ME 323: Mechanics of Materials **Homework 10**

Spring 2024 **Due: April 12, 2024**

Problem 1 (10 points):

A cylindrical vessel (shown in *Figure 1*) has an internal radius of r = 2.5 m, and a wall thickness of t = 15 mm. The internal pressure in the vessel is P = 1.5 MPa and the maximum allowable stress in the walls of the vessel is 400 MPa. Determine:

a) Axial stress σ_a and hoop stress σ_h in the cylindrical part of the vessel.
b) Principal stresses σ_{p1} and σ_{p2}
c) Maximum in-plane shear stress τ_{max}
d) Maximum allowable pressure P_{allow} such that σ_{p1} doesn't exceed σ_{allow}.

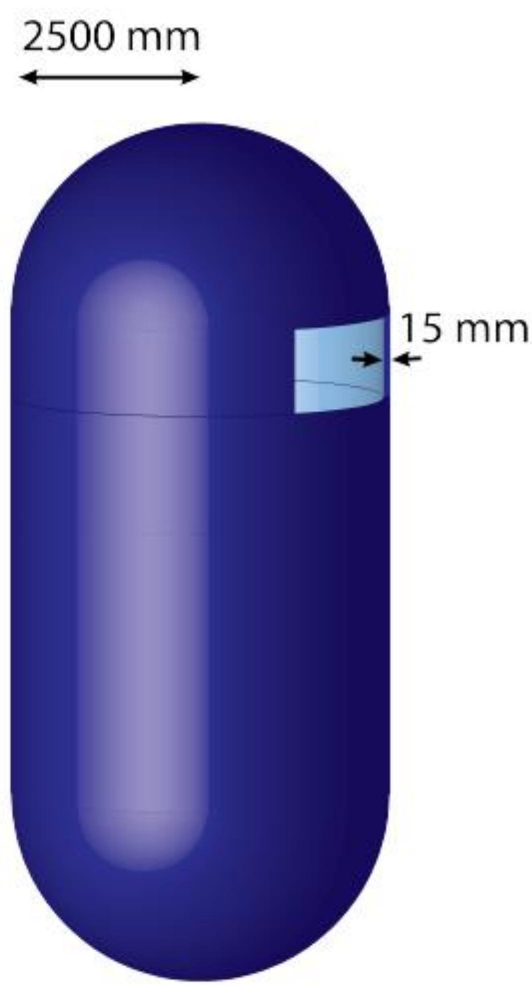


Figure 1: Cylindrical vessel for Problem 1.

Problem 1

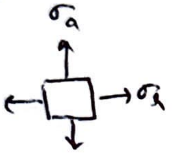
$$\frac{r}{t} = \frac{2500}{15} = 166.6 \Rightarrow \text{Thin-walled vessel}$$
$$\left(\text{or take } \frac{t}{r} << 1\right)$$

a) Axial stress →

$$\sigma_a = \frac{Pr}{2t} = \frac{1.5 \times \cancel{2500}^{1250}}{\cancel{2} \times 15} = 125\text{MPa}$$

$$\sigma_h = \frac{Pr}{t} = 250\text{MPa}$$

b) Principal stresses



$$\sigma_{p_1} = \sigma_h = 250\text{MPa}$$
$$\sigma_{p_2} = \sigma_a = 125\text{MPa}$$

c) Max in-plane shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_h - \sigma_a}{2}\right)^2} = 62.5\text{MPa}$$

d)

$$\sigma_{allow} = \frac{P_{allow} \times r}{t}$$

$$4\cancel{00} = \frac{P_{allow} \times 2\cancel{5}^{5}\cancel{00}}{\cancel{15}_{3}}$$

$$2.4\text{MPa} = P_{allow}$$

Problem 2 (10 points):

The stress element shown below represents the state of stress measured along the x'y' axis in a component loaded under plane stress. No information is known about the stress α except that it is compressive.

(a) Determine the magnitude of the maximum compressive normal stress that can be applied, if the component is made of a material which can withstand a maximum in-plane shear stress of 100 MPa.

(b) Determine the stress components when the element is oriented along the x-y axes.

