ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: Solution
(Last) (First)

PROBLEM #1 (25 pts)

The compressed-air tank is fabricated of steel plate that is welded along a helix that makes an angle of β=60° with respect to the longtigudinal axis of the tank. The inside diameter of the cylinder is 48 inch, the wall thickness is 0.5 inch, and the internal pressure is 200 psi.

(a) Determine the axial stress σ_a and hoop stress σ_h.

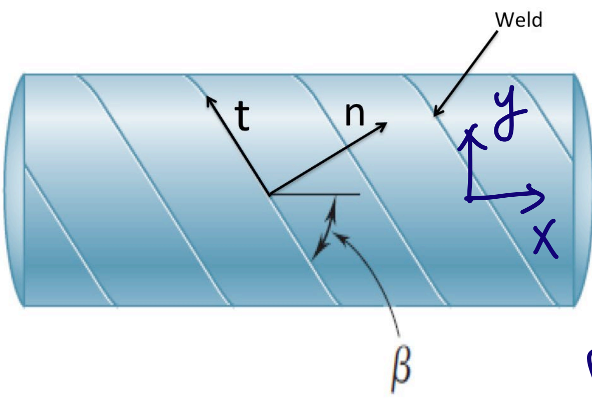
(b) Draw the Mohr's circle on the given graph.

(c) Use Mohr's circle to determine

(i) the principal stresses ($\sigma_1, \sigma_2, \sigma_3$), and the absolute maximum shear stress $\tau_{\max}^{abs}$.

(ii) the stresses in the directions perpendicular (n) and tangent (t) to the weld. Mark the two stress states on Mohr's circle. And show the stresses in a properly oriented stress element.

(d) Knowing that the Young's modulus of steel is E=29×10^6 psi and the Poisson's ratio is ν=0.3, determine the strain ε_t tangent to the weld.



$$\sigma_a = \frac{pR}{2t} = \frac{200 \cdot 24}{2 \cdot 0.5} = 4800\ psi$$

$$\sigma_h = \frac{pR}{t} = \frac{200 \cdot 24}{0.5} = 9600\ psi$$

$\sigma_{p1} = 4800\ psi$, $\sigma_{p2} = 9600\ psi$

$\sigma_1 = 9600\ psi$, $\sigma_2 = 4800\ psi$, $\sigma_3 = 0$

$$\tau_{max.abs} = \frac{\sigma_1 - \sigma_3}{2} = 4800\ psi$$

In n direction:

$\theta = \frac{\pi}{6}$

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos\left(\frac{1}{3}\pi\right)$$

$$= 7200 - 2400 \cdot \frac{1}{2} = 6000\ psi$$

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta = 2400 \cdot \sin\frac{\pi}{3} = 2078\ psi$$

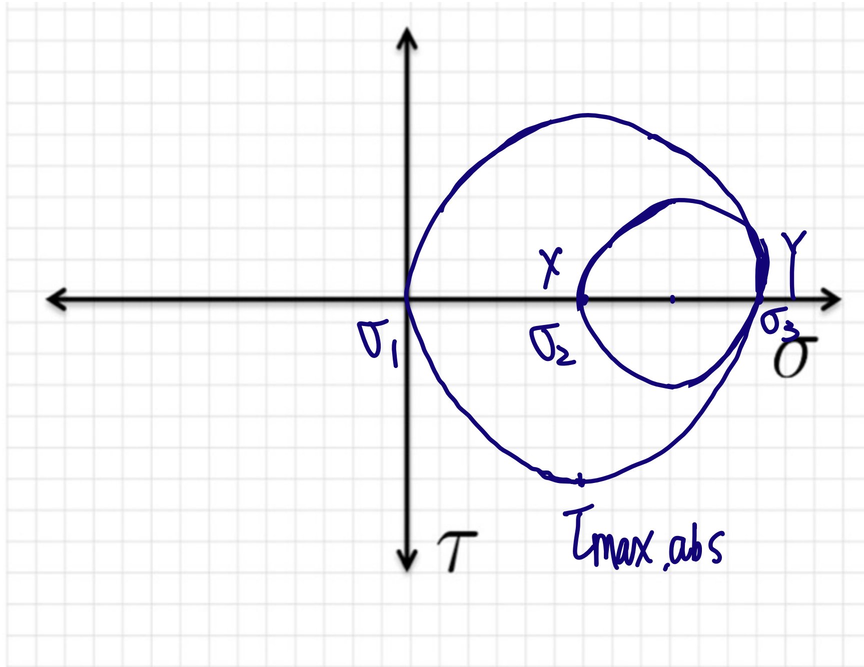
In t direction:

$\theta = \frac{2}{3}\pi$

$$\sigma = 7200 - 2400 \cdot \cos\left(\frac{4\pi}{3}\right) = 8400\ psi$$

$$\tau = 2400 \cdot \sin\left(\frac{4}{3}\pi\right) = -2078\ psi$$

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: ______________________________
(Last) (First)

