ME 323 – Mechanics of Materials
Exam 1
February 28, 2024

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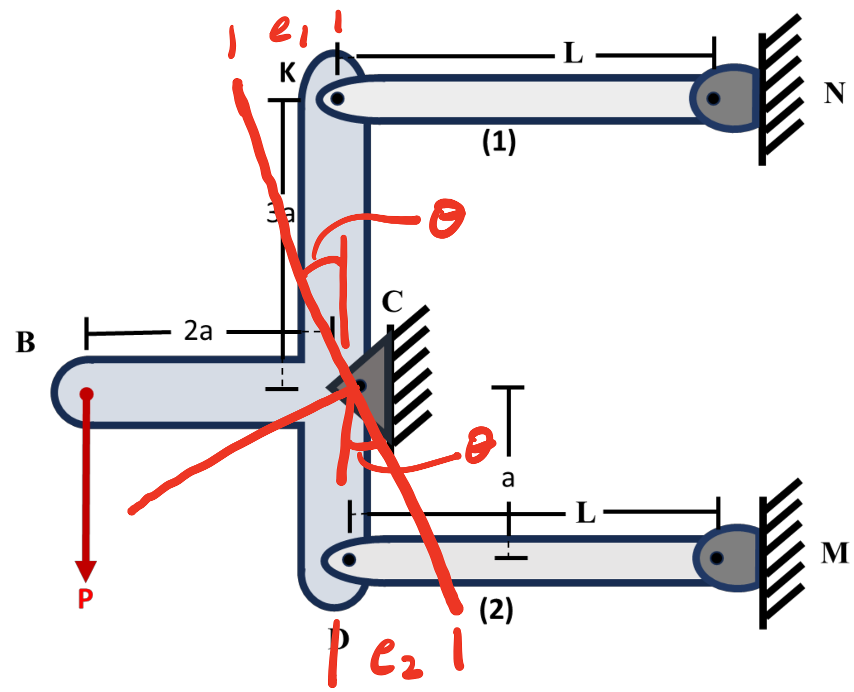
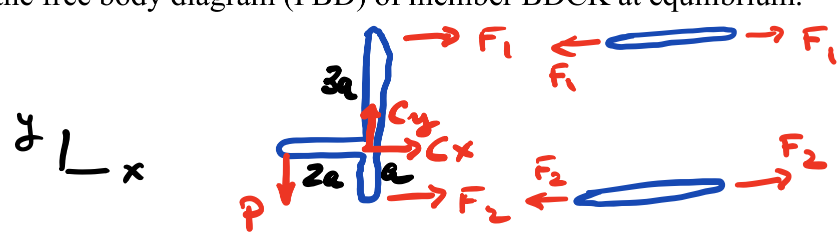
PROBLEM # 1 (25 points)



A rigid member BDCK is supported by two deformable supports, and it is pinned at C. Deformable support (1) consists of rod KN, and deformable support (2) consists of rod DM. Both rods have length *L*, cross-sectional area *A*, Young's modulus *E*, and coefficient of thermal expansion α. The assembly is loaded at B with a vertical force *P*. Rod (1) experiences an increase in temperature equal to ΔT, while the temperature of rod (2) does not change.

a) Draw the free body diagram (FBD) of member BDCK at equilibrium.



b) Write down the equilibrium equations of member BDCK.

$$\Sigma M_c = -F_1(3\cancel{a}) + P(2\cancel{a}) + F_2(\cancel{a}) = 0$$

$$\boxed{F_2 = 3F_1 - 2P} \quad (1)$$

$$\Sigma F_x = C_x + F_1 + F_2 = 0$$

$$\Sigma F_y = C_y - P = 0$$

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c) Is the problem statically determinate or indeterminate?

3eqn's, 4 unknowns ⇒ Indeterminate

d) Write down the force/elongation equations relating the elongations of rods (1) and (2) to the axial loads in rods (1) and (2).

$$e_1 = \frac{F_1 L}{AE} + \alpha \Delta T L \qquad (2)$$

$$e_2 = \frac{F_2 L}{AF} \qquad (3)$$

e) Write down the compatibility equations relating the elongations in rods (1) and (2).

