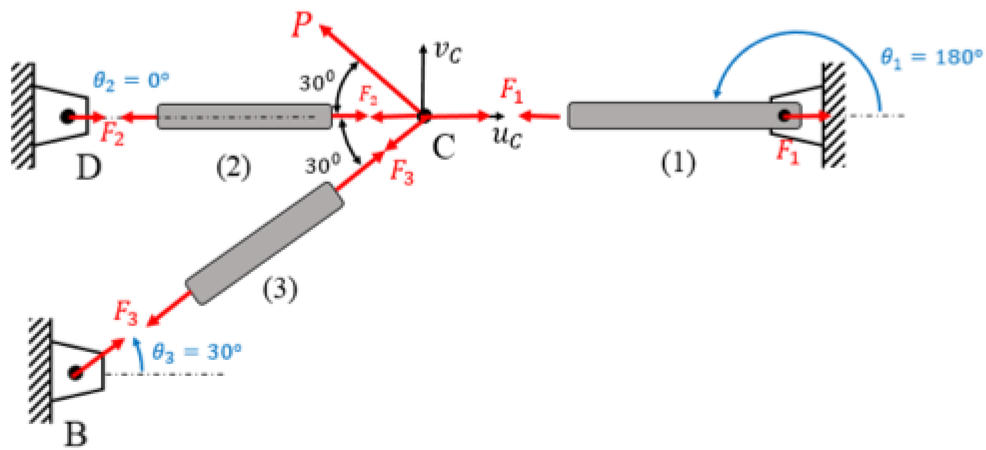
Problem 1

a. The angular orientations of members (1), (2), and (3) w.r.t pinned ends H, D, and B are

$\theta_1 = 180°, \theta_2 = 0°, \theta_3 = 30°$.



From the free body diagram of joint C,

$$\Sigma F_x = \quad F_1 - F_2 - F_3 \cos 30° - P\cos 30° = 0$$

$$\Sigma F_y = \quad F_3 \sin 30° - P \sin 30° = 0$$

Static equilibrium analysis of joint yields 2 equations and three unknowns F_1, F_2, F_3. It means that the problem is indeterminate and needs compatibility conditions to solve for the unknowns.

b. Elongation:

$$e_1 = \frac{F_1(6L)}{(2A)E} = \frac{3F_1L}{EA}; \qquad e_2 = \frac{F_2(4L)}{A(2E)} = \frac{2F_2L}{EA}; \qquad e_3 = \frac{F_3(2L)}{E(2A)} = \frac{F_3L}{EA}$$

c. Compatibility:

$$e_1 = u_C \cos\theta_1 + v_C \sin\theta_1$$

$$\Rightarrow \frac{3F_1L}{EA} = u_C \cos 180° + v_C \sin 180° \Rightarrow \frac{3F_1L}{EA} = -u_C$$

$$e_2 = u_C \cos\theta_2 + v_C \sin\theta_2$$

$$\Rightarrow \frac{2F_2L}{EA} = u_C \cos 0° + v_C \sin 0° \Rightarrow \frac{2F_2L}{EA} = u_C$$

$$e_3 = u_C \cos\theta_3 + v_C \sin\theta_3$$

$$\Rightarrow \frac{F_3 L}{EA} = u_C \cos 30° + v_C \sin 30° \Rightarrow \frac{F_3 L}{EA} = \frac{\sqrt{3}}{2}\, u_C + \frac{1}{2}\, v_C$$