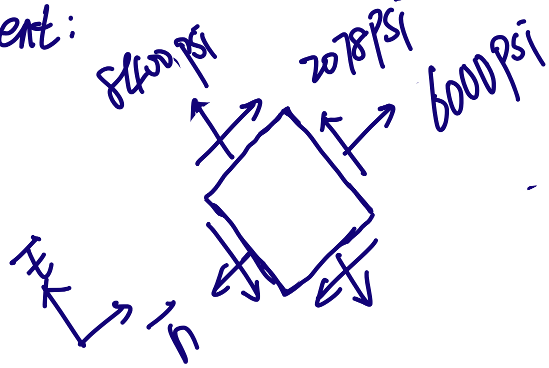


strain in t direction: $$\varepsilon_t = \frac{1}{E}\left[\sigma_t - \nu \sigma_n\right]$$

$$= \frac{1}{29\times10^6}\left[8400 - 0.3\cdot 6000\right]$$

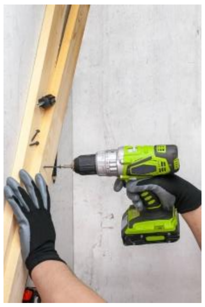
$$= 0.023\%$$

stress element:

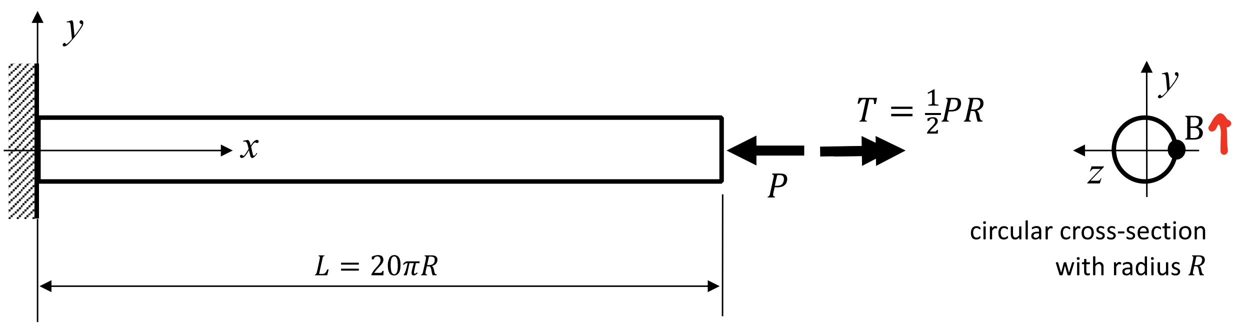


ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM #2 (25 points)



The effect of driving a screwdriver is represented by the cylinder with the applied force P and torque T at the tip. The cylinder has a radius R and a length $L = 20\pi R$, and it is made of a material with a yield strength equal to σ_Y and a Young's modulus equal to $E = 500\sigma_Y$. The cylinder can be modelled as being fixed to a wall at one end (x = 0), and loaded axially by a compressive force P and by a positive torque $T = \frac{1}{2}PR$ on the other end (x = L).



a) Determine the state of stress at point B located at a radial distance R from the axis (see figure) on the cross-section located at $x = L$. Express the stress components in terms of P and R.

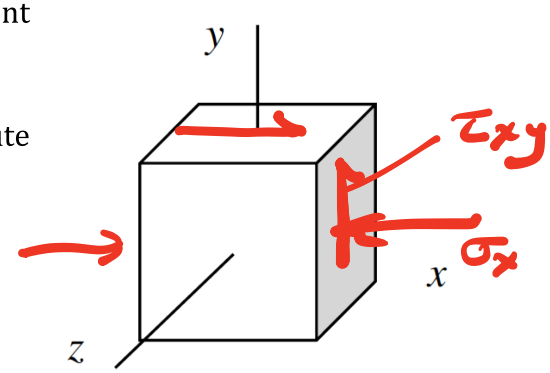
Stresses at B:

$$\sigma_x = -\frac{P}{A} = -\frac{P}{\pi R^2}$$

$$\tau_{xy} = \frac{TR}{I_P} = \frac{(\frac{1}{2}PR)R}{\frac{\pi R^4}{2}} = \frac{P}{\pi R^2}$$

Name: SOLUTION
(Last) (First)

b) Draw the state of stress at point B on the stress element provided.



c) Calculate the principal stresses and the absolute maximum shear stress at point B.

$$\sigma_{av} = \frac{\sigma_x}{2} = -\frac{P}{2\pi R^2}$$

$$R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(-\frac{P}{2\pi R^2}\right)^2 + \left(\frac{P}{\pi R^2}\right)^2} = \frac{\sqrt{5}}{2}\frac{P}{\pi R^2} = 1.118\frac{P}{\pi R^2}$$

$$\sigma_1 = \sigma_{av} + R = 0.618\frac{P}{\pi R^2} > 0$$

$$\sigma_2 = \sigma_{av} - R = -1.618\frac{P}{\pi R^2} < 0$$

$$\tau_{max,abs} = R = 1.118\frac{P}{\pi R^2}$$

d) Determine the Von Mises equivalent stress at point B. Express the stress in terms of P and R.