See Fig. 1 $\quad \theta \simeq \frac{e_1}{3\cancel{a}} = -\frac{e_2}{\cancel{a}} \qquad \boxed{e_1 = -3e} \quad (4)$

$$\frac{F_1 \cancel{L}}{\cancel{AE}} + \alpha \Delta T \cancel{L} \overset{AE}{=} -3\frac{F_2 \cancel{L}}{\cancel{AE}} \qquad F_1 + \alpha \Delta T A E = -3F_2 \quad (5)$$

f) Calculate the axial loads in rods (1) and (2).

From Eq'n (1) : $F_2 = 3F_1 - 2P$ ⇒ Replace in (5)

$$F_1 + \alpha \Delta T A E = -3(3F_1 - 2P) = -9F_1 + 6P$$

$$10F_1 = 6P - \alpha \Delta T A E \qquad \boxed{F_1 = \frac{3}{5}P - \frac{\alpha \Delta T A E}{10}}$$

g) If there was no increase in temperature, that is if $\Delta T = 0$, state whether each rod is in tension or compression.

Replace F_1 in F_2 : $F_2 = 3\left(\frac{3}{5}P - \frac{\alpha \Delta T A E}{10}\right) - 2P$

$$\boxed{F_2 = -\frac{P}{5} - \frac{3}{10}\alpha \Delta T A E}$$

If $\Delta T = 0$ $\quad F_1 = \frac{3}{5}P$ ⇒ tension

$F_2 = -\frac{P}{5}$ ⇒ compression

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h) If the vertical force is removed, that is if $P = 0$, state whether each rod is in tension or compression (due to the thermal loads).

If $P = 0$ $\quad F_1 = -\frac{\alpha \Delta T A E}{10}$ compression since $\Delta T > 0$

$F_2 = -\frac{3}{10} \alpha \Delta T A E$ compression

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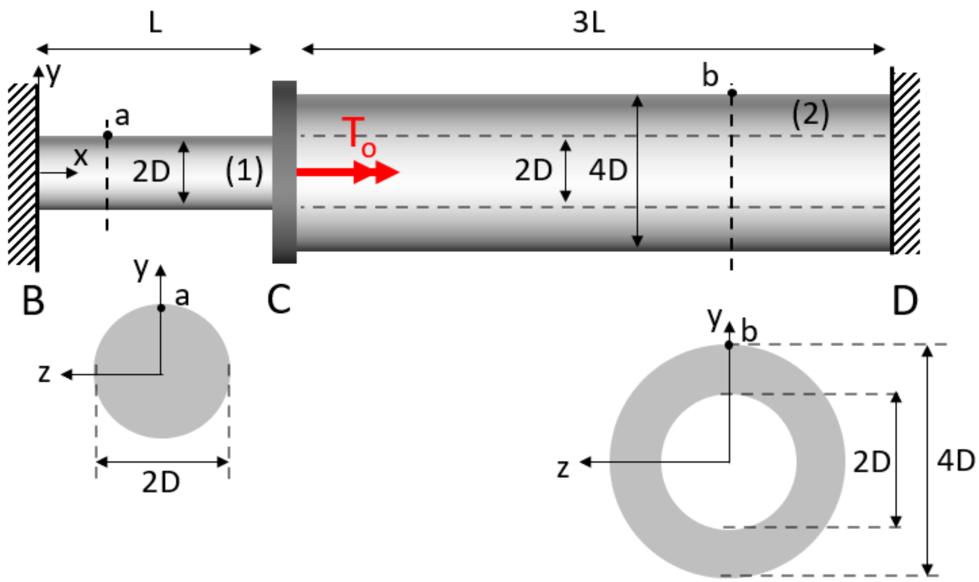
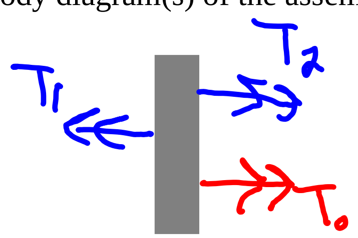
PROBLEM #2 (25 Points): A torsion assembly consists of a solid torsion member between B and C and a hollow torsion member between C and D. A torque of T_o is applied at C, as shown in the image below. Both torsion members have a shear modulus of G.



a) Draw the free body diagram(s) of the assembly.



b) Write the equilibrium equation(s) for the assembly.

$$\Sigma T = T_o + T_2 - T_1 = 0$$

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c) Is the system determinate or indeterminate?

