ME 323: Mechanics of Materials **Homework 4**

Spring 2024 **Due: February 9, 2024**

Problem 1 (10 points):

Figure 1 shows an assembly consisting of an aluminum shell ($E_{Al} = 10.6 \times 10^6$ psi, $\alpha_{Al} = 12.9 \times 10^{-6}/^oF$) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/^oF$). The assembly is unstressed initially. The assembly is now compressed by applying a force P such that its length decreases by 0.01 inch. Considering axial effects only (neglect radial expansion/contraction),

a) Determine the force P
b) Keeping the change in length of the assembly fixed (i.e. length decreases by 0.01 inch from its original value), determine the change in temperature ΔT to the assembly which leads to the axial stress in aluminum to be zero again. (Hint: the applied force P might change accordingly)
c) Upon applying the temperature change in the previous step, determine the resulting axial stress and axial strain in aluminum shell and steel core.

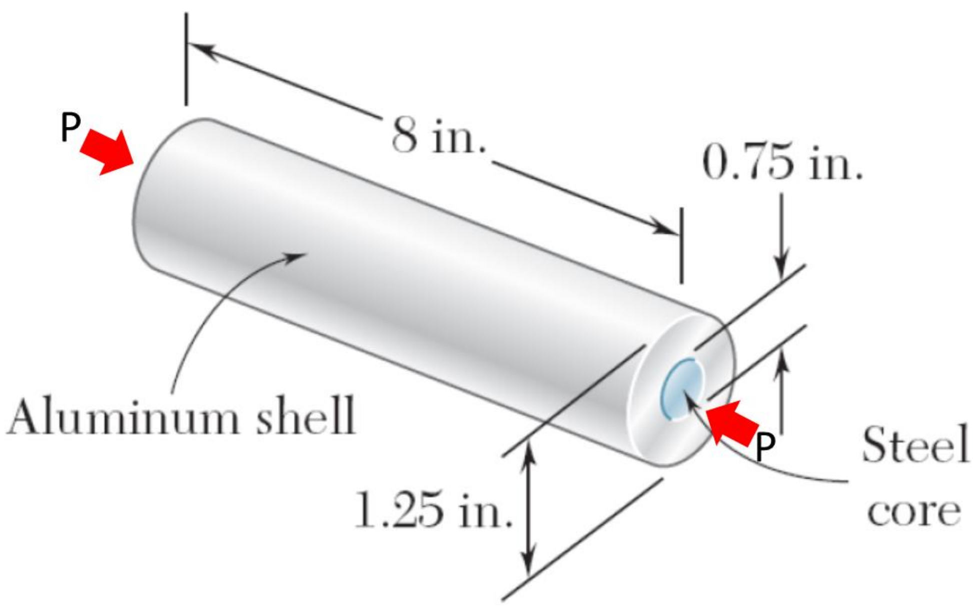
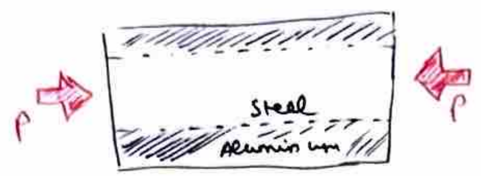
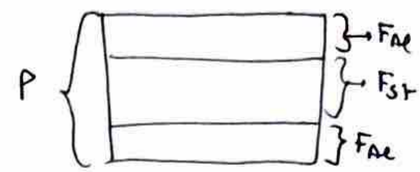


Figure 1: Setup for problem 1

Problem 1





a.) $P = F_{Al} + F_{St}$

$$= \frac{E_{Al} A_{Al} \delta}{L} + \frac{E_{St} A_{St} \delta}{L}$$

$$= \frac{0.01}{8}\left[\left(\frac{\pi}{4} \times (1.25^2 - 0.75^2) \times 10.6\times10^6\right) + \frac{\pi}{4} \times 0.75^2 \times 29\times10^6\right]$$

