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PROBLEM# 1 (25 points).

A point A in a structure Figure 1A is subjected to the in-plane stress state shown in Figure 1B.

(a) Use the stress element in Figure 1B to draw the **Mohr's circle** on the graph paper.

(b) Use the **Mohr's circle** to calculate:

i. The principal stresses in the x-y plane.

ii. The maximum in-plane shear stress and its direction with respect to the x-axis.

iii. The absolute maximum shear stress.

iv. Draw a stress element to show the in-plane maximum shear stress correctly oriented with respect to the x-axis.

(c) Use the **Mohr's circle** to calculate the normal and shear stresses in the x-y directions. Note: The x-axis is oriented at 30° counter clockwise from the x'-axis as shown in Figure 1B.

(d) Use the **Mohr's circle** to calculate the **smallest angle** θ_3 from the **x-axis** where **the normal stress is σ_x''=22 ksi. Indicate the direction (CW or CCW) of θ_3 from the x-axis.**

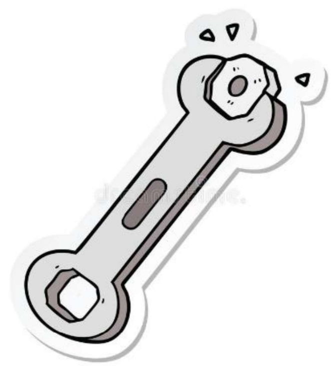

Fig. 1A

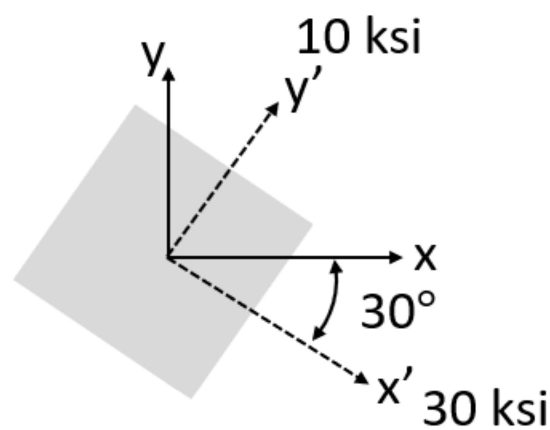

Fig. 1B

See Mohr's Circle

(i) $\sigma_{p1} = \sigma_{x'} = 30$ ksi (zero τ)