Indeterminate

d) Determine the torque in each shaft in terms of the applied torque (T_0).

Torque-twist

$$\Delta\phi_1 = \frac{T_1 L}{G I_{p1}}$$

$$I_{p1} = \frac{\pi}{2}\left(\frac{2D}{2}\right)^4 = \frac{\pi}{2} D^4$$

$$I_{p2} = \frac{\pi}{2}\left[\left(\frac{4D}{2}\right)^4 - \left(\frac{2D}{2}\right)^4\right] = \frac{15\pi}{2} D^4$$

$$\Delta\phi_2 = \frac{T_1 (3L)}{G I_{p2}}$$

Compatibility: $\Delta\phi_1 + \Delta\phi_2 = 0$

$$\frac{T_1 L}{G \frac{\pi}{2} D^4} = -\frac{3 T_2 L}{G \frac{15\pi}{2} D^4}$$

$$T_1 = -\frac{1}{5} T_2$$

$$T_0 + T_2 - \left(-\frac{1}{5} T_2\right) = 0 \Rightarrow T_2 = -\frac{5}{6} T_0$$

$$T_1 = -\frac{1}{5}\left(-\frac{5}{6} T_0\right) = \frac{1}{6} T_0$$

e) Determine the change in angle of connector C in terms of the applied torque (T_0), L, G, and/or D.

$$\phi_C = \phi_B + \Delta\phi_1 \qquad \phi_B = 0$$

$$\phi_C = \Delta\phi_1$$

$$\phi_C = \frac{T_1 L}{G I_{p1}} = \frac{\frac{1}{6} T_0 L}{G \frac{\pi}{2} D^4} = \frac{T_0 L}{3 G \pi D^4}$$

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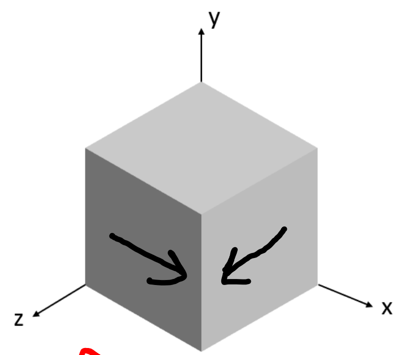
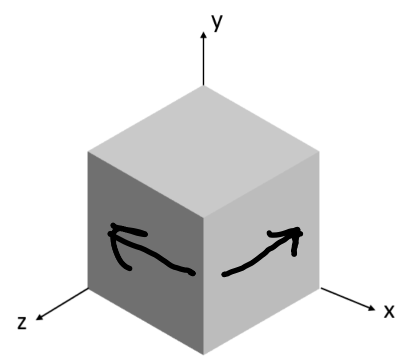
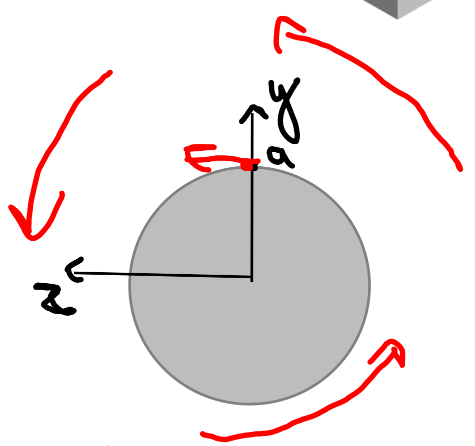
f) Draw the stress elements for points a and b on the structure and determine the magnitude of the shear stress at a and b in terms of the applied torque (T_0), L, G, and/or D.

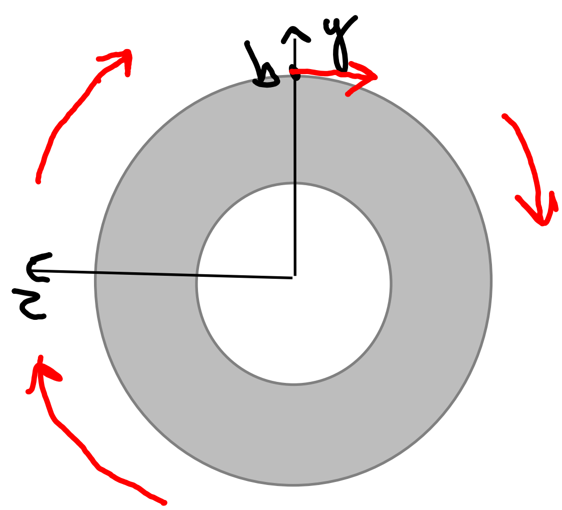
a



b







Positive torque in (1)