$$\sigma_M = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

$$= \frac{P}{\pi R^2}\sqrt{(0.618)^2 - (0.618)(-1.618) + (1.618)^2}$$

$$= 2P/\pi R^2$$

e) Utilizing the ***maximum-distortion-energy failure theory***, determine the smallest radius R that will prevent yielding of the deformable body. Express the smallest radius in terms of P and σ_Y.

$$\sigma_M = \sigma_Y = \frac{2P}{\pi R^2}$$

$$R_{MDE} = \sqrt{\frac{2P}{\pi\sigma_Y}} = 0.798\sqrt{\frac{P}{\sigma_Y}}$$

Name: SOLUTION
(Last) (First)

f) Do you expect the ***Maximum shear stress theory*** to result in a larger or smaller value R than the one predicted in part e)? Explain your answer.

MSS is more conservative than MDE, so it will result in LARGER R.

g) Utilizing ***Euler elastic buckling theory***, determine the smallest radius R that will prevent buckling of the deformable body. Express the smallest radius in terms of P and σ_Y.

$$P_{cr} = \frac{\pi^2 E I}{(L_e)^2}$$

Fixed-free B.C's

$L_e = 2L$

$$P_{cr} = \frac{\pi^2 (500\,\sigma_Y)\left(\frac{\pi R^4}{4}\right)}{(40\pi R)^2}$$

$$= 0.078\,\sigma_Y\,\pi R^2$$

$$R = \sqrt{\frac{P}{0.078\,\pi\,\sigma_Y}} = 2.02\sqrt{\frac{P}{\sigma_Y}}$$

Name: SOLUTION
(Last) (First)

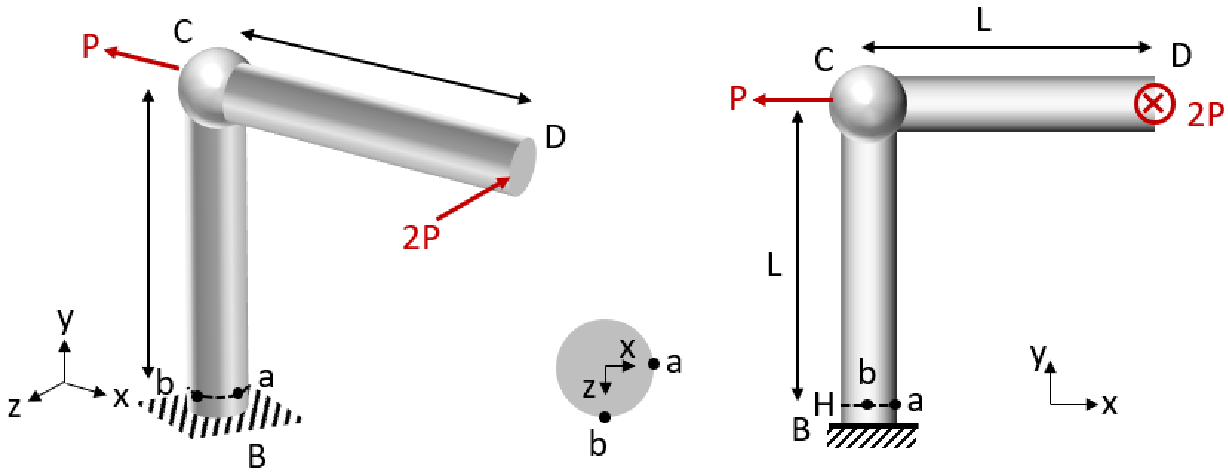
h) Utilizing the results from (e) and (g), determine the smallest radius that will prevent ***both*** yielding ***and*** buckling of the cylindrical body.

$$R_g = 2.02\sqrt{\frac{P}{\sigma_Y}} > R_e = 0.798\sqrt{\frac{P}{\sigma_Y}}$$

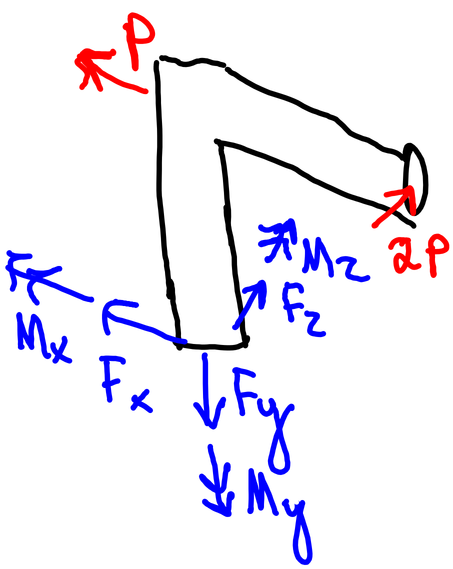
So the larger R_g will prevent both yielding & buckling

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: ______________________________
(Last) (First)

PROBLEM #3 (25 points)
BCD is an "L-shaped" structure made of a solid cylindrical rod with a diameter of d that is fixed to a wall at node B. A force of 2P is applied in the negative z-direction at node D and a force of P is applied in the negative x-direction at node C. The structure is made of a material with a Young's modulus of E and a shear modulus of G.