$\sigma_{p2} = \sigma_{y'} = 10$ ksi

(ii) $\tau_{max, in\ plane} = R = \dfrac{30-10}{2} = 10$ ksi

(iii) $\tau_{max, abs} = \dfrac{\sigma_{p1}}{2} = \dfrac{30}{2} = 15$ ksi

$\sigma_{avg} = \dfrac{30+10}{2} = 20$ ksi

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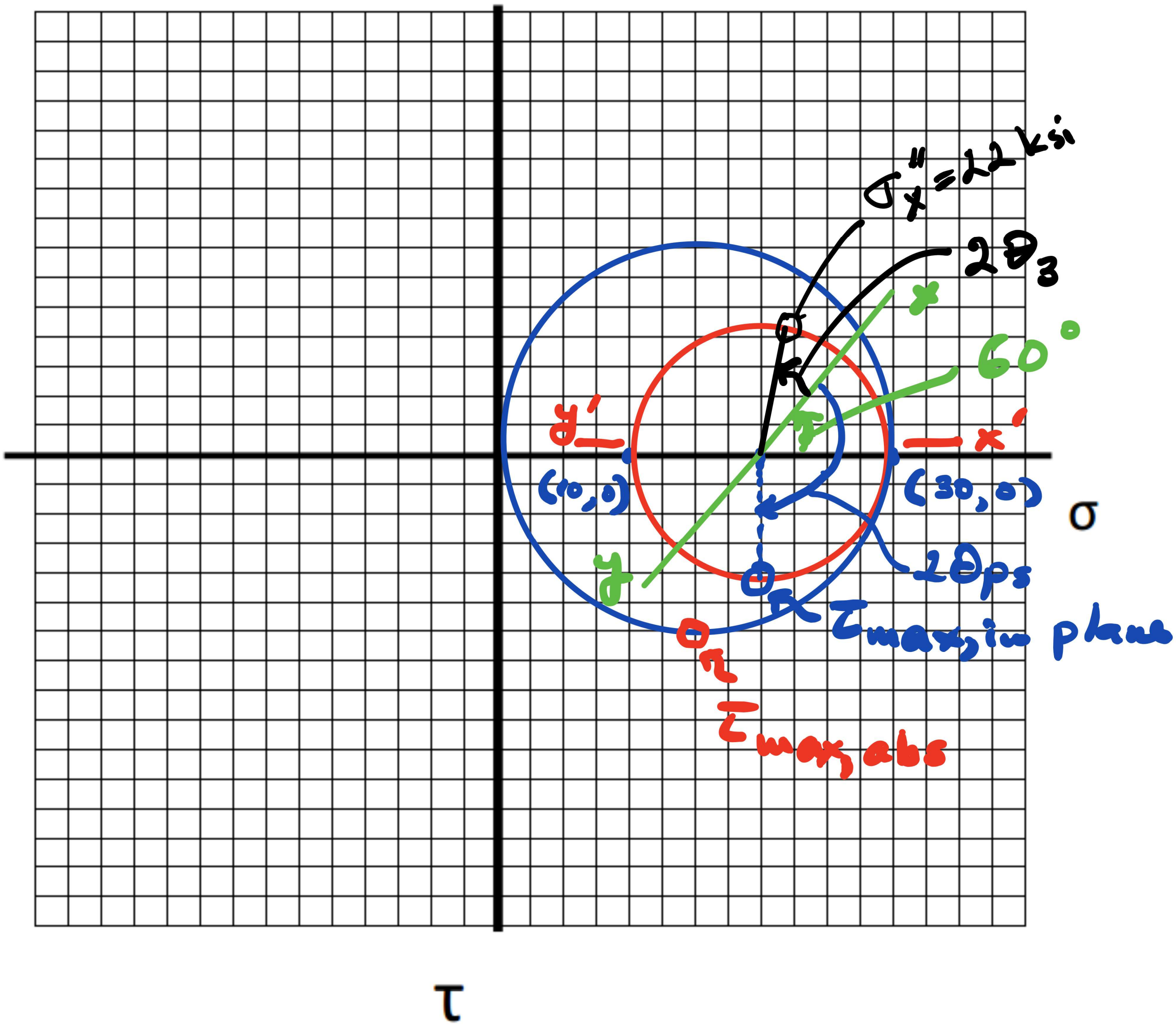
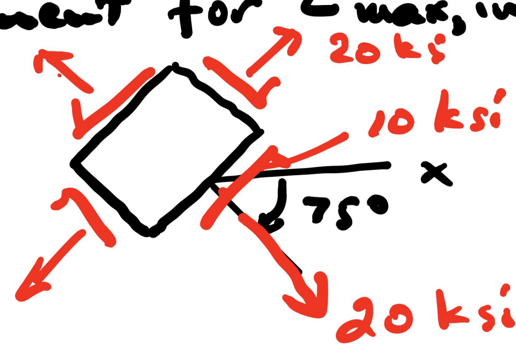


(iv) $2\theta_{ps} = 60° + 90° = 150°$

$\theta_{ps} = 75°$

stress element for $\tau_{max, in\ plane}$



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(c) $\sigma_x = \sigma_{avg} + R\cos 60^\circ$

$= 20 + 10\cos 60^\circ = 25\text{ ksi}$

$\sigma_y = \sigma_{avg} - R\cos 60^\circ = 15\text{ ksi}$

$\tau_{xy} = -R\sin 60^\circ = -8.66\text{ ksi}$

(d) $\sigma_{x''} = 22\text{ ksi} = \sigma_{avg} + R\cos(60^\circ + 2\theta_3)$

$\cos(60 + 2\theta_3) = \frac{22-20}{10} = 0.2$

$60 + 2\theta_3 = 78.46^\circ$

$2\theta_3 = 18.46^\circ$

$\theta_3 = 9.23^\circ$

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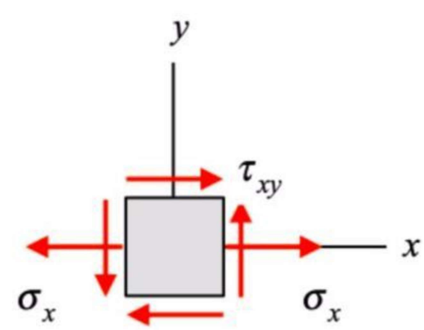
PROBLEM # 2 (25 points)

Consider a state of plane stress in a structural component represented by the stress element provided below where σ_x is yet unknown and $\tau_{xy} = 50$ ksi.

a) Suppose that the structural component is made of a *ductile* material with a yield strength $\sigma_Y = 200$ ksi.
   a. Using the *maximum shear stress theory*, determine the maximum value of σ_x for which the ductile material in the structural component does not fail.
   b. For the maximum value of σ_x found above, does the *maximum distortional energy theory* predict failure of the material?

b) Suppose now that the structural component is instead made of a *brittle* material with equal ultimate strengths in tension and compression of $\sigma_{UT} = \sigma_{UC} = 200$ ksi. Using *Mohr's failure criteria*, determine the maximum value of σ_x for which the brittle material in the structural component does not fail.



$$\sigma_{avg} = \frac{\sigma_x}{2} \qquad R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (50)^2} > \sigma_{avg}$$

$$\sigma_{p_1} = \frac{\sigma_x}{2} + R$$

$$\sigma_{p_2} = \frac{\sigma_x}{2} - R$$

σ_{p_1} & σ_{p_2} have opposite signs.