τ_{xz} = positive

$$\tau = \frac{T\rho}{I_p}$$

$$|\tau_a| = \frac{T_1 D}{\frac{\pi}{2} D^4} = \frac{T_0}{3\pi D^3}$$

Negative torque in (2)

τ_{xz} = negative

$$|\tau_b| = \frac{T_2 (2D)}{\frac{15\pi}{2} D^4} = \frac{2T_0}{9\pi D^3}$$

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PROBLEM #3 (25 Points): Beam AE is supported by a roller at C and a pin at E. It is subject to a distributed load of 500 lb/ft in the segment AB, and a concentrated moment of 2000 lb·ft at D.

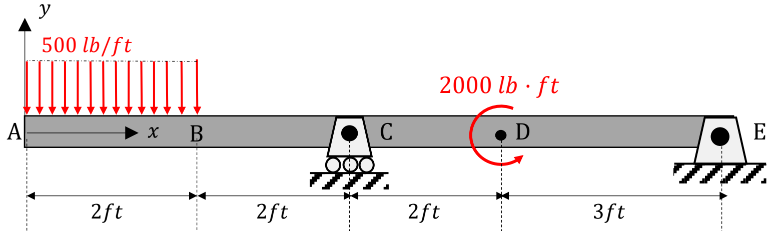
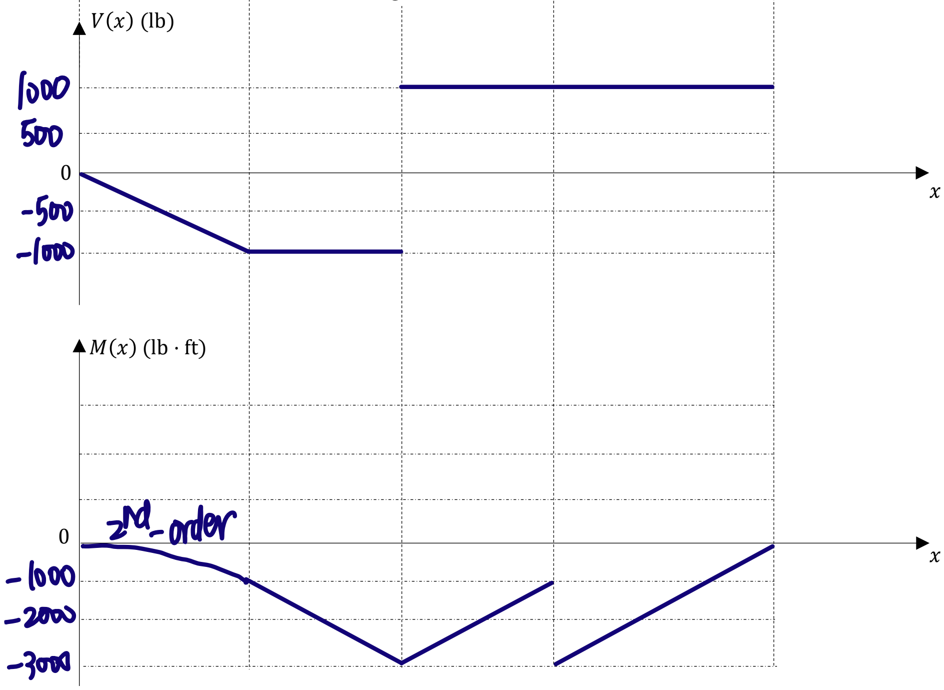
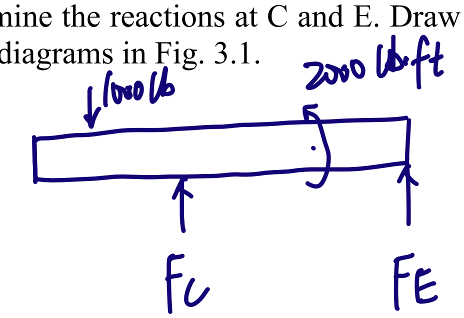


Fig 3.1



a) Determine the reactions at C and E. Draw the shear force and bending moment distributions in the given diagrams in Fig. 3.1.

