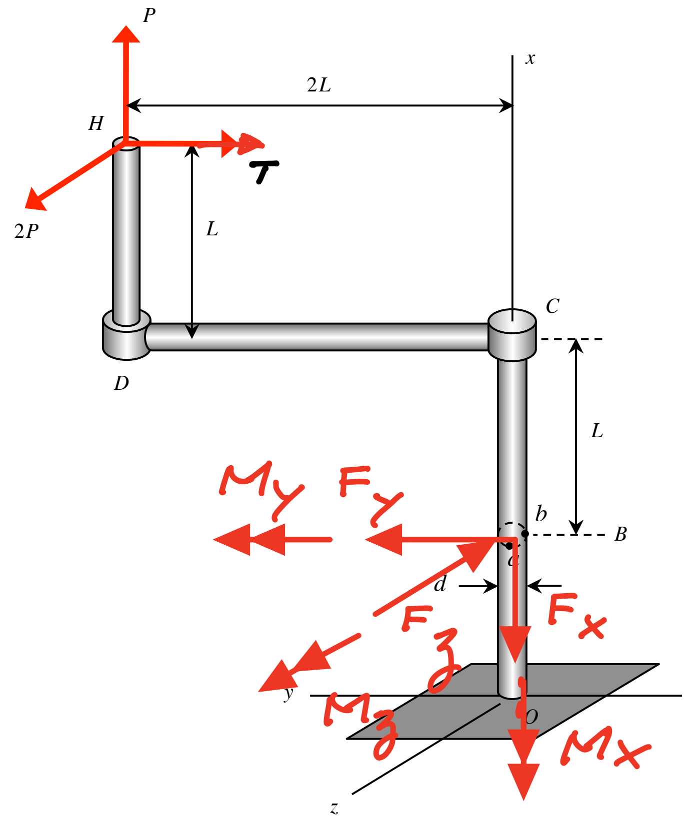


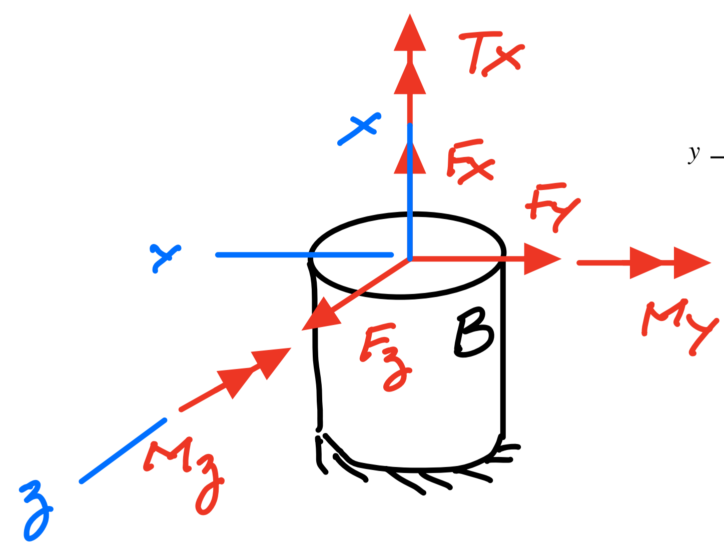
$$\therefore \begin{cases} \sigma_{ave} = \dfrac{\sigma_x + \cancel{\sigma_z}^{\,0}}{2} = \dfrac{\sigma_x}{2} \\[2ex] R = \sqrt{\left(\dfrac{\sigma_x - \cancel{\sigma_z}^{\,0}}{2}\right)^2 + \tau_{xz}^2} = \sqrt{\left(\dfrac{\sigma_x}{2}\right)^2 + \tau_{xz}^2} > \sigma_{ave} \end{cases}$$

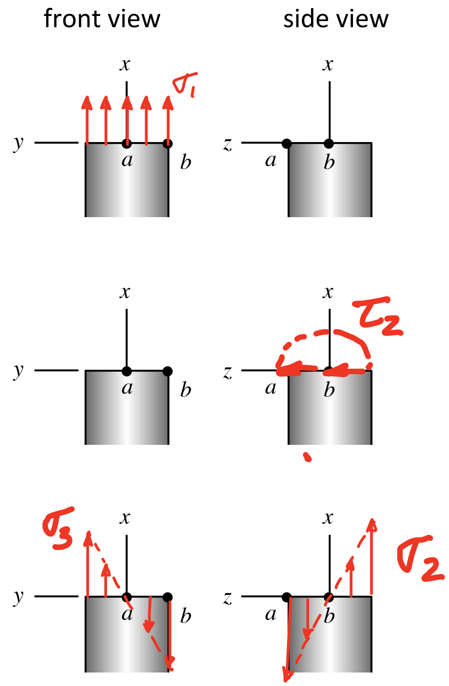
σ_{ave}^2

Since $R > \sigma_{ave} \Rightarrow \sigma_{p1}$ and σ_{p2} have opposite signs

$\Rightarrow \tau_{max,abs} = \tau_{max,in\text{-}plane} = R$







load	"a"	"b"
F_x	$\sigma_1 = \frac{F_x}{A}$	$\sigma_1 = \frac{F_x}{A}$
F_y	0	0
F_z	0	$\tau_2 = \frac{4}{3}\frac{F_z}{A}$
T_x	$\tau_3 = \frac{T_x R}{I_p}$	$\tau_3 = \frac{T_x R}{I_p}$
M_y	$\sigma_2 = \frac{M_y R}{I}$	0
M_z	0	$\sigma_3 = \frac{M_z R}{I}$