(a) Determine the internal reactions at cross-section H.



$$\Sigma F = -F_x\hat{i} - F_y\hat{j} - F_z\hat{k} - P\hat{i} - 2P\hat{k}$$

$$F_x = -P \qquad F_y = 0 \qquad F_z = -2P$$

$$\Sigma M = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = 0$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & L & 0 \\ -P & 0 & 0 \end{vmatrix} = (0, 0, PL)$$

$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ L & L & 0 \\ 0 & 0 & -2P \end{vmatrix} = (-2PL, 2PL, 0)$$

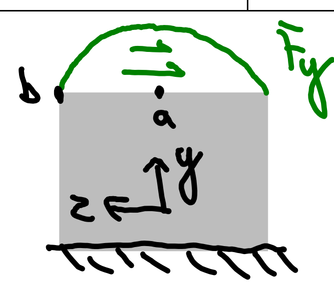
$$0 = -M_x\hat{i} - M_y\hat{j} - M_z\hat{k} - 2PL\hat{i} + 2PL\hat{j} + PL\hat{k}$$

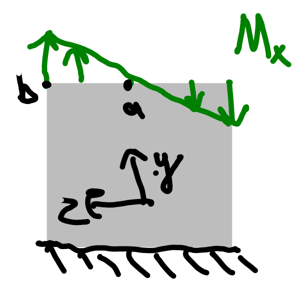
$$M_x = -2PL \qquad M_y = 2PL \qquad M_z = PL$$

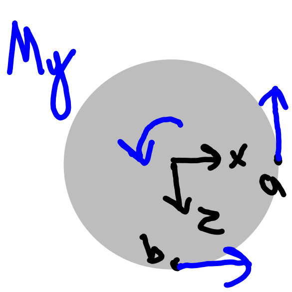
(b) Determine the stresses at points a and b (which are located on the cross section at H) and write them in Table 1.

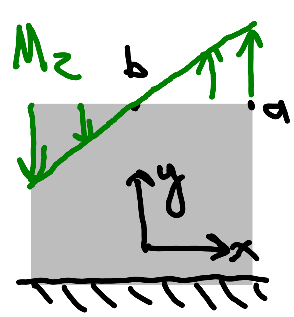
Table 1

Loading	Stress component a	Stress component b
$F_x = -P$	0	$\tau_{xy} = -\frac{4P}{3A} = \frac{-4P}{3\pi r^2} = -\frac{16P}{3\pi d^2}$
$F_y = 0$	0	0
$F_z = -2P$	$\tau_{yz} = -\frac{4(2P)}{3A} = -\frac{32P}{3\pi d^2}$	0
$M_x = -2PL$	0	$\sigma_y = \frac{2PL(\frac{d}{2})}{I} = \frac{64PL}{\pi d^3}$
$M_y = 2PL$	$\tau_{yz} = \frac{-2PL(\frac{d}{2})}{I_p} = \frac{-32PL}{\pi d^3}$	$\tau_{xy} = \frac{2PL(\frac{d}{2})}{I_p} = \frac{32PL}{\pi d^3}$
$M_z = PL$	$\sigma_y = \frac{PL(\frac{d}{2})}{I} = \frac{32PL}{\pi d^3}$	0

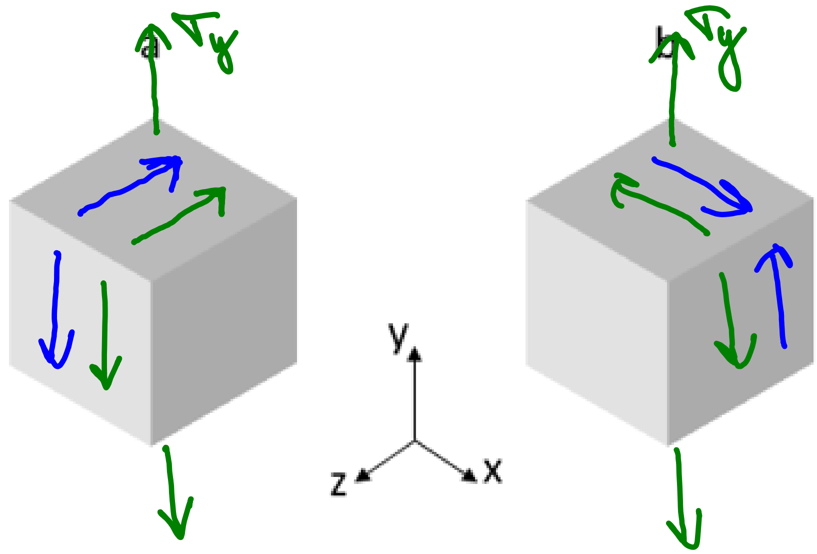






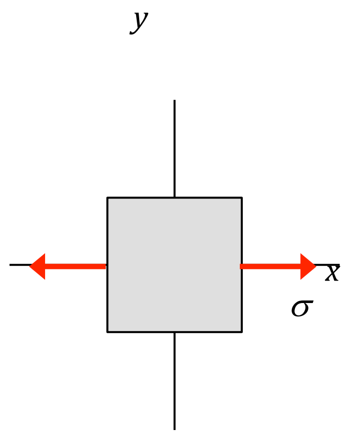


(c) Draw the stress element at points a and b.



ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM #4 - PART A – 2 points



Consider the state of stress shown above in a *ductile* material. Let σ_{MSS} and σ_{MDE} be the values of the normal stress σ above that correspond to failure of the material using the maximum shear stress and maximum distortional energy theories, respectively. Circle the answer below that best describes the relative sizes of σ_{MSS} and σ_{MDE}.

a) $\sigma_{MSS} > \sigma_{MDE}$

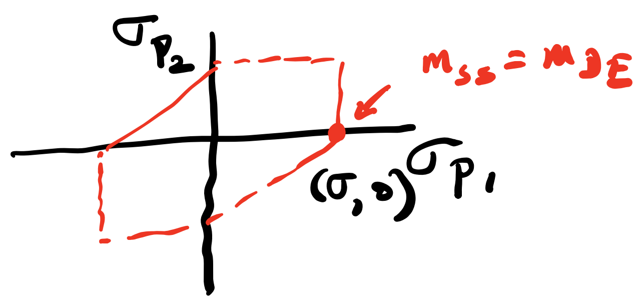
b) $\sigma_{MSS} = \sigma_{MDE}$

c) $\sigma_{MSS} < \sigma_{MDE}$

$\sigma_{av} = \frac{\sigma}{2}$ $\quad R = \frac{\sigma}{2}$

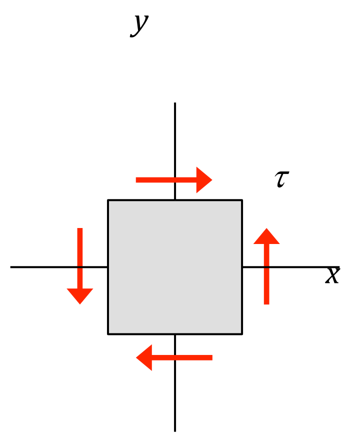
$\sigma_{p_1} = \sigma_{av} + R = \sigma$

$\sigma_{p_2} = \sigma_{av} - R = 0$



ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM #4 - PART B – 2 points



Consider the state of stress shown above in a *ductile* material. Let τ_{MSS} and τ_{MDE} be the values of the shear stress τ above that correspond to failure of the material using the maximum shear stress and maximum distortional energy theories, respectively. Circle the answer below that best describes the relative sizes of τ_{MSS} and τ_{MDE}.

a) $\tau_{MSS} > \tau_{MDE}$

b) $\tau_{MSS} = \tau_{MDE}$

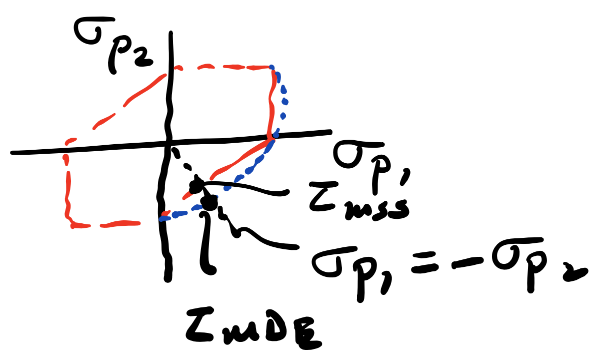
c) $\tau_{MSS} < \tau_{MDE}$

$\sigma_{av} = 0 \qquad R = \tau$

$\sigma_{p1} = \tau \qquad \sigma_{p2} = -\tau$

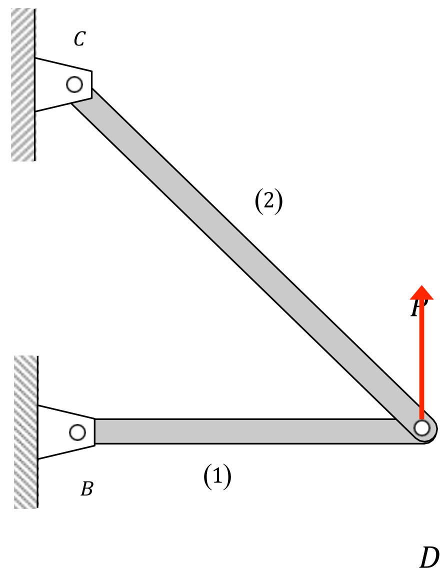
$\sigma_{p1} = -\sigma_{p2}$

$\tau_{MSS} < \tau_{MDE}$



ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM #4 - PART C – 2 points



Consider the truss shown above made up on members (1) and (2).

TRUE or *FALSE*: The stress in member (1) depends on the material makeup of member (2).