FBD:



Equilibrium: $F_C + F_E - 1000 = 0$

$\Sigma M_E = 1000 \cdot 8 - F_C \cdot 5 + 2000 = 0$

$F_C = 2000$ lbs . $F_E = -1000$ lbs

b) The beam has a triangular cross section shown below, where $h = 6$ in. Determine the maximum tensile flexural stress and maximum compressive flexural stress in the beam. (Hints: 1 ft=12 in. The second area moment of the cross section is $I_z = \frac{1}{36}h^4$).

$$\sigma = -\frac{My}{I}$$

Maximum moment:

$$M_{max} = -3000\ lb \cdot ft$$

$$= -36000\ lb \cdot in$$

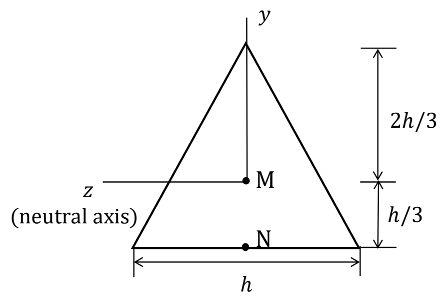


Fig. 3.2. Cross section

$$I = \frac{1}{36} h^4 = \frac{1}{36} \cdot 6^4 = 36\ in^4$$

Max tension: $$(\sigma_T)_{max} = -\frac{-36000 \cdot 4}{36} = 4000\ lb/in^2$$

Max compression: $$(\sigma_C)_{max} = -\frac{36000 \cdot (-2)}{36} = -2000\ lb/in^2$$

c) Points M and N are located at the cross-section D of the beam and their positions are shown in the above figure. Determine the shear stress at the two points.

shear stress at N: $\tau_N = 0$

At M: $$\tau = \frac{VQ}{It}, \quad V = 1000\ lbs$$

$$Q = A^* \cdot y^* = \frac{1}{2} \times 4 \times 4 \cdot \frac{1}{3} \times 4 = \frac{32}{3}\ in^3$$

$$I = 36\ in^4, \quad t = 4\ in$$

$$\tau_M = \frac{1000 \cdot \frac{32}{3}}{36 \cdot 4} = 74.07\ lb/in^2$$

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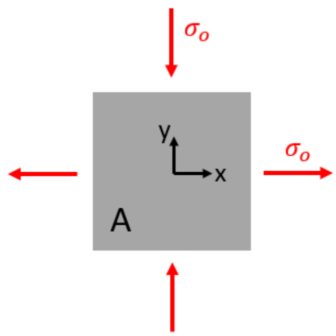
<u>PROBLEM #4 (25 points)</u>:

Part A (6 points): The plane stress elements shown below have stresses in the directions shown, but no stresses in the z directions. The Poisson's ratio is between 0 and 0.5 (0<ν<0.5).

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \cancelto{0}{\sigma_z}\right)\right)$$
$$\epsilon_x = \frac{1}{E}\left(\sigma_0 + \nu\sigma_0\right)$$



(i) (**2 points**) For stress state A: In the absence of temperature changes, circle whether each of the strains are <0, =0, or >0:

ϵ_x	<0	=0	>0
ϵ_y	<0	=0	>0
ϵ_z	<0	=0	>0
γ_{xy}	<0	=0	>0

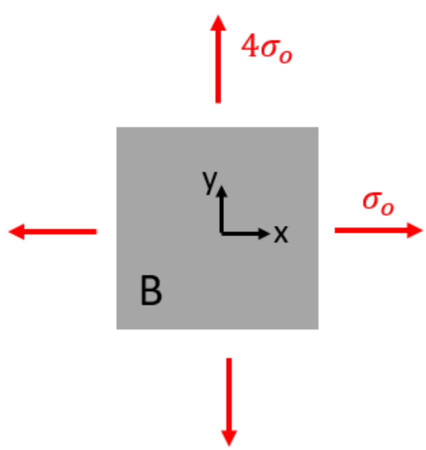
$$\epsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \cancelto{0}{\sigma_z}\right)\right)$$
$$\epsilon_y = \frac{1}{E}\left(-\sigma_0 - \nu\sigma_0\right)$$

$$\epsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right) = 0$$

(ii) (**2 point**) For stress state A: If the temperature is increased, circle whether each of the strains are <0, =0, or >0:

ϵ_z	<0	=0	>0
γ_{xy}	<0	=0	>0

$$\epsilon_z = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_x + \sigma_y\right)\right) + \alpha\Delta T$$
$$\epsilon_z = \alpha\Delta T$$



(iii) (**2 points**) For stress state B: In the absence of temperature changes, what condition would enable the strain in the x direction to be zero (circle one)?