d. From previous equations

$$\frac{3F_1 L}{EA} = -\frac{2F_2 L}{EA}$$

$$\Rightarrow F_2 = \frac{-3}{2} F_1$$

From the equilibrium equations,

$$F_3 = P$$

and

$$F_1 + \frac{3}{2}F_1 - P\cos 30° - P\cos 30° = 0$$

$$\Rightarrow F_1 = \frac{2\sqrt{3}}{5} P$$

$$\Rightarrow F_2 = \frac{-3}{2} F_1 = \frac{-3\sqrt{3}}{5} P$$

$$u_C = \frac{2F_2 L}{EA} = \frac{-6\sqrt{3}}{5}\frac{PL}{EA}$$

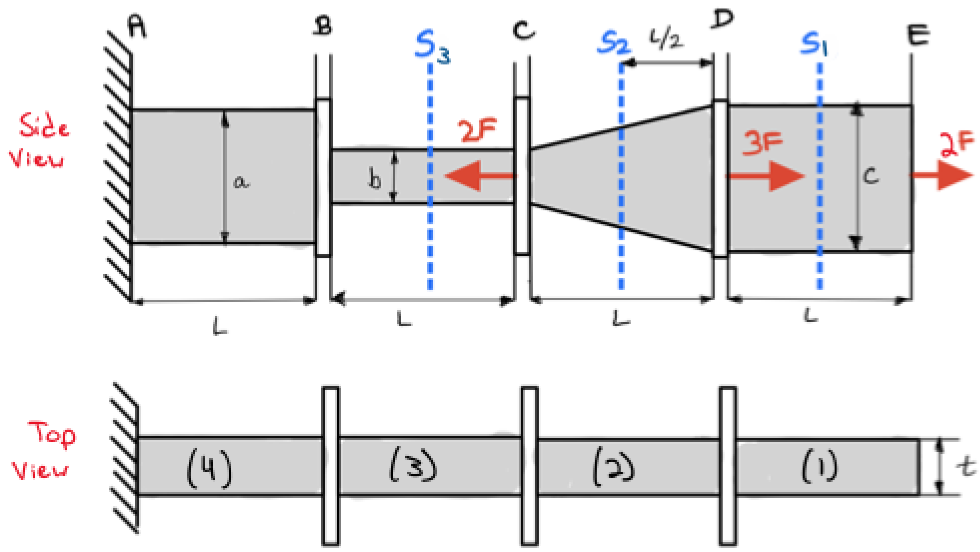
$$\frac{F_3 L}{EA} = \frac{\sqrt{3}}{2}\, u_C + \frac{1}{2}\, v_C \Rightarrow \frac{PL}{EA} - \frac{\sqrt{3}}{2}\left(\frac{-6\sqrt{3}}{5}\frac{PL}{EA}\right) = \frac{1}{2} v_C$$

$$\Rightarrow v_C = \frac{28}{5}\frac{PL}{EA}$$

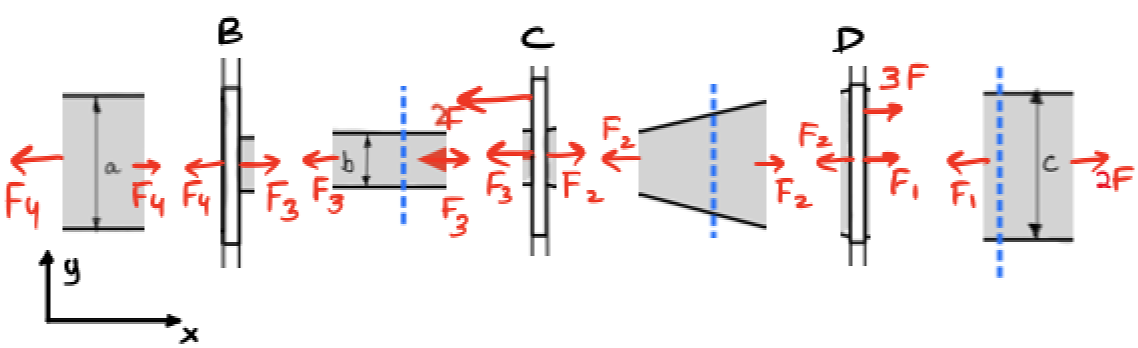
Problem 3.2 ***(10 points)***

The axial bar shown in Fig.1 has four sections. Each section is connected to the neighbouring sections by rigid connectors (at B, C and D). The size of the rigid connectors is negligible. The first section AB, second section BC and the last section DE have uniform rectangular cross sections with width a,b and c respectively. The section CD has width varying linearly from b to c. The length of each section is L. All the sections have same thickness t. Three loads F,3F and 2F are applied to the bar as shown in the figure. Assume the Young's Modulus of all the sections is E.

(a) Find expressions for stresses at sections S1, S2 and S3. (S2 is midway of section CD as shown)
(b) Find expressions for displacement at points B and C



Free body Diagram :



Now,

At connector B: $\overset{+}{\rightarrow}\sum F_x = 0$: $F_3 - F_4 = 0$

$$F_4 = F_3$$

connector C: $\overset{+}{\rightarrow}\sum F_x = 0$: $F_2 - F_3 - 2F = 0$

$$F_3 = F_2 - 2F$$

Connector D: $\overset{+}{\rightarrow}\sum F_x = 0$: $F_1 + 3F - F_2 = 0$

$$F_2 = F_1 + 3F$$

But looking at Section S_3, we know that

$F_1 = 2F$

$\therefore$ $F_2 = 5F$, $F_3 = 3F$ and $F_4 = 3F$

(a) Stresses:

$$\sigma_{S_3} = \frac{F_3}{A_3} = \frac{3F}{b \times t}$$

$$\sigma_{S_2} = \frac{F_2}{A_2} = \frac{5F}{\left(\frac{b+c}{2}\right)t} = \frac{10F}{(b+c)t}$$

$$\sigma_{S_1} = \frac{F_1}{A_1} = \frac{2F}{ct}$$

$$b + \frac{c-b}{L} \cdot x = a' + b'x$$

(b) $u_B = u_{AB} + u_A = \frac{F_4 (L_{AB})}{A_4 E} = \frac{3F \cdot L}{(at)E}$ ($u_A \to 0$)

$$\frac{5F}{E}\int \frac{d(x)}{A(x)} = \frac{5F}{Et}\int \frac{d(x)}{a'+b'x}$$

$$\frac{5F}{Et}\left[\frac{1}{b'}\left(\ln(b'L+a') - \ln(a')\right)\right]$$

$$\frac{5F}{Et}\left[\frac{L}{c-b}\left(\ln(c-b+b) - \ln(b)\right)\right]$$

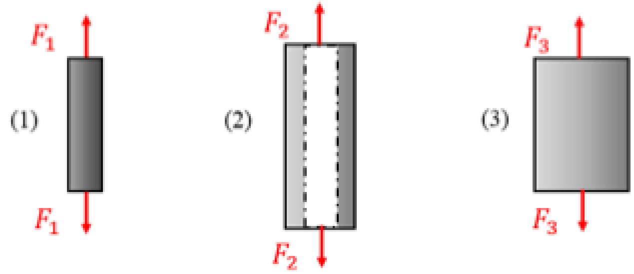
$$\frac{5FL}{E \cdot t(c-b)}\left(\ln\frac{c}{b}\right)$$

$$u_c = u_B + u_{BC} = \frac{3FL}{atE} + \frac{3FL}{btE}$$

$$u_d = u_c + u_{cD} = \frac{3FL}{atE} + \frac{3FL}{btE} + \frac{5FL}{Et(c-b)}\ln\left(\frac{c}{b}\right)$$

Problem 3

a) Let F_1, F_2 and F_3 represent the tensile axial forces carried by rods (1),(2) and (3), respectively.



From the FBD above,

$$\Sigma F_D = F_1 - F_2 - 2W = 0$$

$$\Sigma F_C = F_2 - F_3 - W = 0$$

$$\Rightarrow F_2 = F_3 + W; \qquad F_1 = F_2 + 2W = F_3 + 3W \text{[FBD + Equilibrium: 3pt]}$$

Force-Elongation:

$$e_1 = \frac{F_1 L_1}{E_1 A_1} = \frac{F_1 L}{E\frac{\pi d^2}{4}} = \frac{4F_1 L}{E\pi d^2} \text{[0.5pt]}$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} = \frac{F_2 \frac{3}{2} L}{E\frac{3\pi d^2}{4}} = \frac{2F_2 L}{E\pi d^2} \text{[0.5pt]}$$

$$e_3 = \frac{F_3 L_3}{E_3 A_3} = \frac{F_3 L}{E\frac{9\pi d^2}{4}} = \frac{4F_3 L}{9E\pi d^2} \text{[0.5pt]}$$

Compatibility:

$$u_B = u_H + e_1 + e_2 + e_3 = 0$$

$$=> e_1 + e_2 + e_3 = 0 \text{ [1pt]}$$

Combine this with the equilibrium equation to get:

$$\frac{4F_1L}{E\pi d^2}+\frac{2F_2L}{E\pi d^2}+\frac{4F_3L}{9E\pi d^2}=0 => \frac{4(F_3+3W)L}{E\pi d^2}+\frac{2(F_3+W)L}{E\pi d^2}+\frac{4F_3L}{9E\pi d^2}=0[0.5pt]$$

$$=> F_3=-\frac{63}{29}W$$

$$=> \sigma_3=\frac{F_3}{A_3}=\frac{-\frac{63}{29}W}{\frac{9\pi d^2}{4}}=-\frac{28W}{29\pi d^2}\text{ (in compression)}[1pt]$$

b) The equilibrium equation becomes:

$$\Sigma F_D=F_1-F_2-2W+P=0$$

$$F_1=F_2+2W-P=F_3+3W-P[1pt]$$

Take the compatibility equation:

$$\frac{4F_1L}{E\pi d^2}+\frac{2F_2L}{E\pi d^2}+\frac{4F_3L}{9E\pi d^2}=0 => \frac{4(F_3+3W-P)L}{E\pi d^2}+\frac{2(F_3+W)L}{E\pi d^2}+\frac{4F_3L}{9E\pi d^2}=0$$

$$=> F_3=-\frac{63}{29}W+\frac{18}{29}P$$

$$=> \sigma_3=\frac{F_3}{A_3}=\frac{-\frac{63}{29}W+\frac{18}{29}P}{\frac{9\pi d^2}{4}}[1pt]$$

