Problem 7.1 ( 10 points) The beam shown below has a Young's modulus E and second area moment of inertia I. Determine the following using the second-order integration method: Give your answers in terms of E, I, L, $P_{0}$ and $M_{0}$
(a) Bending moment $\mathrm{M}(\mathrm{x})$ along the length of the beam.
(b) The slope of the beam $\theta(x)$.
(c) The deflection of the beam $\mathrm{v}(\mathrm{x})$.


Problem 7.2 ( 10 points) A cantilever beam ABC with a circular cross section has a roller support at B and is subjected to a uniformly distributed loads $w_{0}$ between B and C , a concentrated load P at C , and an external couple $M_{0}$ at B . Using the superposition method, determine:
(a) The reaction at point B .
(b) The analytical expression for the deflection of the beam $v_{x}$.
(c) The deflection at point C .


Data: $L_{1}=1 \mathrm{~m}, L_{2}=2 \mathrm{~m}$, cross section diameter $\mathrm{d}=40 \mathrm{~mm}, \mathrm{E}=200 \mathrm{GPa}, \mathrm{P}=150 \mathrm{~N}, M_{0}=100 \mathrm{~N} . \mathrm{m}$, and $w_{0}=80 \mathrm{~N} / \mathrm{m}$. Note: Please refer to the superposition tables on the blog for necessary beam deflection equations.

## Problem 7.3 (10 points)

A The beam shown below has a Young's modulus of 50 GPa and has a square cross-section of side length 1 cm . Determine the following using the second-order integration approach:
(a) Bending moment $\mathrm{M}(\mathrm{x})$ along the length of the beam.
(b) Slope of the beam $\vartheta(x)$.
(c) The deflection of the beam $\mathrm{v}(\mathrm{x})$.
(d) Slope of the beam at support B.


## Problem 7.4 (5 points)

1. Consider the cantilevered beam with distributed loads as shown below.


Select the general form that the equation for flexural energy due to bending will take (note that $f_{i}(x)$ represents some continuous function $\mathrm{M}(\mathrm{x}) /(2 \mathrm{EI})$ for the length of integration):
(a) $U=\int_{0}^{L} f_{1}(x) d x+\int_{L}^{2 L} f_{2}(x) d x+\int_{2 L}^{3 L} f_{3}(x) d x$
(b) $U=\int_{0}^{L} f_{1}(x) d x+\int_{L}^{1.5 L} f_{2}(x) d x+\int_{1.5 L}^{3 L} f_{3}(x) d x$
(c) $U=\int_{0}^{L} f_{1}(x) d x+\int_{L}^{1.5 L} f_{2}(x) d x+\int_{1.5 L}^{2 L} f_{3}(x) d x+\int_{2 L}^{3 L} f_{4}(x) d x$
(d) $U=\int_{0}^{L} f_{1}(x) d x+\int_{0}^{1.5 L} f_{2}(x) d x+\int_{0}^{2 L} f_{3}(x) d x+\int_{0}^{3 L} f_{4}(x) d x$

