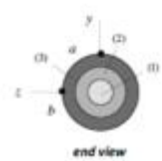
Problem 5.1 – 10 points

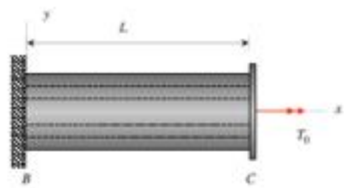
A composite shaft is made up of tubular elements (2) and (3), and a core (1). All elements are attached to a fixed wall at B and to a rigid connector at end C. The material making up elements (1), (2) and (3) have shear moduli of $G_1 = G$, $G_2 = 2G$ and $G_3 = G$, respectively. A torque T_0 is applied to connector C.

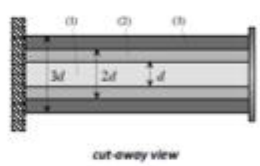
a) Determine the torques in the three elements and the angle of rotation of connector C.

b) Make an accurate sketch of the shear strain and shear stress distributions on a cross section of the shaft on the axes provided below for both the strain and stress distribution plots. Clearly indicate the slopes of the distribution curves.

c) Show the components of stress on stress elements at points "a" and "b" on the shaft.

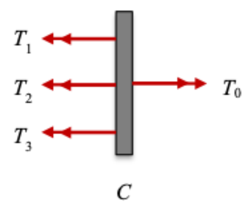






Solution:

a) The torque FBD of connector C:



From the torque FBD,

$$\Sigma M_C:\ T_0 - T_1 - T_2 - T_3 = 0$$ [2pt])

Compatibility:

$$\Delta\phi_1 = \Delta\phi_2 = \Delta\phi_3 = \phi_C - \phi_B \qquad [3pt]$$

$$\Rightarrow \frac{T_1 L_1}{G_1 I_{p1}} = \frac{T_2 L_2}{G_2 I_{p2}} = \frac{T_3 L_3}{G_3 I_{p3}}$$

$$\Rightarrow \frac{T_1 L}{G\frac{\pi}{32}(d)^4} = \frac{T_2 L}{2G\frac{\pi}{32}\{(2d)^4 - (d)^4\}} = \frac{T_3 L}{G\frac{\pi}{32}\{(3d)^4 - (2d)^4\}} \quad \#(2.2)$$

From #2.1, #2.2,

$$T_1 = \frac{T_2}{30} = \frac{T_3}{65}$$

Thus,

$$T_1 = \frac{T_0}{96}\ ;\ T_2 = \frac{5T_0}{16}\ ;\ T_3 = \frac{65T_0}{96} \quad [0.5pt]$$

$$\phi_C = \phi_B + \Delta\phi_1 = 0 + \frac{T_0}{96}\cdot\frac{L}{G\frac{\pi}{32}(d)^4}$$

$$\Rightarrow \phi_c = \frac{T_0 L}{3\pi d^4 G} \quad [0.5pt]$$

b)

The shear strain of a cross section at L'

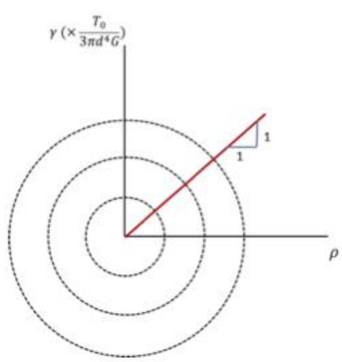
$$\gamma = \frac{\rho\Delta\phi}{L'}\ ;\ \Delta\phi = \frac{T_0 L'}{3\pi d^4 G} \quad \Rightarrow \gamma = \frac{T_0}{3\pi d^4 G}\rho$$

For the shear stress,

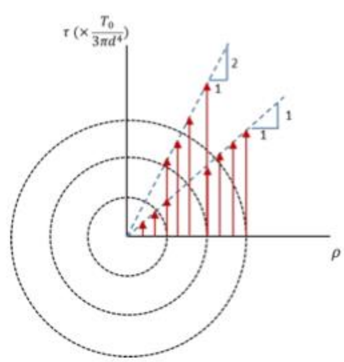
$$\tau_1 = \frac{T_1\rho_1}{I_{p1}} = \frac{T_0}{3\pi d^4}\rho_1, \quad 0 < \rho_1 < \frac{d}{2}$$

$$\tau_2 = \frac{T_2\rho_2}{I_{p2}} = \frac{2T_0}{3\pi d^4}\rho_2, \quad \frac{d}{2} < \rho_2 < d$$

$$\tau_3 = \frac{T_3\rho_3}{I_{p3}} = \frac{T_0}{3\pi d^4}\rho_3, \quad d < \rho_3 < \frac{3d}{2}$$