Statically determinate – stresses do NOT depend on material properties

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: ______________________________
(Last) (First)

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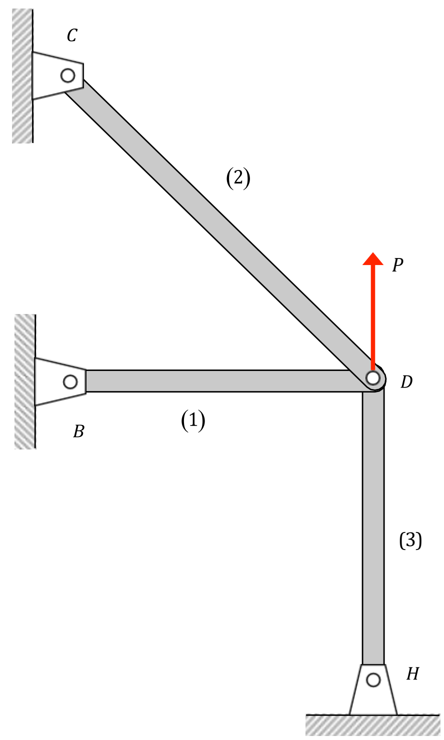
PROBLEM #4 - Part D - 2 points



Consider the truss shown above made up on members (1), (2) and (3).

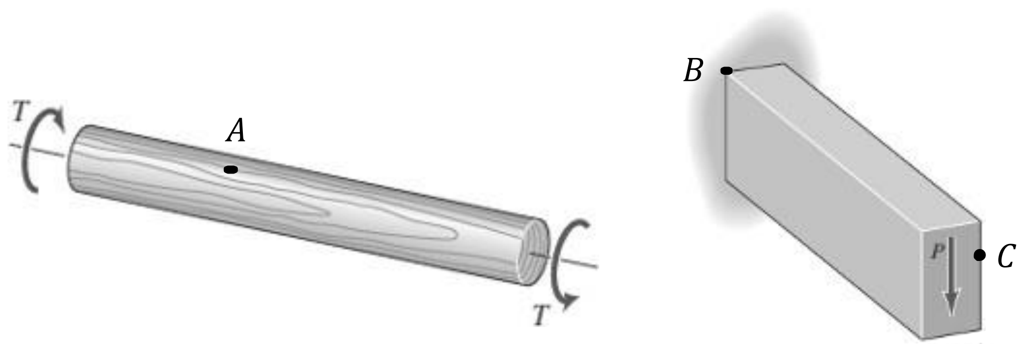
TRUE or *FALSE*: The stress in member (1) depends on the material makeup of members (2) and (3).

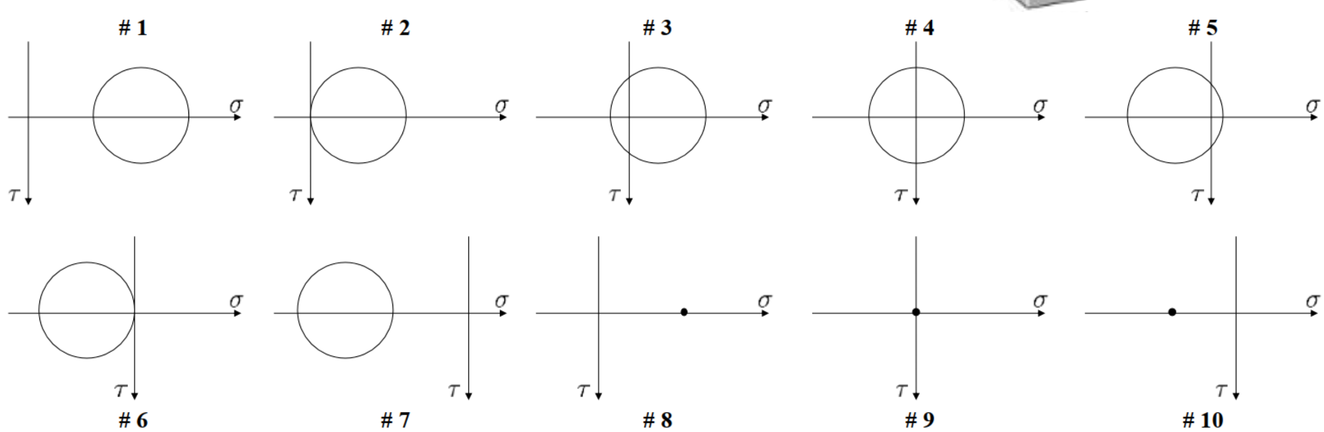
Statically INdeterminate :
stresses do depend on material properties.

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM # 4 *Part E* – 6 points

A cylindrical shaft is subjected to a torque *T*, and a beam of rectangular cross-section is fixed at one end and loaded with vertical load *P* on its other end.