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(a) - a. MSS - Ductile material

$$\tau_{max,abs} = R = \frac{\sigma_y}{2} = \frac{200}{2} = 100 \text{ ksi}$$

$$R^2 = \left(\frac{\sigma_x}{2}\right)^2 + (50)^2 = 100^2$$

$$\left(\frac{\sigma_x}{2}\right)^2 = 100^2 - 50^2 = 7500$$

$$\sigma_x^2 = 4(7500) = 30{,}000$$

$$\boxed{\sigma_x = 173.2 \text{ ksi}}$$

(a) b. MSS is more conservative than MDE. So with the same stress, MDE does NOT predict failure

(b) Brittle material

Since σ_{p1} & σ_{p2} have opposite signs use <u>Mohr's criterion</u>:

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$$\frac{\sigma_{p1}}{\sigma_{UT}} - \frac{\sigma_{p2}}{\sigma_{UC}} = 1$$

$$\sigma_{p1} - \sigma_{p2} = 200 \text{ ksi}$$

$$\left[\cancel{\frac{\sigma_x}{2}} + R\right] - \left[\cancel{\frac{\sigma_x}{2}} - R\right] = 200$$

$$2R = 200$$

$$R = 100 \text{ ksi}$$

$$\left(\frac{\sigma_x}{2}\right)^2 + 50^2 = 100^2$$

$$\sigma_x^2 = 30{,}000$$

$$\boxed{\sigma_x = 173.2 \text{ ksi}}$$

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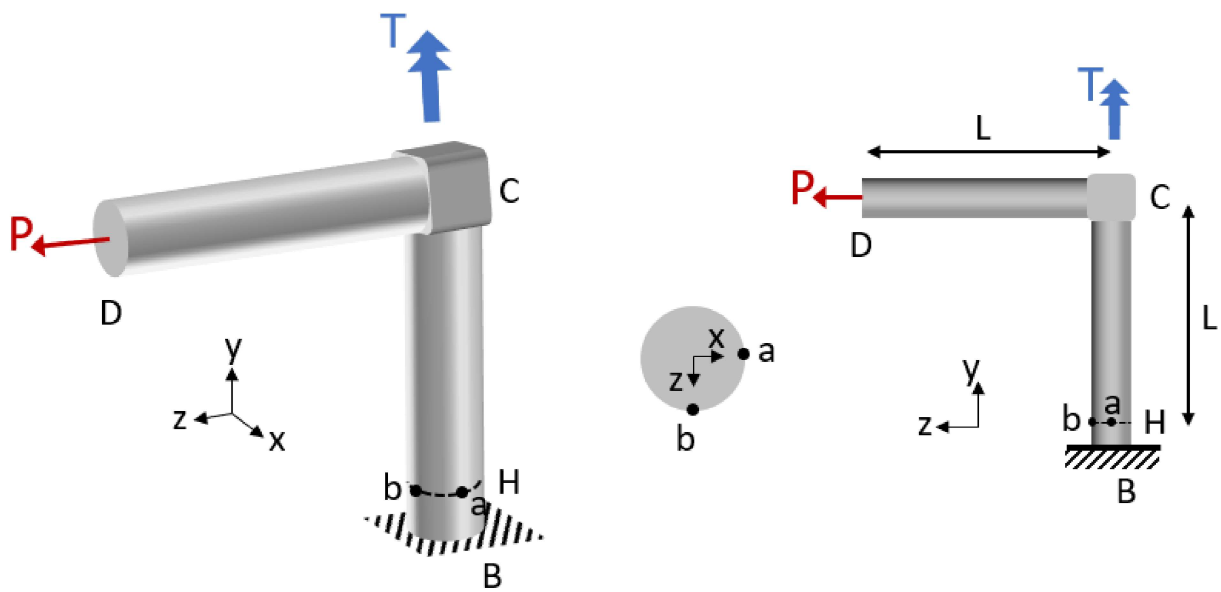
PROBLEM #3 (25 points)

BCD is an "L-shaped" structure made of a solid cylindrical rod with a diameter of d that is fixed to a wall at node B. A force of P is applied in the z-direction at node D and a torque of T is applied in the y-direction at node C. The structure is made of a material with a Young's modulus of E and a shear modulus of G.