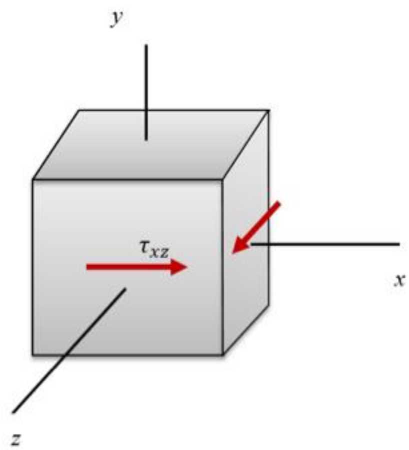
[1pt]



[1pt]

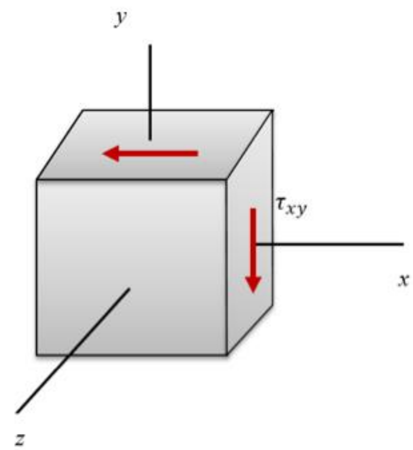
c)

$$|\tau_{a,(xz)}| = |\tau_{b,(xy)}| = \frac{T_3 \frac{3d}{2}}{I_{p3}} = \frac{T_0}{2\pi d^3}$$



Point "a"

[1pt]



Point "b"

[1pt]

Problem 5.2(10 points)

A beam is supported by a pin at A and a roller at D. It is subjected to a concentrated load P at C. The cross-section of the beam is shown below.

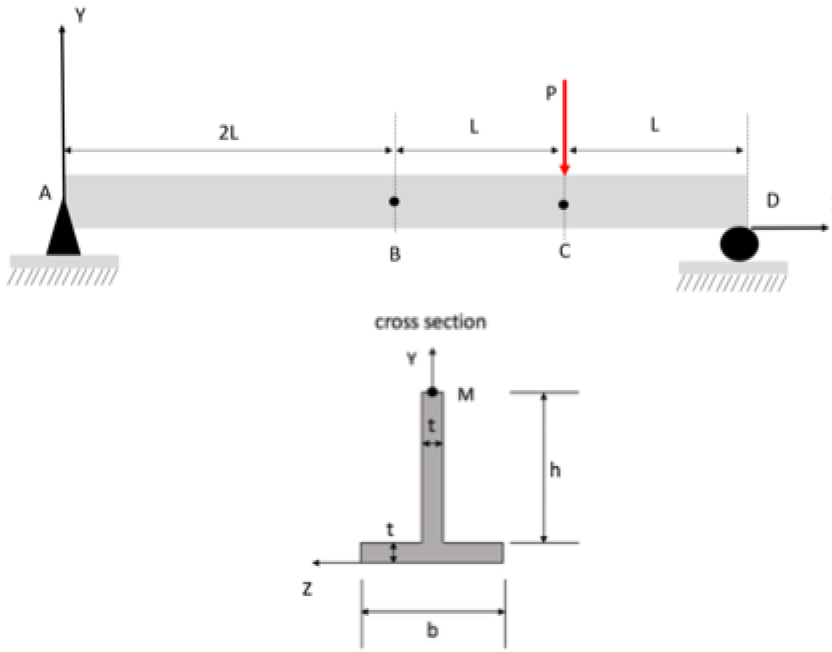
(a) Determine the centroid of the given cross-section.

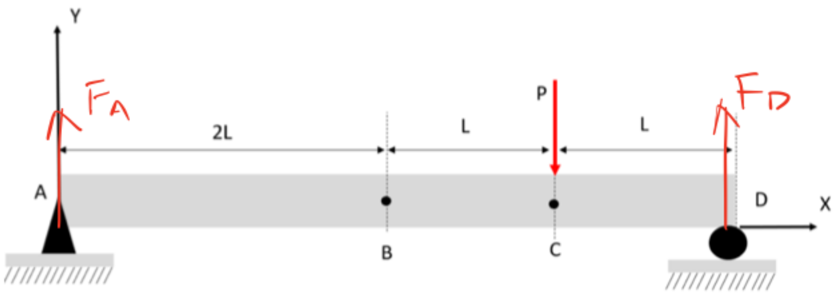
(b) Determine the second moment of area of the cross-section.

(c) Determine the shear stress at the centroid of the cross-section at B.

(d) Determine the flexural stress at point M on the cross-section at B.

