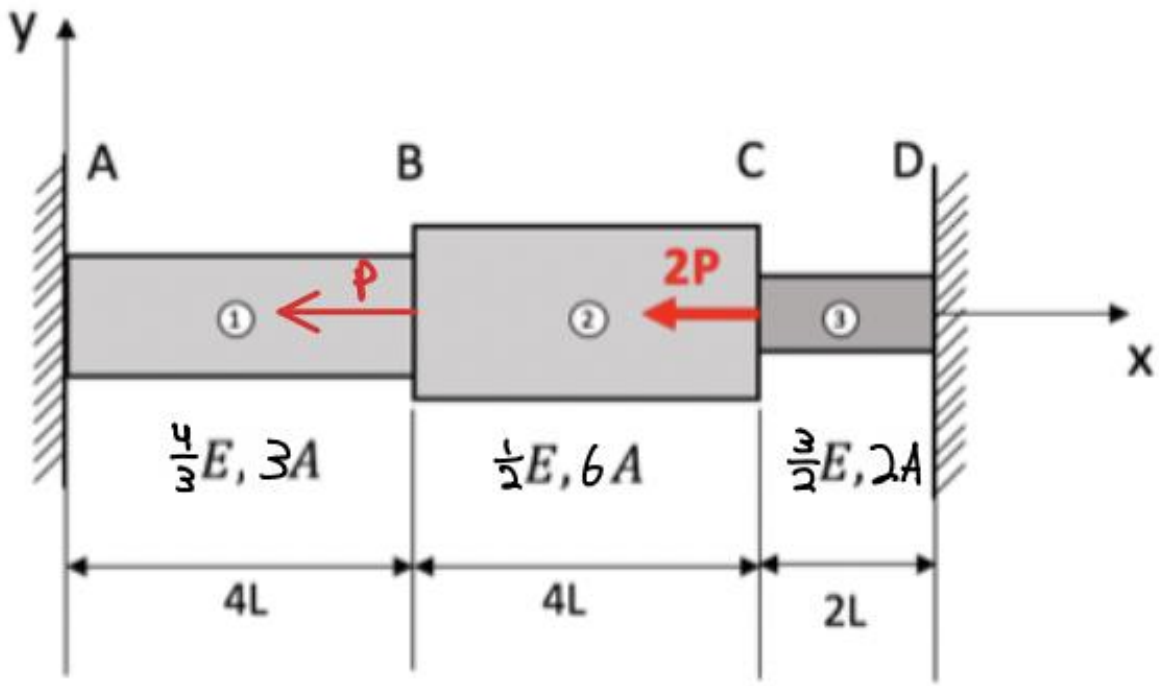
Problem 9.1 (10 points)

A three-segment rod AD is fixed to walls at A and D. An external load 2P is applied at C and an external load P applied at B. The properties are shown in the figure. Segments 1, 2, and 3 have Young's modulus 4E/3, E/2, and 3E/2, respectively. Segments 1, 2, and 3 have cross-sectional area 3A, 6A, and 2A, respectively.

a) Use three finite elements (one element per segment), write down the stiffness matrix [K] and the forcing vector [F].

b) Enforcing the boundary conditions, write the reduced system of equations and solve for the displacements at B and C.

c) Determine the reactions at A and D.



Problem 9.2 (10 points)

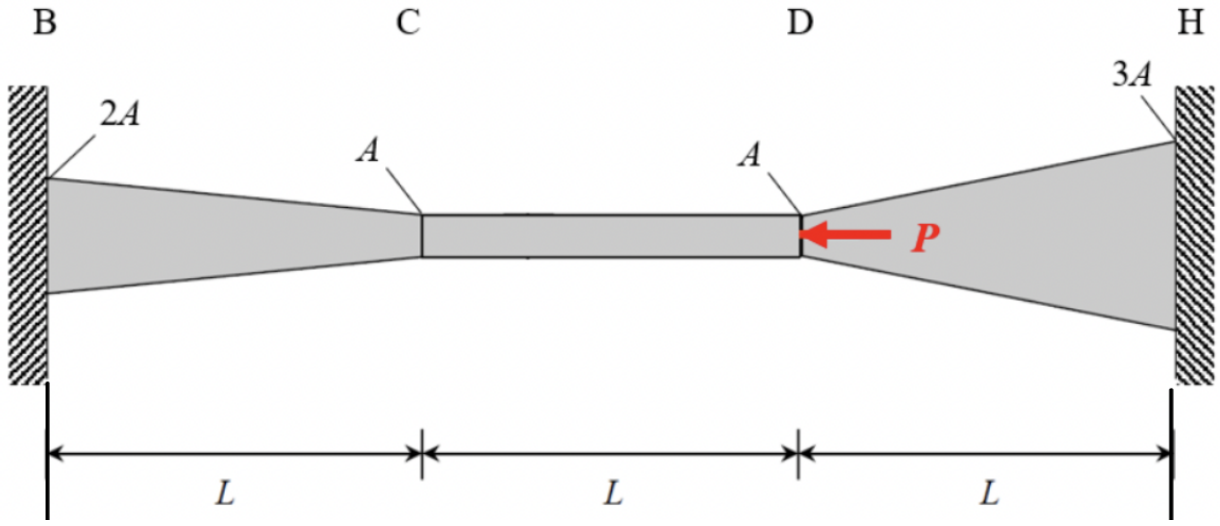
A rod is made of three segments: BC, CD, and DH. All segments have length L and are made of a material with Young's modulus E. The cross-sectional area of segment BC decreases linearly from 2A at B to A at C. The cross-sectional area of segment C is constant. The cross-sectional area of segment DH increases linearly from A at D to 3A at H. A force P acts to the left at point D.