$\nu = 0$
$\nu = 0.1$
$\nu = 0.25$
$\nu = 0.33$
$\nu = 0.5$
$\nu = 1.0$

$$\epsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right)$$
$$0 = \frac{1}{E}\left(\sigma_0 - \nu\left(4\sigma_0\right)\right)$$
$$0 = 1 - 4\nu$$
$$\nu = \frac{1}{4}$$

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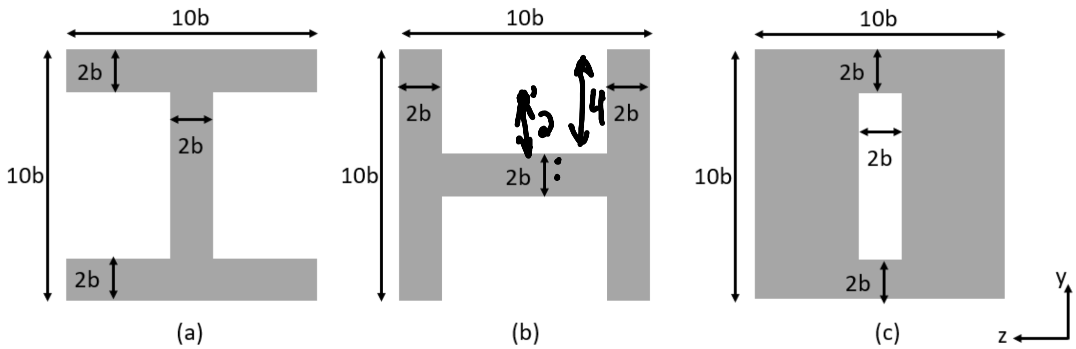
Part B (3 points):



(**3 points**) For transverse loading in the y direction, choose the correct order for the second area moments of the cross sections of beams a, b, and c:

$I_a > I_b > I_c$
$I_b > I_c > I_a$
$I_c > I_a > I_b$
$I_c > I_b > I_a$

Conceptual:

(a) has more area far from the centroid than (b)

$\Rightarrow I_{(a)} > I_{(b)}$

(c) has more area than (a) $\Rightarrow I_{(c)} > I_{(a)}$

Calculations:

$$I_a = \frac{(10b)(10b)^3}{12} - 2\left[\frac{(4b)(6b)^3}{12}\right] = 689.\bar{3}\, b^4$$

$$I_b = \frac{(10b)(10b)^3}{12} - 2\left[\frac{(6b)(4b)^3}{12} + (6b)(4b)(3b)^2\right] = 337.\bar{3}\, b^4$$

$$I_c = \frac{(10b)(10b)^3}{12} - \frac{(2b)(6b)^3}{12} = 797.\bar{3}$$

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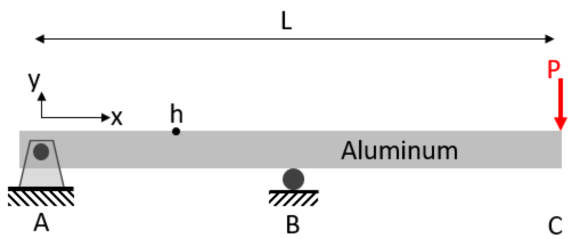
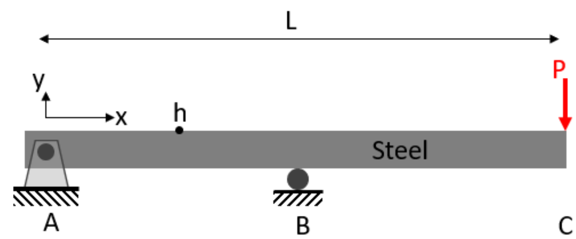
Part C (4 points): Two beams (f) and (g) with cylindrical cross-sections have the same loading conditions but are made of different materials. Point h is at the most positive point of the cross-section in the y-direction.