Use: t=0.04 m, b=0.07 m, h=0.08 m, L=1 m, P=30 N.





a). $\bar{y} = \dfrac{t \cdot b \cdot \frac{t}{2} + t \cdot h \cdot (t + \frac{h}{2})}{tb + th} = 0.052\ m$

$\bar{z} = \frac{1}{2} b = 0.035\ m.$

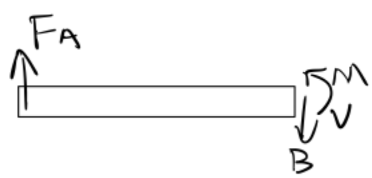
b). $I_z = \dfrac{bt^3}{12} + bt\,(\bar{y} - \frac{t}{2})^2 + \frac{1}{12} th^3 + ht\,(t + \frac{h}{2} - \bar{y})^2$

$= 7.456 \times 10^{-6}\ m^4$

c). $\Sigma F_y = 0 = F_A + F_D - P$

$\Sigma M_A = 0 = F_D \cdot 4L - P \cdot 3L$

$\Rightarrow \begin{cases} F_A = 7.5N \\ F_D = 22.5N \end{cases}$



B : $\Sigma F_y = 0 = F_A - V$

$V = 7.5N.$

$\Sigma M_B = M - F_A \cdot 2L$

$M = 15\ Nm$

$Q = y^* A^* = \frac{1}{2} (h + t - \bar{y})^2 \cdot t = 9.248 \times 10^{-5}\ m^3$

$\tau = \dfrac{V\ Q}{I_z\, t} = 2325.6 Pa$

d) $\sigma_M = - \dfrac{My}{I} = - \dfrac{15 \cdot (h + t - \bar{y})}{I_z} = -1.368 \times 10^5\ Pa$

Problem 5.3 *(10 points)*

A design decision is to be taken based on whether a beam can support higher normal stresses in a box configuration or an H-configuration(see the two figures below), considering plane of bending is the XY plane. Assuming that both cross sections are symmetric about the origins placed at the respective centroids:

(a) Determine the second area moment of inertia of the two cross sections, I_{ZZ}^{BOX} and I_{ZZ}^{H}

(b) Determine which cross section would experience a lower magnitude of maximum normal stress for the same loading conditions.

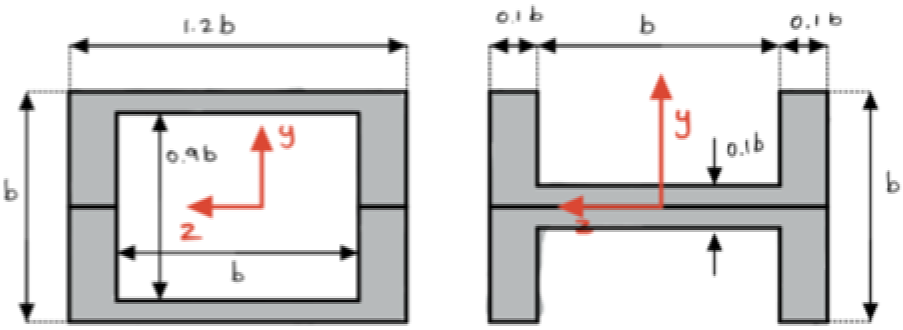
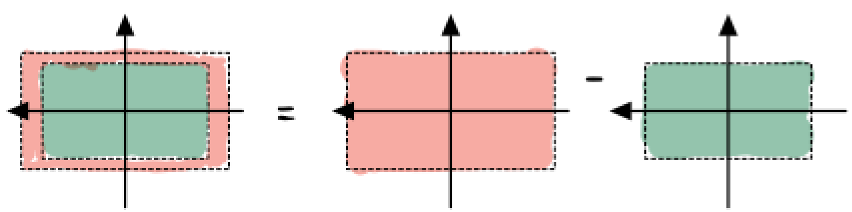


Figure 2: Left: Box-configuration Right: H-Configuration

(a) Box configuration

$$I_{zz} = I_{zz}\,[\text{Big rectangle}] - I_{zz}\,[\text{Small / Inner Rectangle}]$$



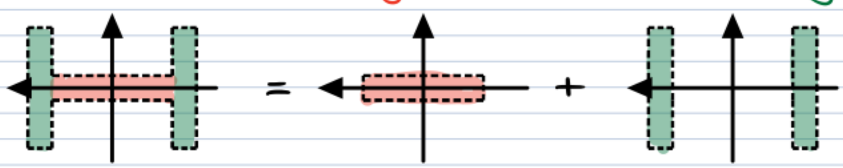
$$= \frac{1}{12}(bh^3)_{BRect} - \frac{1}{12}(bh^3)_{S\,Rect}$$

$$= \frac{1}{12}(1.2b)(b)^3 - \frac{1}{12}(b)(0.9b)^3$$

$$I_{zz} = 0.1\, b^4 - 0.054\, b^4$$
$$= 0.046\, b^4$$

H configuration

$$I_{zz} = I_{zz}\ \text{Rectangle H} + 2\, I_{zz}\ \text{V. Rectangle}$$



$$= \frac{1}{12}(b)(0.1b)^3 + 2 \times \frac{1}{12}(0.1b)(b)^3$$
$$= 0.0168\, b^4$$