(c) Draw a stress element oriented along the maximum in-plane shear stress directions. (Show the angle of this rotated element with respect to the axis x')

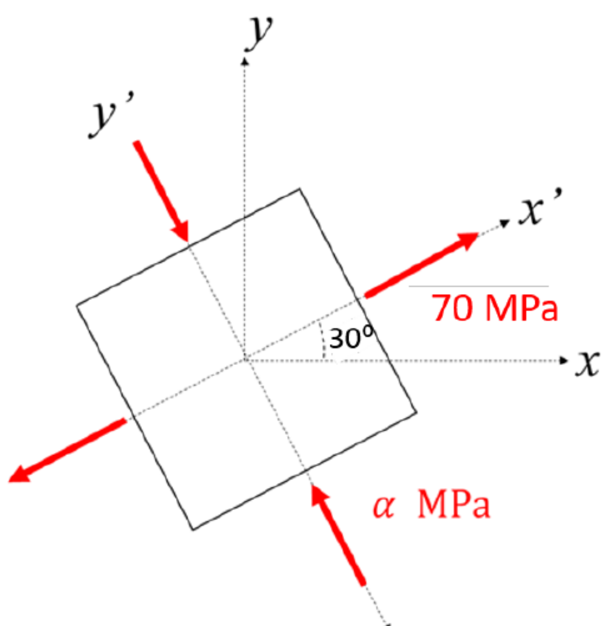


Figure 2: Stress element for Problem 2

(a) Let the stress along the y'direction be $\sigma_{y'}$. In the $x'y'$ orientation as seen in the problem figure, the element is in oriented along the principal stress direction (as shear stress $\tau_{x'y'}$ is 0). Hence $\sigma_{x'}(= 70\ MPa)$ and $\sigma_{y'}$ are principal stresses.

Given max shear stress that the material can withstand is 100 MPa, we have

$$|\tau_{max}| = 100 \text{ MPa}$$

$$=> \frac{|\sigma_{y'} - \sigma_{x'}|}{2} = 100 \text{ MPa}$$

$$=> \sigma_{y'} - 70 = \pm 200$$

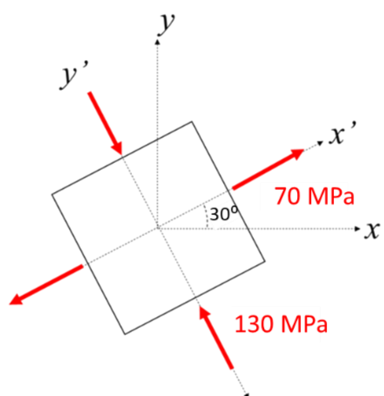
$$=> \sigma_{y'} = +270 \text{ MPa } or - 130 \text{ MPa}$$

Since $\sigma_{y'}$ is a compressive state of stress

$$\sigma_{y'} = -130 \text{ MPa}$$

Hence magnitude of $\sigma_{y'}$, i.e.,

$$\alpha = 130 \text{ MPa}$$



(b) The stress components along the xy axes can be obtained by rotating $\theta = -30°$

$$\sigma_x = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\theta) + \tau_{x'y'}\sin(2\theta)$$

$$\sigma_x = 20 \text{ MPa}$$

$$\sigma_y = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\theta) - \tau_{x'y'}\sin(2\theta)$$

$$\sigma_y = -80 \text{ MPa}$$

$$\tau_{xy} = -\left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\sin(2\theta) + \tau_{x'y'}\cos(2\theta)$$

$$\tau_{xy} = 86.603 \text{ MPa}$$

(c) Max shear stress is oriented at an angle of $\pm 45°$ to the principal axis. In this problem, the principal axis is oriented along $x'y'$ axis. For the solution we will consider a rotation of $\beta = -45°$ to get to max shear orientation

$$\tau_{x''y''} = \tau_{max} = -\left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\sin(2\beta)) + \tau_{x'y'}\cos(2\beta)$$

$$\tau_{max} = 100 \text{ MPa}$$

The correspnding normal stress along x'' is

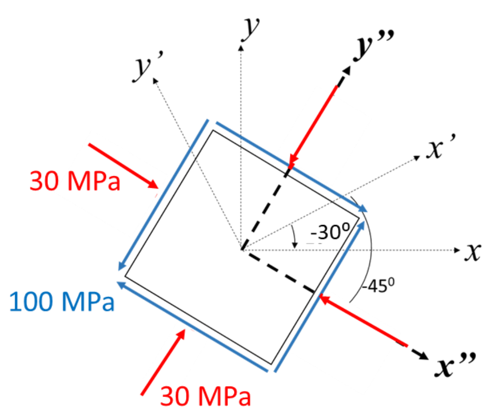
$$\sigma_{x''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} + \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\beta) + \tau_{x'y'}\cos(2\beta)$$

$$\sigma_{x''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = -30 \text{ MPa} = \sigma_{ave}$$

And along y'' axis is

$$\sigma_{y''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} - \left(\frac{\sigma_{x'} - \sigma_{y'}}{2}\right)\cos(2\beta) - \tau_{x'y'}\cos(2\beta)$$

$$\sigma_{y''} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = -30 \text{ MPa} = \sigma_{ave}$$



Problem 3 (10 points):