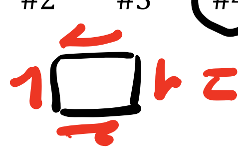


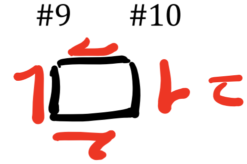


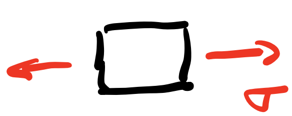
Referring to the ten Mohr's circles shown above, circle the number of the correct **in-plane** Mohr's

circle for the state of stress at

- Point A: #1 #2 #3 #4 #5 #6 #7 #8 #9 #10
- Point B: #1 #2 #3 #4 #5 #6 #7 #8 #9 #10
- Point C: #1 #2 #3 #4 #5 #6 #7 #8 #9 #10

Point A : 

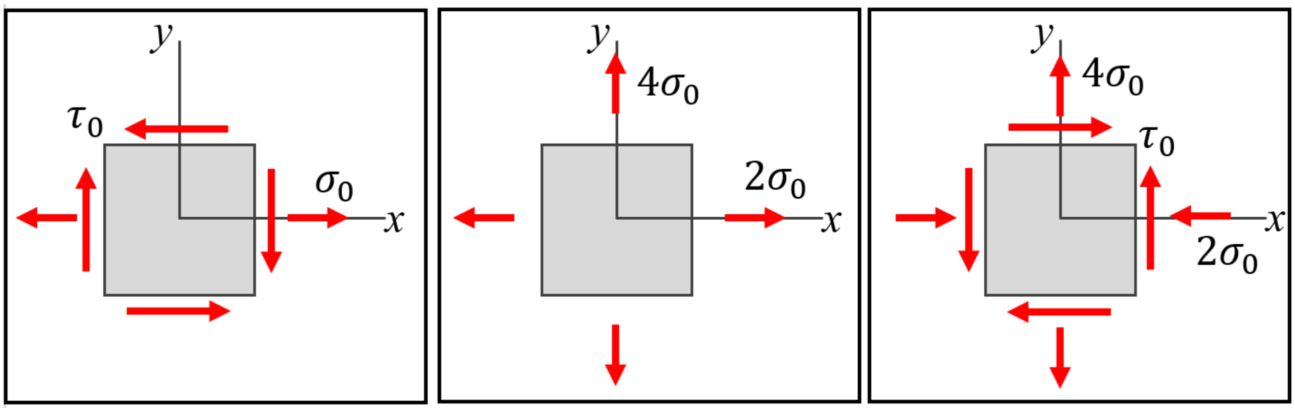
Point C : 

Point B : 

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM #4 - Part F

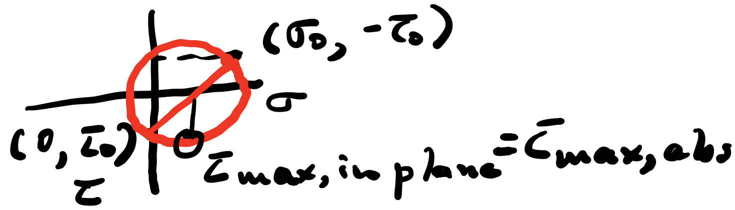
Stress state 1 **Stress state 2** **Stress state 3**



Consider the states of stress shown above. Let $|\tau|_{max,in-plane}$ and $|\tau|_{max,abs}$ represent the maximum magnitude in-plane and absolute shear stress, respectively, for a given stress state. Choose the correct responses below for the above three states of stress.

4.F.1 (2 points) – stress state 1

a) $|\tau|_{max,in-plane} > |\tau|_{max,abs}$
b) $|\tau|_{max,in-plane} = |\tau|_{max,abs}$
c) $|\tau|_{max,in-plane} < |\tau|_{max,abs}$
d) More information is needed on σ_0 and τ_0 in order to answer this.

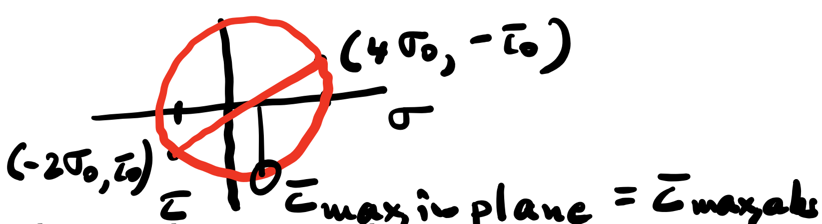


4.F.2 (2 points) – stress state 2

e) $|\tau|_{max,in-plane} > |\tau|_{max,abs}$
f) $|\tau|_{max,in-plane} = |\tau|_{max,abs}$
g) $|\tau|_{max,in-plane} < |\tau|_{max,abs}$
h) More information is needed on σ_0 and τ_0 in order to answer this.

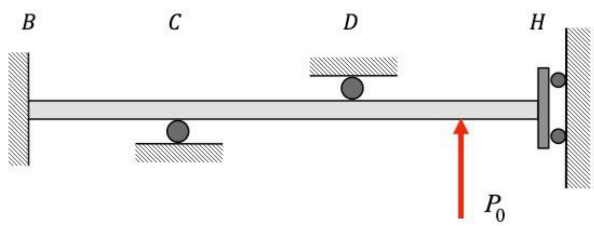
4.F.3 (2 points) – stress state 3

i) $|\tau|_{max,in-plane} > |\tau|_{max,abs}$
j) $|\tau|_{max,in-plane} = |\tau|_{max,abs}$
k) $|\tau|_{max,in-plane} < |\tau|_{max,abs}$
l) More information is needed on σ_0 and τ_0 in order to answer this.



ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

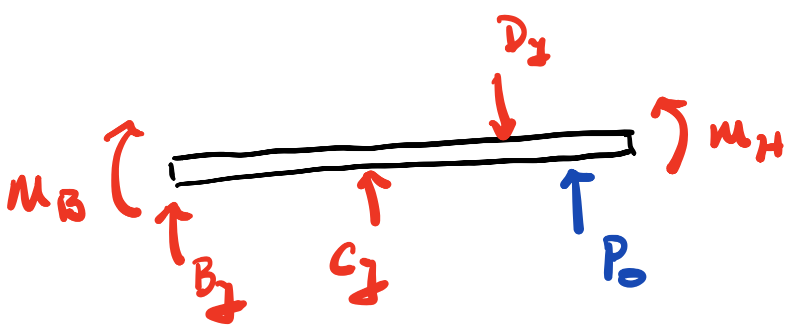
PROBLEM #4 - Part G – 2 points



Consider the beam shown above.

What is the number of redundant reaction loads on the beam? (circle one)

1 2 3 4



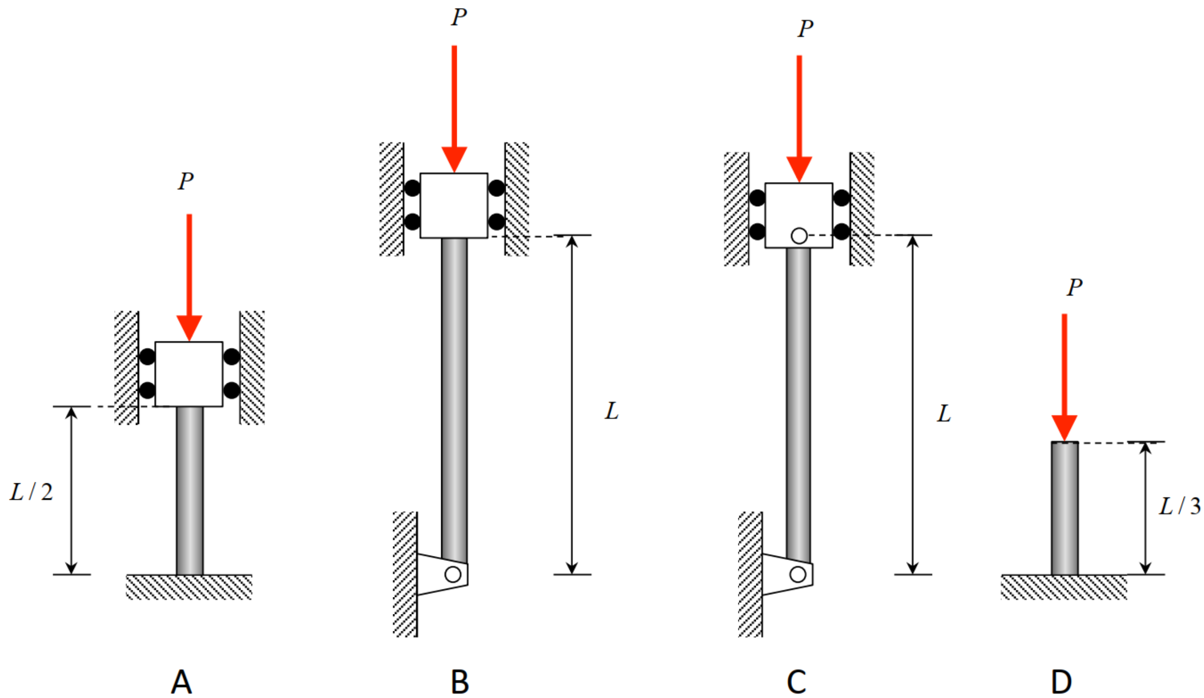
5 unknown reactions : M_B, B_y, C_y, D_y, M_H

2 equations

# of redundant loads = $5-2=3$

ME323 - Mechanics of Materials
Exam 3
May 2, 2024
Name: SOLUTION
(Last) (First)

PROBLEM NO. 4 - PART H – 3 points



Consider the four columns (A, B, C and D) shown above with differing boundary conditions and lengths. The loading is the same for each column, each column is made up of the same material having a Young's modulus E and each column has the same circular cross section of radius R.

a) Which column has the ***largest*** critical Euler buckling load? A B C D

b) Which column has the ***second largest*** critical Euler buckling load? A B C D

c) Which column has the ***smallest*** critical Euler buckling load? A B C D

$$P_{cr} = \pi^2 \frac{EI}{(L_e)^2}$$

A : $L_e = (0.5)\left(\frac{L}{2}\right) = 0.25L$ ⇒ Largest P_{cr}

B : $L_e = 0.7L$ ⇒ 3rd largest P_{cr}

C : $L_e = L$ ⇒ Smallest P_{cr}

D : $L_e = 2\left(\frac{L}{3}\right) = \frac{2L}{3}$ ⇒ 2nd largest P_{cr}