(b) If both the beams experience same loading condition, [$V(x)$ & $M(x)$ are the same], then the normal $\underset{[max]}{}$ stress at a point is given by:

$$|\sigma_{max}| = \left| \frac{M_{max} * y_{max}}{I_z} \right|$$

Now,

$y_{max,\ box\ config} = +0.5b$ $\qquad$ $y_{max,\ H\ config} = 0.5b$

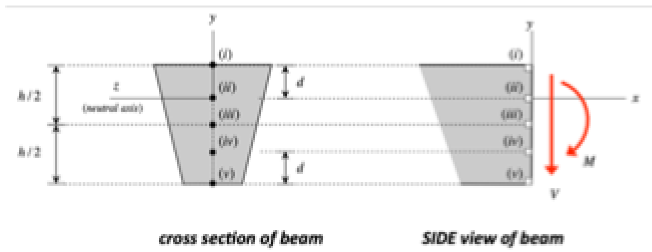
$$\sigma_{max,\ box\ config} = \frac{M_{max}(0.5b)}{0.046\, b^4} \qquad \sigma_{max,\ H\text{-}config} = \frac{M_{max}(0.5b)}{0.0168 b^4}$$

$$= 10.86\, b^{-3} M_{max} \quad < \quad = 29.76\, b^{-3} M_{max}$$

<u>Conclusion</u>: Box is experiencing the same loads but has lower normal stresses developed.

Hence it can support larger loads before failure and is a better choice.

Problem 5.4
Part 1
A shear force V and bending moment M act at a cross section of a trapezoidal cross-sectioned beam. Consider the five points (i), (ii), (iii), (iv) and (v) on the beam cross section, as shown above. Match up the state of stress at each of these five points with the stress elements (a) through (o) shown below. If you choose "(o) NONE of the above", provide a sketch of the correct state of stress for your answer.



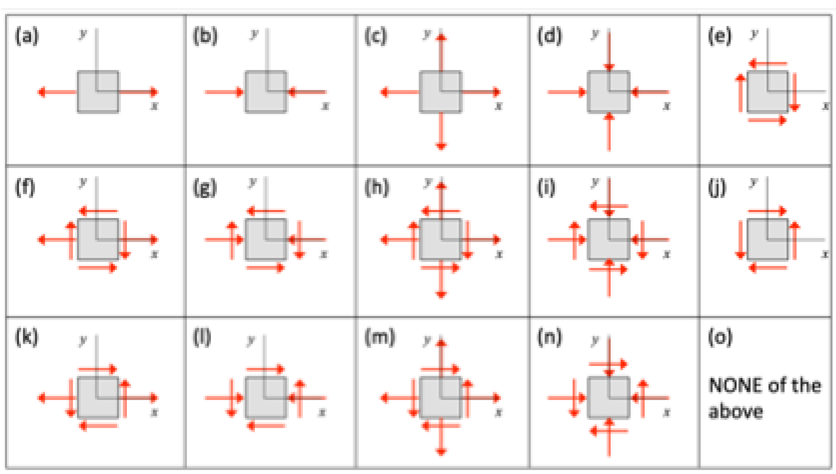
The state of stress at point (i) is a

The state of stress at point (ii) is e

The state of stress at point (iii) is g

The state of stress at point (iv) is g

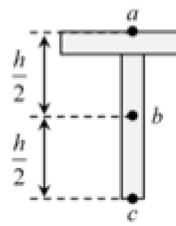
The state of stress at point (v) is b



I = Same at all points
M = Same at all points

Part 2

1. A beam has the T-shaped cross section shown below. The internal bending moment in the beam is known to be M.



Centroid for a T c.s is closer to the top $\therefore |\bar{y}^*|$ is greater at 'c'.

Let $|\sigma_a|$, $|\sigma_b|$, and $|\sigma_c|$ be the magnitudes of the normal stress at points a, b, and c, respectively. It is desired to determine $|\sigma_{max}|$, the maximum magnitude of the normal stress on the cross section.

Choose the correct statement.

- ~~(a)~~ $|\sigma_{max}| = |\sigma_a|$
- ~~(b)~~ $|\sigma_{max}| = |\sigma_b|$
- (c) $|\sigma_{max}| = |\sigma_c|$ ✓
- ~~(d)~~ $|\sigma_{max}| = |\sigma_a| = |\sigma_c|$
- ~~(e)~~ $|\sigma_{max}| = |\sigma_a| = |\sigma_b| = |\sigma_c|$

$\therefore |\sigma_c| > |\sigma_a| > |\cancel{\sigma}_b|$ 0

↓
max