(a) Determine the internal reactions at cross-section H. See page 9.
(b) Draw the relevant stress distributions on the cross-sections on the next page.
(c) Determine the stresses at point a and b (which are located on the cross section at H) and write them in Table 1.
(d) Draw the stress element at points a and b.

from page 9:

$F = (0, 0, P)$

$M = (PL, T, 0)$

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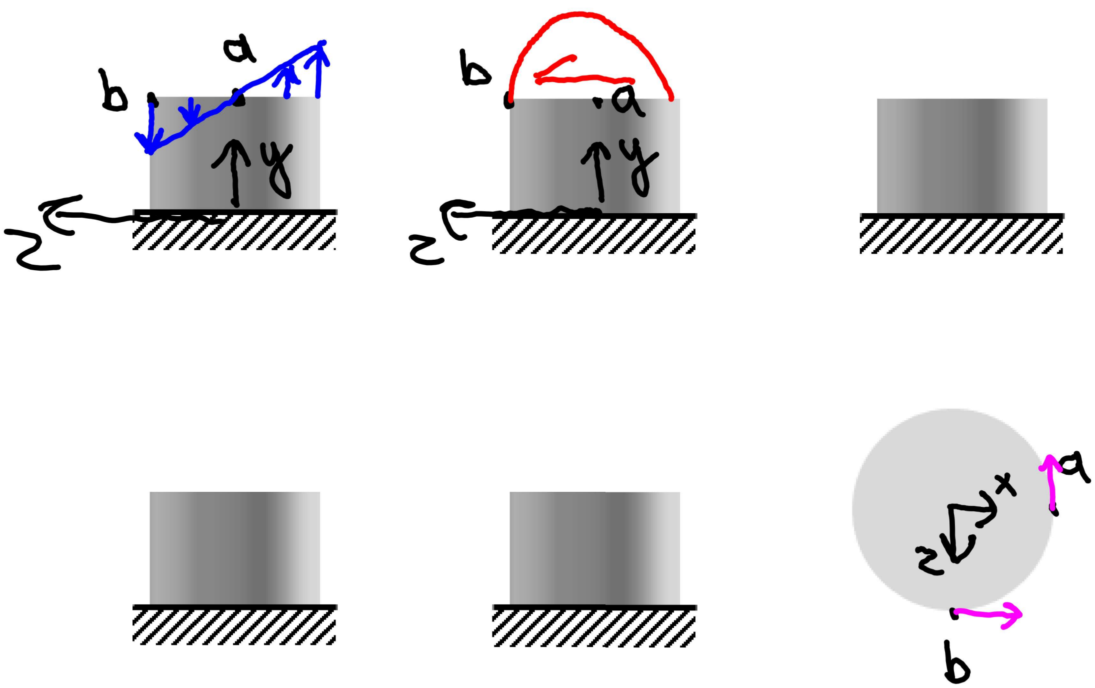

Table 1

Force Component	a	b
F_z	$\tau_{yz} = \frac{VQ}{It} = \frac{4V}{3A} = \frac{4P}{3(\pi(\frac{d}{2})^2)} = \frac{16P}{3\pi d^2}$	0
M_x	0	$\sigma_y = -\frac{M_z}{I} = \frac{-PL(\frac{d}{2})}{\frac{\pi}{4}(\frac{d}{2})^4} = \frac{-32PL}{\pi d^3}$
$M_y = T_y$	$\tau_{yz} = \frac{-T\rho}{I_p} = \frac{-T(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4} = \frac{-16T}{\pi d^3}$	$\tau_{yx} = \frac{T\rho}{I_p} = \frac{T(\frac{d}{2})}{\frac{\pi}{2}(\frac{d}{2})^4} = \frac{16T}{\pi d^3}$