$$\boxed{P = 26421.285\ N}$$

b.) $\sigma_{al} = 0 = \frac{F_{Al}}{A_{Al}} \Rightarrow F_{Al} = 0$

$$\delta = \frac{F_{Al} L}{A_{Al} E_{Al}} + \alpha_{Al} L \Delta T$$

$$\Delta T = -\frac{0.01}{12.9\times10^{-6} \times 8} = \boxed{-96.899^\circ F}$$

c.) Strains in aluminium shell and stell core:

$$\varepsilon_{St} = \varepsilon_{Al} = \frac{\delta}{L} = -\frac{0.01 in}{8 in} = \boxed{-0.00125}$$

$$\boxed{\sigma_{Al} = 0}$$

$$\delta = e_{St} = \frac{F_{St} L}{A_{St} E_{St}} + \alpha_{St} L \Delta T$$

$$F_{St} = \frac{A_{St} E_{St}}{L}\left(\delta - \alpha_{St} L \Delta T\right)$$

$$\sigma_{St} = \frac{F_{St}}{A_{St}} = \frac{E_{St}}{L}\left(\delta - \alpha_{St} L \Delta T\right)$$

$$= \frac{29\times10^6}{8}\left(-0.01 - 6.5\times10^{-6} \times 8 \times (-96.90)\right)$$

$$\boxed{\sigma_{St} = -17984.35\ psi}$$

Problem 2 (10 points):

A composite shaft is made up of elements 1, 2, and 3 (each having shear modulus G). Elements 2 and 3 are tubular shafts joined together by rigid connector C. Element 1 is a solid core that connects between a fixed wall at B with element 3 at rigid connector D. Element 1 is NOT joined to connector C and passes through a hole in C (refer to *figure 2*). Torques T_0 and $2T_0$ act on D and C respectively. Determine the maximum shear stress in each shaft because of this loading. You may express your answers in fractions using the given variables.

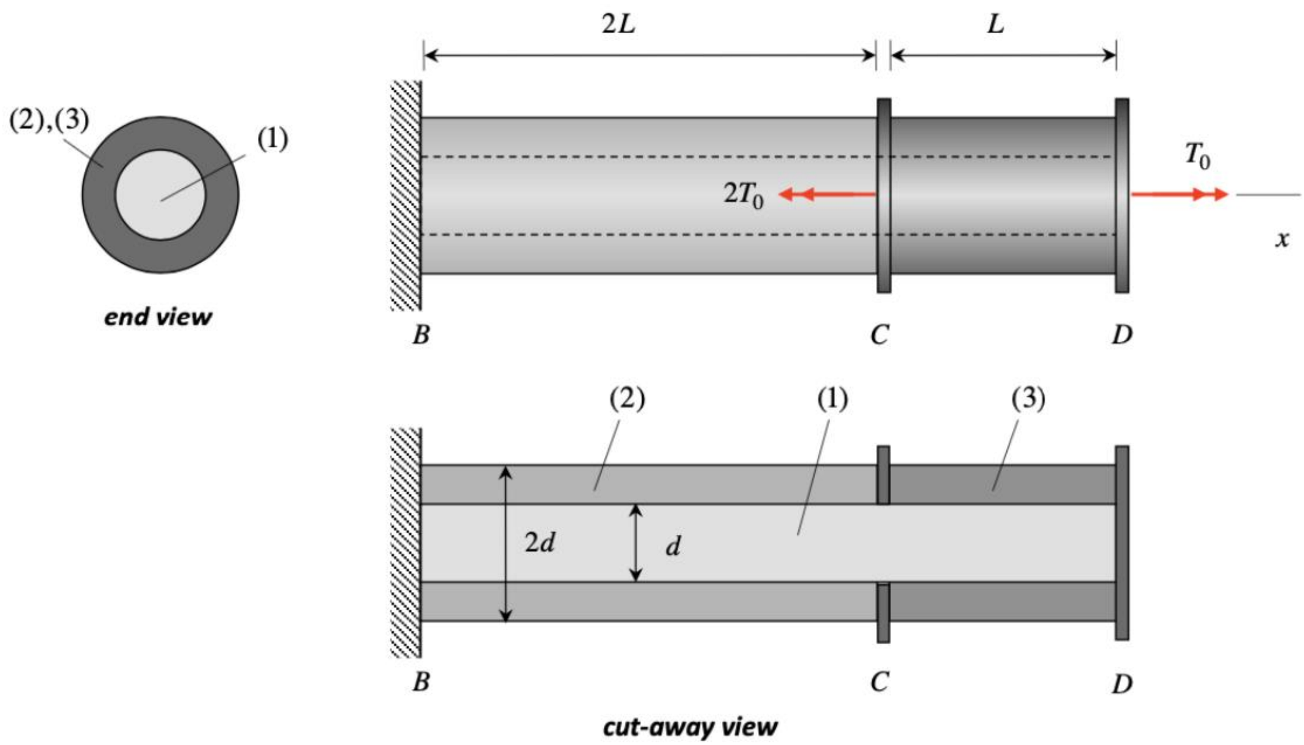
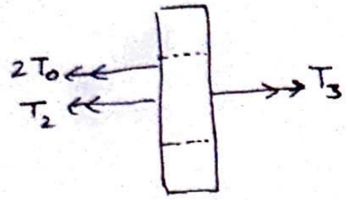