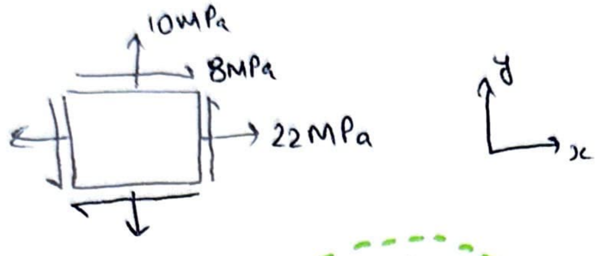
For the following state of plane stress:
σ_x=22 MPa; σ_y=10 MPa; τ_{xy}=8 MPa; with other stresses zero

(a) Sketch the stresses on a stress element.
(b) Draw the Mohr's circle for this loading condition. Determine value of σ_{p1}, σ_{p2}, and in-plane τ_{max}.
(c) Find the value of the maximum absolute shear stress $\tau_{max,abs}$.
(d) Determine the value of the stresses if the element is rotated counterclockwise by 45°.

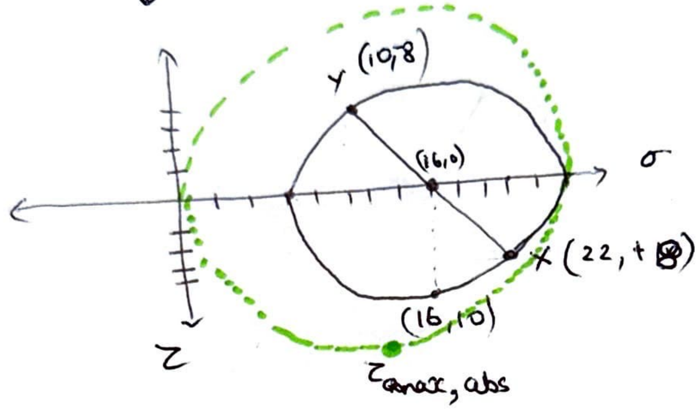
Problem 3

$\sigma_x = 22\,MPa, \quad \sigma_y = 10\,MPa, \quad \tau_{xy} = 8\,MPa$

a.)



b.)



$R = 10\,MPa$

$\sigma_{p_1} = 26\,MPa$

$\sigma_{p_2} = 6\,MPa$

$\tau_{max} = 10\,MPa$

c.) $\tau_{max,abs} = \dfrac{\sigma_{p1}}{2} = 13\,MPa$

d.) Element rotated counterclockwise by 45°

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= 16 + 0 + 8$$

$$= 24\,MPa$$

$$\tau_{x'y'}' = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -6\,MPa$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= 16 - 0 - 8$$

$$= 8\,MPa$$

Problem 4 (2.5 + 2.5 points):

I. A moment M (about positive z) and torque T (about positive x) are applied to a circular rod as shown in *Figure 4.1*. Choose the correct in plane Mohr's circle, from the given options, for the stress states at Point a and Point b. Justify. (Note that **location of Point a is at (L/2, 0, -R)** where R is the radius of the cross section.)