(f)



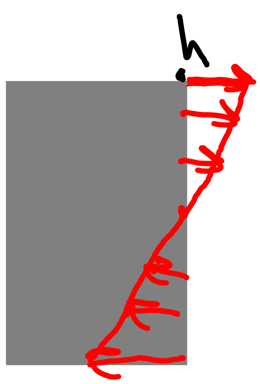
(g)

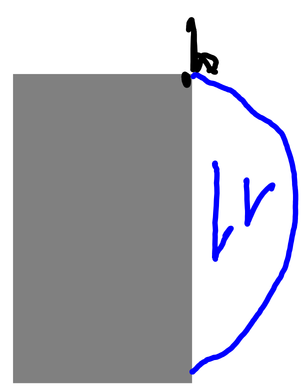
(i) (**2 points**) What is the relationship between the flexural stresses of the beams at point h (circle one):

$\sigma_f(h) = \sigma_g(h) = 0$ $\quad$ $\sigma_f(h) = \sigma_g(h) \neq 0$ $\quad$ $\sigma_f(h) \neq \sigma_g(h)$

(ii) (**2 points**) What is the relationship between the shear stresses of the two beams at point h (circle one):

$\tau_f(h) = \tau_g(h) = 0$ $\quad$ $\tau_f(h) = \tau_g(h) \neq 0$ $\quad$ $\tau_f(h) \neq \tau_g(h)$





Shear and moment do not depend on materials properties for determinate beams $\Rightarrow$ $\sigma_f(h) = \sigma_g(h)$ and $\tau_f(h) = \tau_g(h)$

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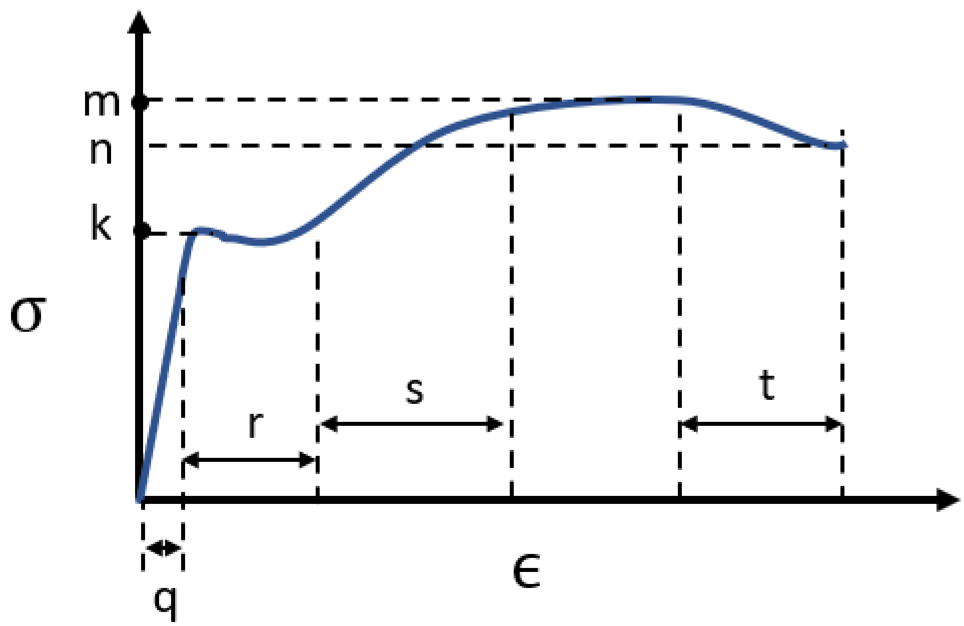
Part D (4 points): A stress-strain curve is shown below for steel.