Use a three-element (four-node) finite element model to do the following:

a) Construct the global stiffness matrix [K] in terms of E, A, and L.
b) Construct the force vector [F] in terms of P.
c) Enforce the displacement boundary conditions.
d) Solve for the nodal displacements in terms of PL/EA.
e) Determine the reactions at walls B and H in terms of P.
f) Compare the results from different numbers of elements (use 5 and 7).

Use the code provided in the lecture book or write your own.

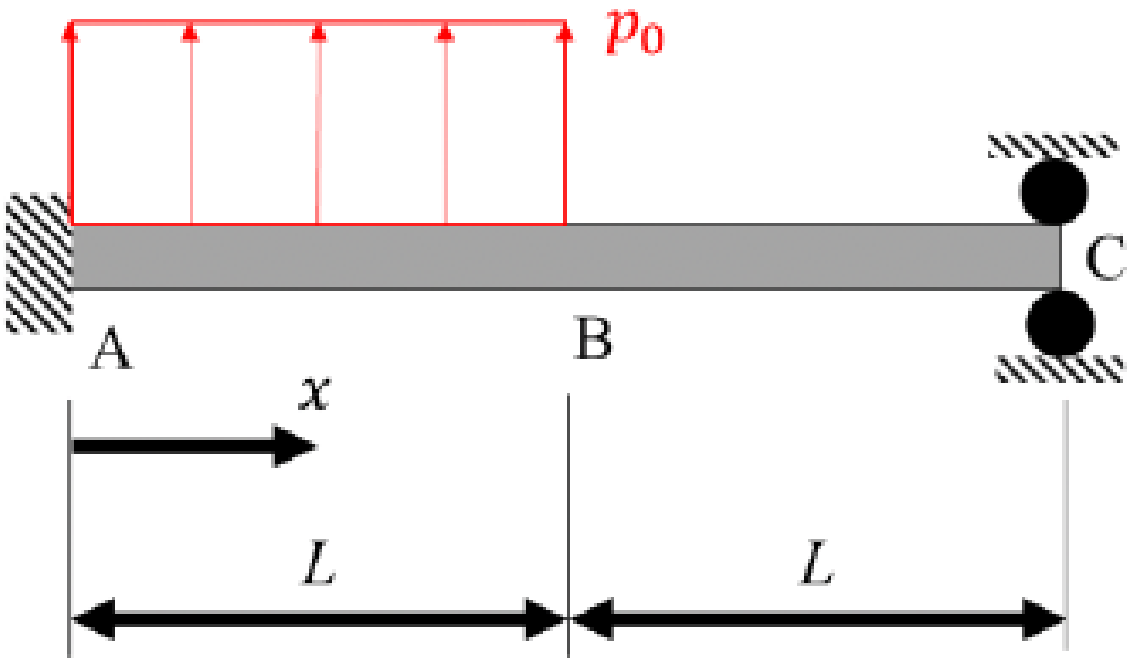
Note: Attach your modified code to your submission.



Problem 9.3 (10 points)

A beam ABC of length 2L is loaded with a distributed load over half its length as shown in the figure below. The beam is fixed at A and supported by a roller at C. The beam has a modulus of elasticity E and second area moment of the cross-section I. Determine the following: Give your answers in terms of p_0, E, I, x, L.

a) The reaction forces at A, B, and C. (Hint: make use of the principle of superposition to avoid integration).
b) The bending moment M(x).
c) The shear force V(x).
d) Plot both the shear force and bending moment diagrams.



Problem 9.4 (2.5 + 2.5 points)

1) A shaft has a stiffness matrix [K] = $\begin{bmatrix} K1 & -K1 & 0 \\ -K1 & K1+K2 & -K2 \\ 0 & -K2 & K2 \end{bmatrix}$. The right end of the shaft is free. Simplify the matrix.
   a. $\begin{bmatrix} K1 & -K1 \\ -K1 & K1+K2 \end{bmatrix}$
   b. $\begin{bmatrix} K1+K2 & -K2 \\ -K2 & K2 \end{bmatrix}$
   c. $\begin{bmatrix} K1 & -K1 \\ -K1 & K1+K2 \\ 0 & -K2 \end{bmatrix}$

2) What is the order of the reduced stiffness matrix for a shaft composed of 4 elements and with both ends fixed.
   a. 4
   b. 3
   c. 5
   d. 6