Useful Equations

Bending deformation:

$$\frac{dV}{dx} = w(x) \qquad \frac{dM}{dx} = V(x) \qquad M = EIv'' \qquad \Delta V = P \qquad \Delta M = -M_0$$

$$\sigma(x,y) = \frac{-Ey}{\rho} = \frac{-M_{zz}y}{I_{zz}} \qquad I_{zz} = \frac{bh^3}{12} \text{ (rectangle)}, \quad I_{zz} = \frac{\pi r^4}{4} \text{ (circle)}$$

Strain energy density:

$$\bar{u} = \frac{1}{2}[\sigma_x(\varepsilon_x - \alpha\Delta T) + \sigma_y(\varepsilon_y - \alpha\Delta T) + \sigma_z(\varepsilon_z - \alpha\Delta T) + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}]$$

Energy methods:

$$U = \frac{1}{2}\int_0^L \frac{F^2(x)}{EA}dx \qquad U = \frac{1}{2}\int_0^L \frac{f_s V^2(x)}{GA}dx \qquad U = \frac{1}{2}\int_0^L \frac{M^2(x)}{EI}dx \qquad U = \frac{1}{2}\int_0^L \frac{T^2(x)}{GI_p}dx$$

Work-energy principle: $U = W$

Castigliano's 2nd theorem:

$$\delta_{P_i} = \frac{\partial U}{\partial P_i} \qquad \theta_{M_i} = \frac{\partial U}{\partial M_i} \qquad \phi_{T_i} = \frac{\partial U}{\partial T_i}$$

$$\delta_{P_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial P_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial P_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial P_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial P_i}dx$$

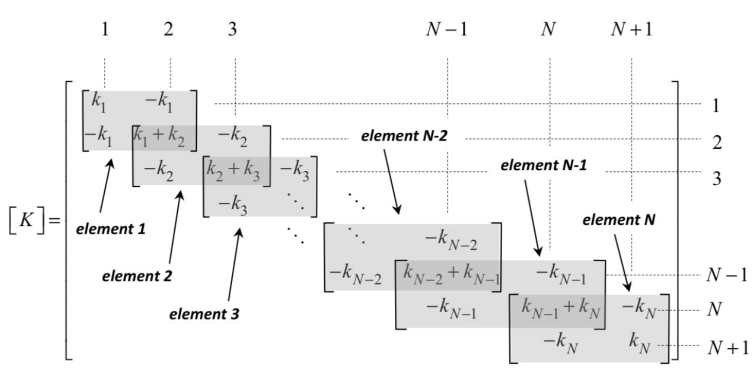
$$\theta_{M_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial M_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial M_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial M_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial M_i}dx$$

$$\phi_{T_i} = \int_0^L \frac{M(x)}{EI}\frac{\partial M(x)}{\partial T_i}dx + \int_0^L \frac{F(x)}{EA}\frac{\partial F(x)}{\partial T_i}dx + \int_0^L \frac{T(x)}{GI_p}\frac{\partial T(x)}{\partial T_i}dx + \int_0^L \frac{f_s V(x)}{AG}\frac{\partial V(x)}{\partial T_i}dx$$

f_s=6/5 (rectangular cross section), f_s=10/9 (circular cross section)

Finite element method:

$$k_i = \frac{(EA)_i}{L_i}$$



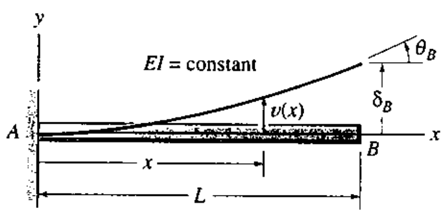
APPENDIX

E

DEFLECTIONS AND SLOPES OF BEAMS; FIXED-END ACTIONS

R.R. CRAIG

E.1. Deflections and Slopes of Uniform Cantilever Beams*



Notation

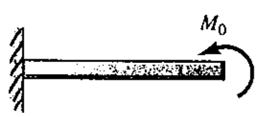
$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\delta_B \equiv v(L)$ = deflection at end B

$\theta_B \equiv v'(L)$ = slope at end B

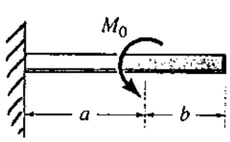
1



$$v = \frac{M_0 x^2}{2EI} \qquad v' = \frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 L^2}{2EI} \qquad \theta_B = \frac{M_0 L}{EI}$$

2

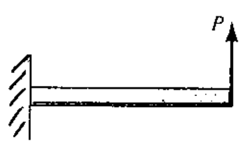


$$v = \frac{M_0 x^2}{2EI} \qquad v' = \frac{M_0 x}{EI} \qquad 0 \le x \le a$$

$$v = \frac{M_0 a}{2EI}(2x - a) \qquad v' = \frac{M_0 a}{EI} \qquad a \le x \le L$$

$$\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$$

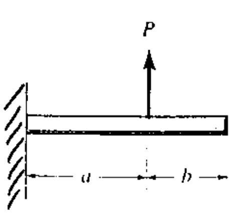
3



$$v = \frac{Px^2}{6EI}(3L - x) \qquad v' = \frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

4

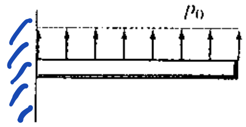


$$v = \frac{Px^2}{6EI}(3a - x) \qquad v' = \frac{Px}{2EI}(2a - x) \qquad 0 \le x \le a$$

$$v = \frac{Pa^2}{6EI}(3x - a) \qquad v' = \frac{Pa^2}{2EI} \qquad a \le x \le L$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

5

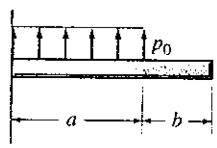


$$v = \frac{p_0 x^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$v' = \frac{p_0 x}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{p_0 L^4}{8EI} \qquad \theta_B = \frac{p_0 L^3}{6EI}$$

6



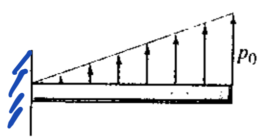
$$v = \frac{p_0 x^2}{24EI}(6a^2 - 4ax + x^2) \qquad 0 \le x \le a$$

$$v' = \frac{p_0 x}{6EI}(3a^2 - 3ax + x^2) \qquad 0 \le x \le a$$

$$v = \frac{p_0 a^3}{24EI}(4x - a) \qquad v' = \frac{p_0 a^3}{6EI} \qquad a \le x \le L$$

$$\delta_B = \frac{p_0 a^3}{24EI}(4L - a) \qquad \theta_B = \frac{p_0 a^3}{6EI}$$

7

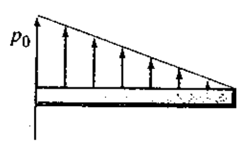


$$v = \frac{p_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$v' = \frac{p_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

$$\delta_B = \frac{11 p_0 L^4}{120EI} \qquad \theta_B = \frac{p_0 L^3}{8EI}$$

8

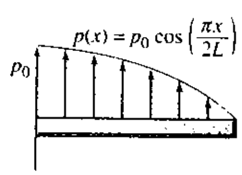


$$v = \frac{p_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$v' = \frac{p_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

$$\delta_B = \frac{p_0 L^4}{30EI} \qquad \theta_B = \frac{p_0 L^3}{24EI}$$

9



$$v = \frac{p_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 L x^2 - \pi^3 x^3\right)$$

$$v' = \frac{p_0 L}{\pi^3 EI}\left(2\pi^2 L x - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$

$$\delta_B = \frac{2 p_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{p_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

*Beam-deflection theory is covered in Chapter 7. The sign convention used here is the same as in Chapter 7.