(**4 points**) Indicate which of the following terms correspond with the points k, m, n and the regions q, r, s, t in the diagram of the stress strain curve above:

Necking region: t

Ultimate stress: m

Yield stress: k

Elastic region: q

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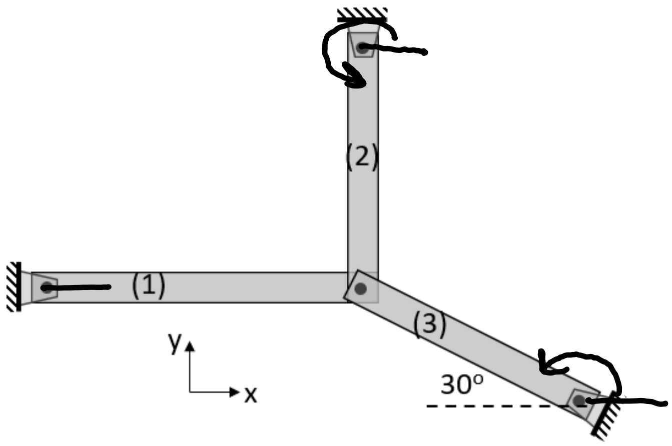
Part E (3 points): In the planar truss below with three members, what angles are used in the equations for the elongation of each member?



θ_1 = 0°
θ_2 = 270°
θ_3 = 150°

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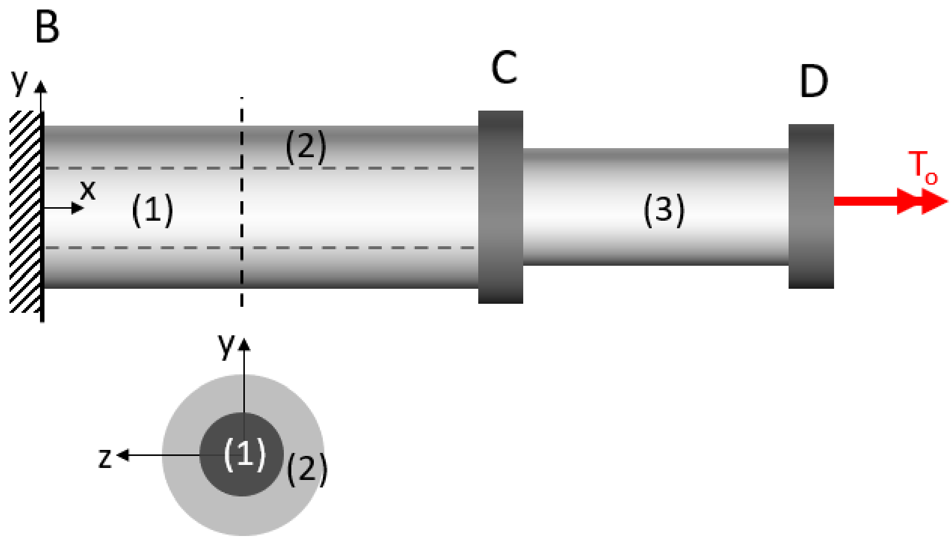
Part F (5 points): A torsion assembly has a bimetallic torsion member from B to C, a solid torsion member from C to D, and a torque of T_o applied at D.



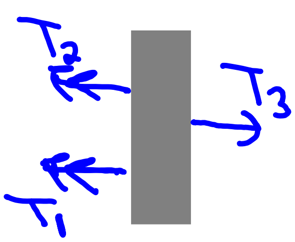
If the shear modulus of member (1) was increased:

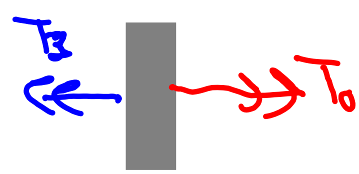
(i) (**2 points**) The maximum shear stress in (3) would (circle one):

Increase Stay the same Decrease

(ii) (**3 points**) The maximum shear stress in (2) would (circle one):

Increase Stay the same Decrease





$T_3 - T_2 - T_1 = 0$

$T_0 - T_2 - T_1 = 0$ Indeterminate

$T_0 - T_3 = 0$

$T_0 = T_3$

$\Delta\phi_1 = \Delta\phi_2$

$$\frac{T_1 L}{G_1 I_{p1}} = \frac{T_2 L}{G_2 I_{p2}}$$

$\uparrow G_1 \Rightarrow \downarrow T_2$

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$$\frac{(T_0 - T_2)L}{G_1 I_{p1}} = \frac{T_2 L}{G_2 I_{p2}}$$

$$T_0 = T_2\left(\frac{G_1}{G_2}\right)\left(\frac{I_{p1}}{I_{p2}}\right) + T_2$$

$$T_2 = \frac{T_0}{1 + \left(\frac{G_1}{G_2}\right)\left(\frac{I_{p1}}{I_{p2}}\right)}$$