ME 323: Mechanics of Materials ***Homework Set H21***

Summer 2023 ***Assigned/Due: July 12/July 14***

A rectangular cross-section rod is made up of three sections: BC, CD and DH. Sections BC and DH have a linear taper in their thickness, whereas CD has a constant thickness h. The dimension of the cross-section into the page is a constant value of *b*. The material is homogeneous throughout with a Young's modulus of E. For this problem:

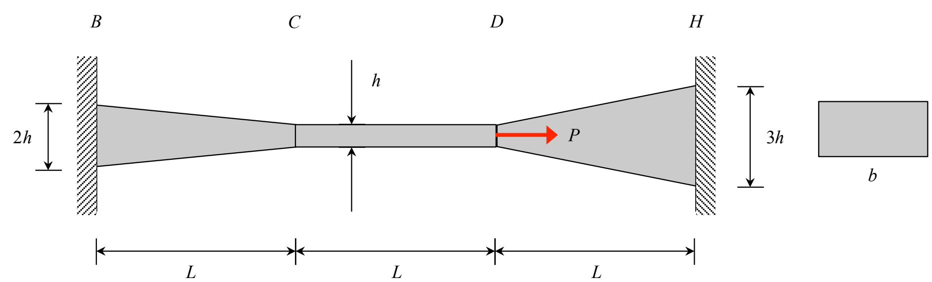
a) Draw a free body diagram of the entire rod.

b) Assemble the stiffness matrix [K] and force vector {F} for a four-node (three-element) finite element model for the rod.

c) Enforce the boundary conditions on the stiffness matrix and force vector.

d) Solve for the displacements at locations C and D on the rod. You may use Matlab or Mathematica to solve this set of equilibrium equations. State your answers in terms of *P*, *E*, *L*, *h* and *b*.

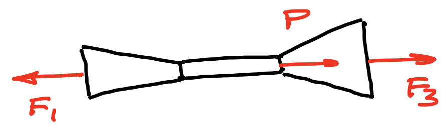
e) Determine the reactions on the rod due to the wall supports.



$$\begin{cases} k_1 = \dfrac{(EA)_1}{L} = \dfrac{E}{L}\left[\dfrac{1}{2}(2hb+hb)\right] = \dfrac{3}{2}\dfrac{Ehb}{L} \\ k_2 = \dfrac{(EA)_2}{L} = \dfrac{Ehb}{L} \\ k_3 = \dfrac{(EA)_3}{L} = \dfrac{E}{L}\left[\dfrac{1}{2}(hb+3hb)\right] = 2\dfrac{Ehb}{L} \end{cases}$$

$$[K] = \begin{bmatrix} k_1 & -k_1 & & \\ -k_1 & k_1+k_2 & -k_2 & \\ & -k_2 & k_2+k_3 & -k_3 \\ & & -k_3 & k_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{2} & -\frac{3}{2} & & \\ -\frac{3}{2} & \frac{5}{2} & -1 & \\ & -1 & 3 & -2 \\ & & -2 & 2 \end{bmatrix} \frac{Ehb}{L}$$



$$\{F\} = \begin{Bmatrix} \cancel{F_1} \\ 0 \\ P \\ \cancel{F_3} \end{Bmatrix}$$

To enforce BC's, strike out 1^{st} & 4^{th} row of $[K]$ & $\{F\}$ and the 1^{st} & 4^{th} column of $[K]$

$$\therefore \begin{cases} \left(\frac{5}{2}\cancel{\frac{Ehb}{L}}\right) u_2 - \cancel{\frac{Ehb}{L}} u_3 = 0 \\ -\frac{Ehb}{L} u_2 + 3\frac{Ehb}{L} u_3 = P \end{cases}$$

$$u_3 = \frac{5}{2} u_2$$

$$\left(-1 + \frac{15}{2}\right)\frac{Ehb}{L} u_2 = P \Rightarrow \boxed{u_2 = \frac{2}{13}\frac{PL}{Ehb}}$$

$$\& \quad \boxed{u_3 = \frac{5}{2}\left(\frac{2}{13}\frac{PL}{Ehb}\right) = \frac{5}{13}\frac{PL}{Ehb}}$$

From the 1^{st} and 4^{th} equilibrium equations above:

$$-F_1 = -\frac{3}{2}u_2 \quad \Rightarrow \quad F_1 = \frac{3}{2}u_2 = \frac{3}{13}\frac{PL}{Ehb}$$

$$F_3 = -2u_3 = -\frac{10}{13}\frac{PL}{Ehb}$$