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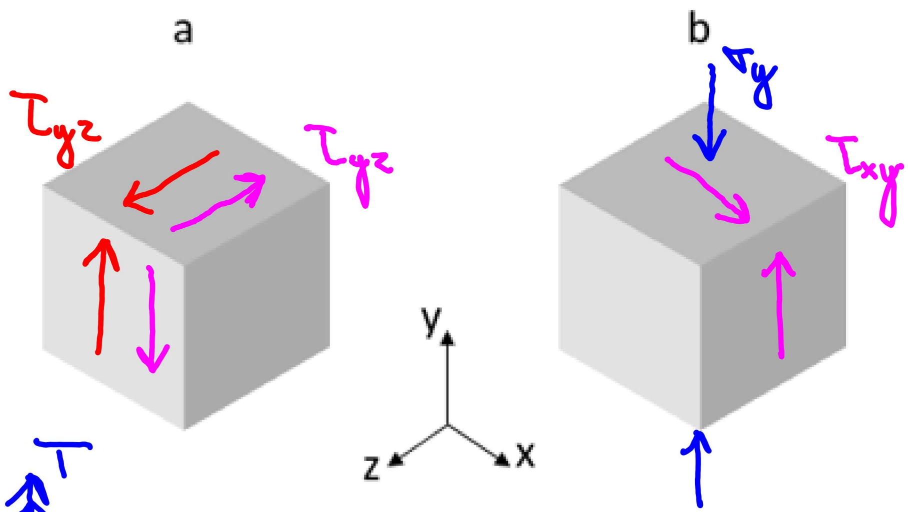

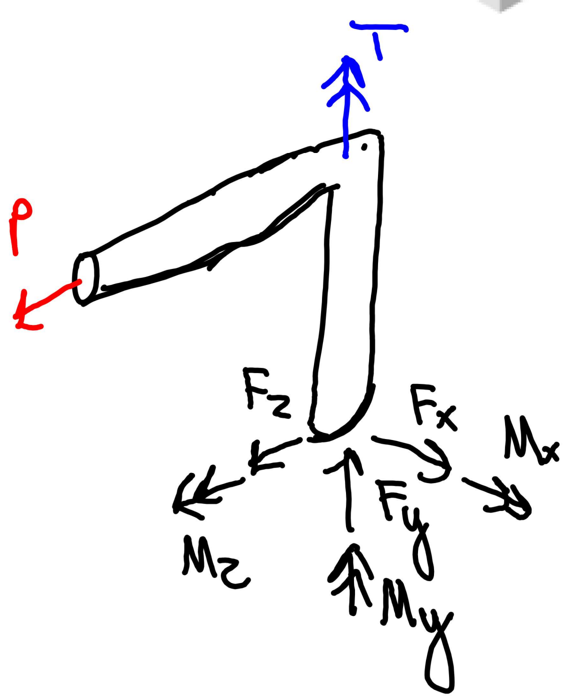

$$\Sigma F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} + P\hat{k} = 0$$

$$F_{negative} = (0, 0, -P)$$

$$\Sigma M = M_x\hat{i} + M_y\hat{j} + M_z\hat{k} + T\hat{j} + \vec{r}_1 \times \vec{F}_1 = 0$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & L & L \\ 0 & 0 & P \end{vmatrix} = (PL, 0, 0)$$

$$0 = M_x\hat{i} + M_y\hat{j} + M_z\hat{k} + PL\hat{i} + T\hat{j}$$

$$M_{negative} = (-PL, -T, 0)$$

$$F_{positive} = (0, 0P)$$

$$M_{positive} = (PL, T, 0)$$

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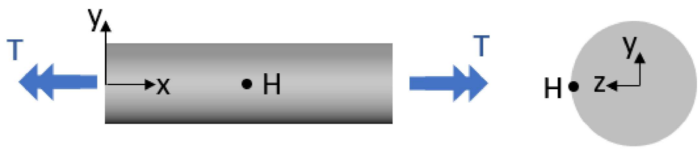
PROBLEM #4 – PART A (12 points)

Two torsion members have the same dimensions but are made of different materials. Member (a) is made of a brittle material with $\sigma_{UC} = \sigma_{UT} = 120\ MPa$ and G=40 GPa. Member (b) is made of a ductile material with $\sigma_Y = 120\ MPa$ and G=120 GPa. Point H is located at the most positive z-direction.



Torsion member (a)
$\sigma_{UC} = \sigma_{UT} = 120\ MPa$
G = 40 GPa

Torsion member (b)
$\sigma_Y = 120\ MPa$
G = 120 GPa

$\tau = \frac{TR}{I_p}$
all geometric

(a) Circle the correct relationship between the maximum shear stresses in the two sections. (**1 point**)

$|\tau_{max,a}| < |\tau_{max,b}|$

$\boxed{|\tau_{max,a}| = |\tau_{max,b}|}$

$\boxed{|\tau_{max,a}| > |\tau_{max,b}|}$

some people interpreted the question to be asking about max T at failure

(b) Circle the correct relationship between the angular changes in the two sections. (**1 point**)

$|\phi_a| < |\phi_b|$

$|\phi_a| = |\phi_b|$

$\boxed{|\phi_a| > |\phi_b|}$

$\phi = \frac{TR}{GI_p}$

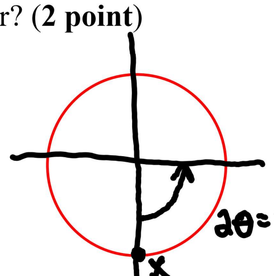
(c) In member (a), at the stress element at H, at what angle relative to the given x-axis is failure expected to occur? (**2 point**)

$0\ degrees$

$15\ degrees$ CCW

$\boxed{45\ degrees \text{ CCW}}$

$90\ degrees$ CCW



$2\theta = 90°$.