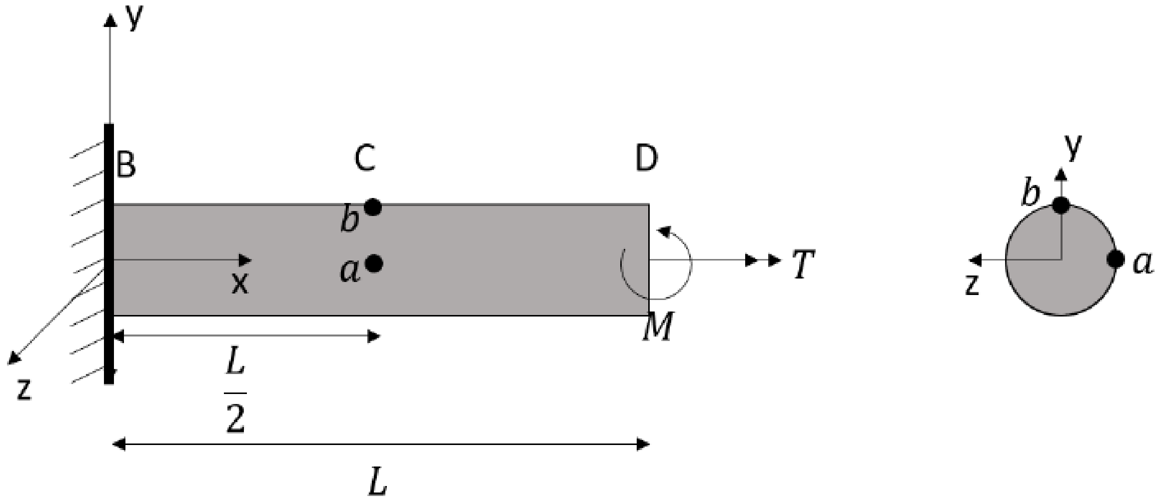
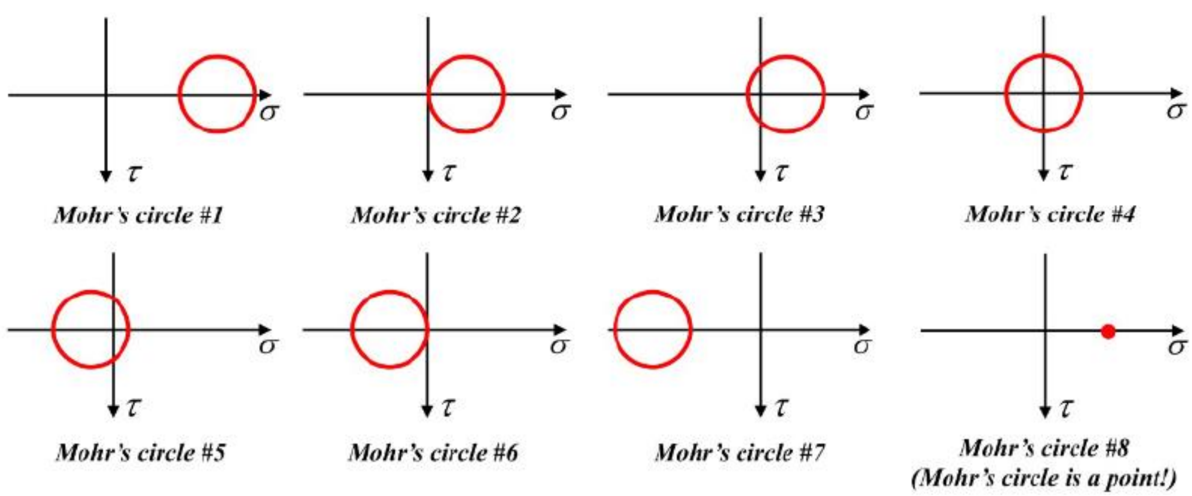


Figure 4.1: Loading of circular rod for Problem 4.I



Point a - The correct Mohr's circle is #4

$$\tau_{xy} > 0$$

$$\sigma_x = 0, \qquad \sigma_y = 0$$

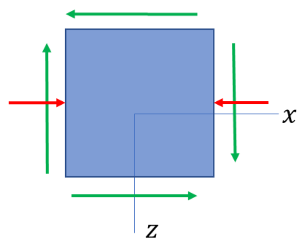
$$\sigma_{avg} = \frac{(\sigma_x + \sigma_y)}{2} = 0$$

Point b – The correct Mohr's circle is #5

$$\tau_{xz} > 0$$

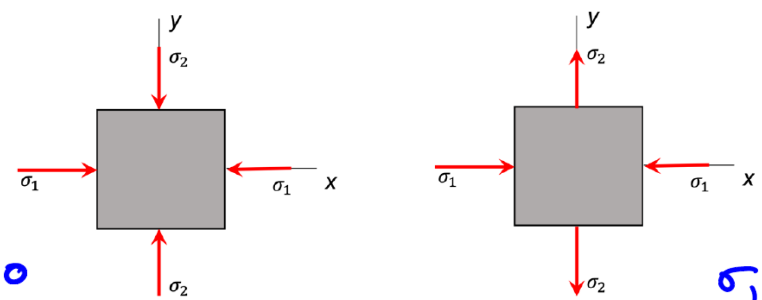
$$\sigma_x < 0, \qquad \sigma_z = 0$$

$$\sigma_{avg} = \frac{(\sigma_x + \sigma_y)}{2} < 0$$



II.

Consider stress states (a) and (b) shown above, with $|\sigma_1| > |\sigma_2|$. Let $(|\tau|_{max,abs})_a$ and $(|\tau|_{max,abs})_b$ represent the absolute maximum shear stress corresponding to stress states (a) and (b), respectively. Choose the response below that describes the relative sizes of these stresses.



$\sigma_2 < \sigma_1 < 0$

$\sigma_1 < 0 < \sigma_2$

Stress state (a)

Stress state (b)

i. $(|\tau|_{max,abs})_a > (|\tau|_{max,abs})_b$

ii. $(|\tau|_{max,abs})_a = (|\tau|_{max,abs})_b$

iii. $(|\tau|_{max,abs})_a < (|\tau|_{max,abs})_b$

Once, all 3 mohr's circles are drawn, (b) produces a larger $\tau_{max,abs}$