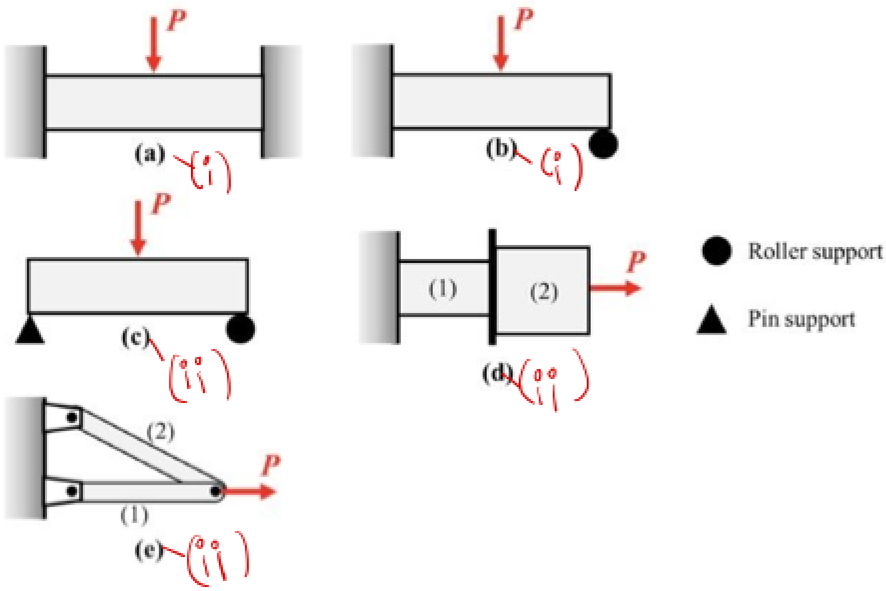
Since that the magnitude of the compressive stress in (3) reduced by 50% from that found in part a):

$$\frac{-\frac{63}{29}W+\frac{18}{29}P}{\frac{9\pi d^2}{4}}=0.5\times\frac{-\frac{63}{29}W}{\frac{9\pi d^2}{4}}$$

$$=> \frac{18}{29}P=0.5\times\left(\frac{63}{29}W\right) => P=\frac{7}{4}W[1pt]$$

Problem 3.4

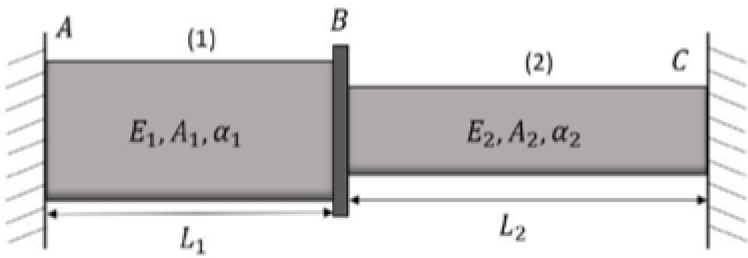
1. Match the following six structures (a)-(e) with correct option given in i-iii.



i. Statically indeterminate structure
ii. Statically determinate structure
iii. Insufficient information

Number of variables vs number of equation

3. Consider a bar made of two sections fixed at both ends. For section (1) let the length be L_1, area A_1, Young's modulus E_1 and coefficient of thermal expansion α_1. The corresponding values for section (2) are L_2, A_2, E_2 and α_2. It is known that $L_1 < L_2$, $E_1 > E_2$, $A_1 > A_2$ and $\alpha_1 > \alpha_2$. The bar is free of stress at temperature T_1. Let σ_1 and σ_2 represent the axial stresses in section 1 and 2, respectively, after the rise in temperature. The temperature is raised from T_1 to T_2 ($T_2 > T_1$).



If δ_1 is the change in length of section 1 and δ_2 is the change in length of section 2, which of the following statements is true?

(a) $\delta_1 = \delta_2 = 0$
(b) $\delta_1 + \delta_2 = 0$
(c) $\delta_1 = \delta_2 \neq 0$
(d) $\delta_1 = \frac{E_1}{E_2}\delta_2$

$\delta_C = \delta_A + \delta_L + \delta_2$

$\delta_C = \delta_A = 0$

(Elongation at the support should be zero)