(d) In member (b), at the stress element at H, at what angle relative to the given x-axis is failure expected to occur? (**2 point**)

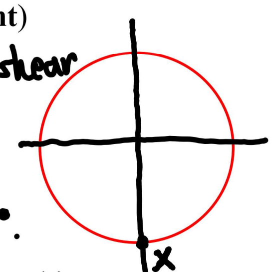
$\boxed{0\ degrees}$

$15\ degrees$ CCW

$45\ degrees$ CCW

$\boxed{90\ degrees \text{ CCW}}$

max shear when
$2\theta = 0$
$2\theta = 180°$.



(e) Circle the correct relationship between the torque (T) at which the two torsion members will fail. (**3 points**)

$\sigma_{p1} = R = \frac{TR}{I_p}$

$\sigma_{p2} = R$

failure when $\sigma_{p1} = \sigma_{UT}$

$120\ MPa = \frac{TR}{I_p}$

$T_{failure,a} = \frac{120 I_p}{R}$

$|T_{failure,a}| < |T_{failure,b}|$

$|T_{failure,a}| = |T_{failure,b}|$

$\boxed{|T_{failure,a}| > |T_{failure,b}|}$

$\tau_{max,abs} = R = \frac{\sigma_Y}{2}$

$R = \frac{TR}{I_p} = \frac{120}{2}$

$T_{failure,b} = \frac{60 I_p}{R}$

(f) In member (b), what is the relationship between the failure torque predicted by the maximum shear stress theory (MSS) and the maximum distortional energy theory (MDE)? (**3 points**)

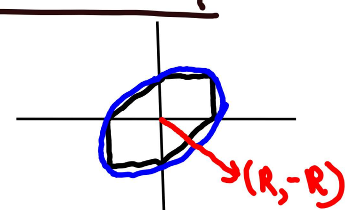
$|T_{failure,MSS}| > |T_{failure,MDE}|$

$|T_{failure,MSS}| = |T_{failure,MDE}|$

$\boxed{|T_{failure,MSS}| < |T_{failure,MDE}|}$

$\sigma_{p1} = R$

$\sigma_{p2} = -R$



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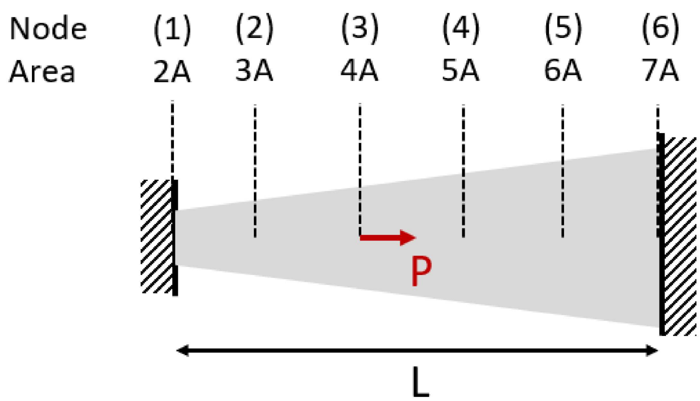
PROBLEM 4 – PART B (5 points)

An axial member with a Young's modulus of E is analyzed using the finite element method with five elements of equal length.



(a) The entry at the first row and first column of the stiffness matrix has which value? (circle one) (**1 points**)

2EA/L	5EA/2L	5EA/L
15EA/2L	$\boxed{25EA/2L}$	25EA/L

$$\frac{(EA)_{avg}}{L} = \frac{\left(\frac{2EA+3EA}{2}\right)}{L/5} = \frac{5EA}{2}\left(\frac{5}{L}\right)$$

(b) What is the size of the reduced stiffness matrix? (circle one) (**2 points**)

6x1	$\boxed{4x4}$	4x6
5x5	6x6	25x25

6 nodes
⇒ 6x6 stiffness matrix

2 BC's
⇒ 4x4 reduced stiffness matrix.

(c) If the number of nodes was doubled, the error compared to an analytical solution (error = analytical result – FEM result) would be: (circle one) (**2 points**)

$\boxed{\text{error}_{6\text{nodes}} > \text{error}_{12\text{nodes}}}$

$\text{error}_{6\text{nodes}} = \text{error}_{12\text{nodes}}$

$\text{error}_{6\text{nodes}} < \text{error}_{12\text{nodes}}$

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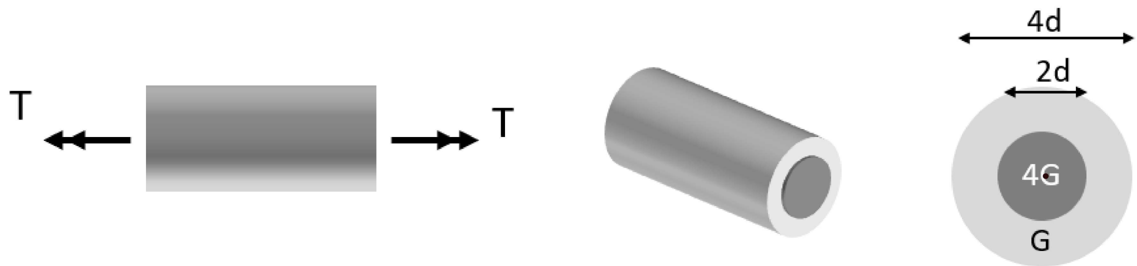
PROBLEM 4 – PART C (2 points)

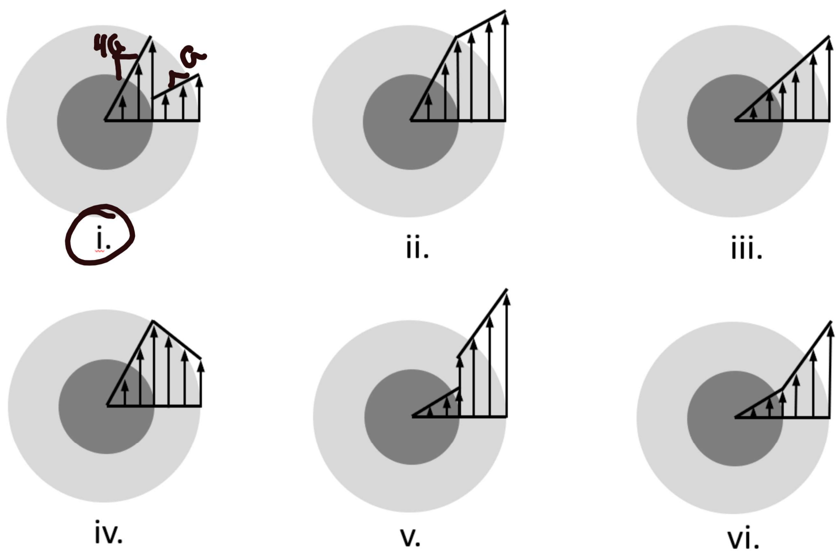
A bimetallic bar consists of an inside metal that has a shear modulus of 4G and an outer metal that has a shear modulus of G. The bar is subjected to a torque of T.



The distribution of stresses on the cross-section of the bar is (circle one):

$T = G\rho \frac{d\phi}{dx}$

$\Rightarrow$ stress depends linearly on G



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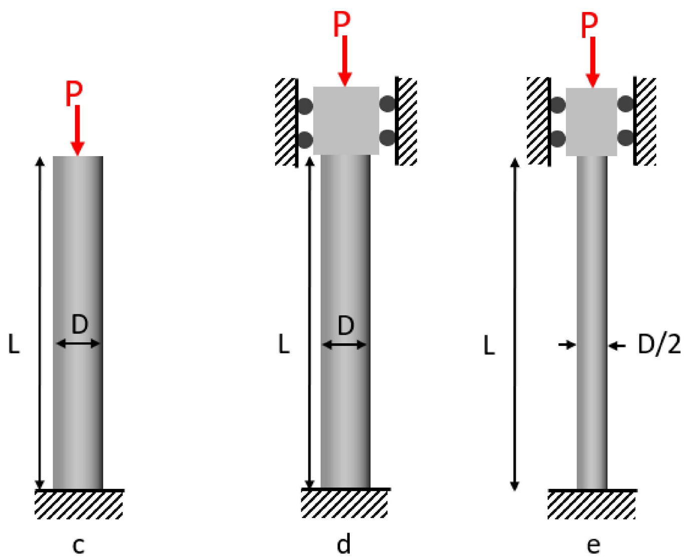
PROBLEM 4 – PART D (4 points)

Cylindrical columns c, d, and e are made of the same material with a Young's modulus of E. All columns are known to experience Euler buckling. A compressive axial load is applied to each column.



The relative values of the critical buckling loads are:

(a) Choose one (**2 points**).

$P_{cr,c} > P_{cr,e}$

$\boxed{P_{cr,c} = P_{cr,e}}$

$P_{cr,c} < P_{cr,e}$

(b) Choose one (**2 points**).

$\boxed{P_{cr,d} > P_{cr,e}}$

$P_{cr,d} = P_{cr,e}$

$P_{cr,d} < P_{cr,e}$

c

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

$$P_{cr} = \frac{\pi^2 E\left(\frac{\pi}{4}\left(\frac{D}{2}\right)^4\right)}{(2L)^2}$$

$$P_{cr} = \frac{\pi^3 ED^4}{256L^2}$$

d

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

$$P_{cr} = \frac{\pi^2 E\left(\frac{\pi}{4}\left(\frac{D}{2}\right)^4\right)}{(0.5L)^2}$$

$$P_{cr} = \frac{\pi^3 ED^4}{16L^2}$$

e

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2}$$

$$P_{cr} = \frac{\pi^2 E\left(\frac{\pi}{4}\left(\frac{D}{4}\right)^4\right)}{(0.5L)^2}$$

$$P_{cr} = \frac{\pi^3 ED^4}{256L^2}$$

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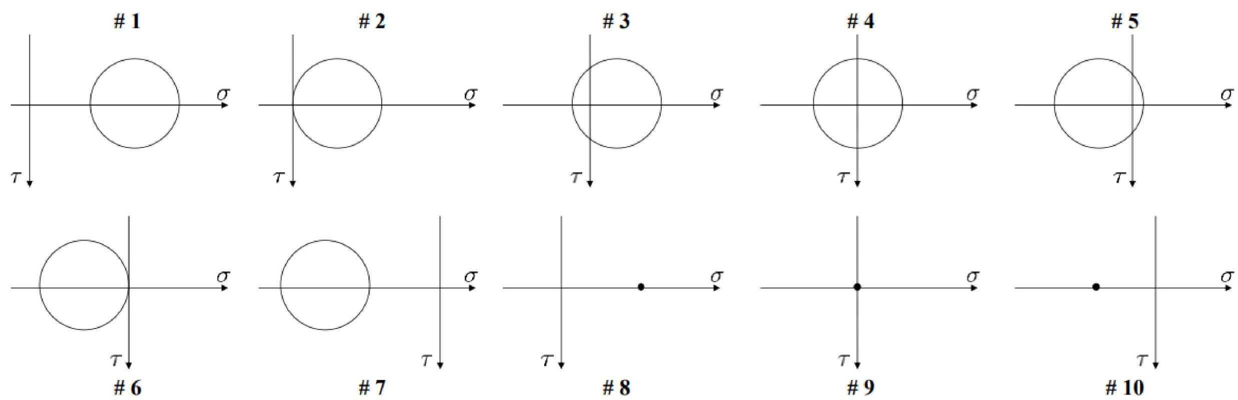
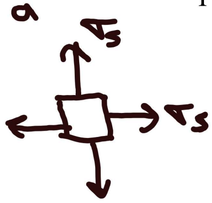
PROBLEM 4 – PART E (2 points)

The following Mohr's circles are used for Part E:



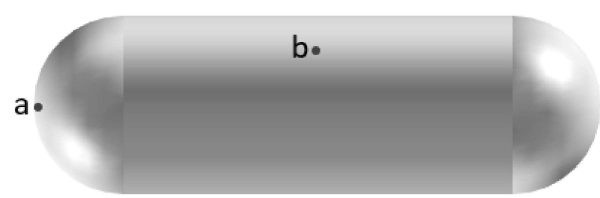
A pressure vessel with hemispherical caps is subjected to a positive pressure of P.

a

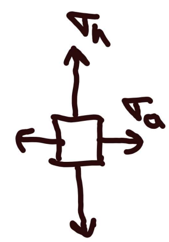


$\sigma_{p1} = \sigma_s$

$\sigma_{p2} = \sigma_s$



b)



$\sigma_{p1} = \sigma_h$

$\sigma_{p2} = \sigma_a$

$\sigma_{p2} = \frac{\sigma_{p1}}{2}$

(a) Choose the Mohr's circle that corresponds to the loading at point a. (**1 Points**)

 # 1 # 2 # 3 # 4 # 5 # 6 # 7 # 8 # 9 # 1 0

(b) Choose the Mohr's circle that corresponds to the loading at point b. (**1 Points**)

 # 1 # 2 # 3 # 4 # 5 # 6 # 7 # 8 # 9 # 1 0