Figure 2: Composite shaft for problem 2

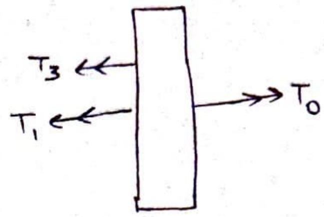
Problem 3 :

FBDs

Connector C



Connector D



Moment equilibrium : $\Sigma M_C = 0 = T_3 - 2T_0 - T_2 = 0$ (1)

$\Sigma M_D = 0 = T_0 - T_3 - T_1 = 0$ (2)

Torque Twist equations :

$$\Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{P_1}} = \frac{3 T_1 L}{G \frac{\pi}{32} d^4} \quad (3)$$

$$\Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{P_2}} = \frac{2 T_2 L}{G \frac{\pi}{32}\{(2d)^4 - d^4\}} \quad (4)$$

$$\Delta\phi_3 = \frac{T_3 L_3}{G_3 I_{P_3}} = \frac{T_3 L}{G \frac{\pi}{32}\{(2d)^4 - d^4\}} \quad (5)$$

Compatibility :

$$\phi_C = \cancel{\phi_B}^{\,0} + \Delta\phi_2 = \Delta\phi_2$$

$$\phi_D = \phi_C + \Delta\phi_3 = \Delta\phi_2 + \Delta\phi_3$$

$$\phi_D = \cancel{\phi_B}^{\,0} + \Delta\phi_1 = \Delta\phi_1$$

$$\Rightarrow \Delta\phi_1 = \Delta\phi_2 + \Delta\phi_3 \quad (6)$$

using (3), (4), (5) $\Rightarrow 45 T_1 = 2T_2 + T_3$ (7)

Solving (1), (2) and (7) $\Rightarrow$ $\boxed{T_1 = \frac{-T_0}{48}}$ $\boxed{T_2 = \frac{-47 T_0}{48}}$ $\boxed{T_3 = \frac{49 T_0}{48}}$

$$|\tau_{max\,1}| = \left|\frac{T_1 \rho_{1\,max}}{I_{P_1}}\right| = \frac{T_0}{3\pi d^3} \quad \text{at } d/2$$

$$|\tau_{max\,2}| = \left|\frac{T_2 \rho_{2\,max}}{I_{P_2}}\right| = \frac{94 T_0}{45\pi d^3} \quad \text{at } d$$

$$|\tau_{max\,3}| = \left|\frac{T_3 \rho_{3\,max}}{I_{P_3}}\right| = \frac{98 T_0}{45\pi d^3} \quad \text{at } d$$

Problem 3 (10 points):

In order to record the rotation of shaft A in digital form, a coder F is connected to it with the help of a gear train as shown in *figure 3*. The gear train consists of 4 gears and 3 solid shafts of diameter d. Two gears have radius r, while the other two have radius nr. Given that the rotation of the coder F is prevented, determine the angle through which end A rotates in terms of T, l, G, I_p, and n.

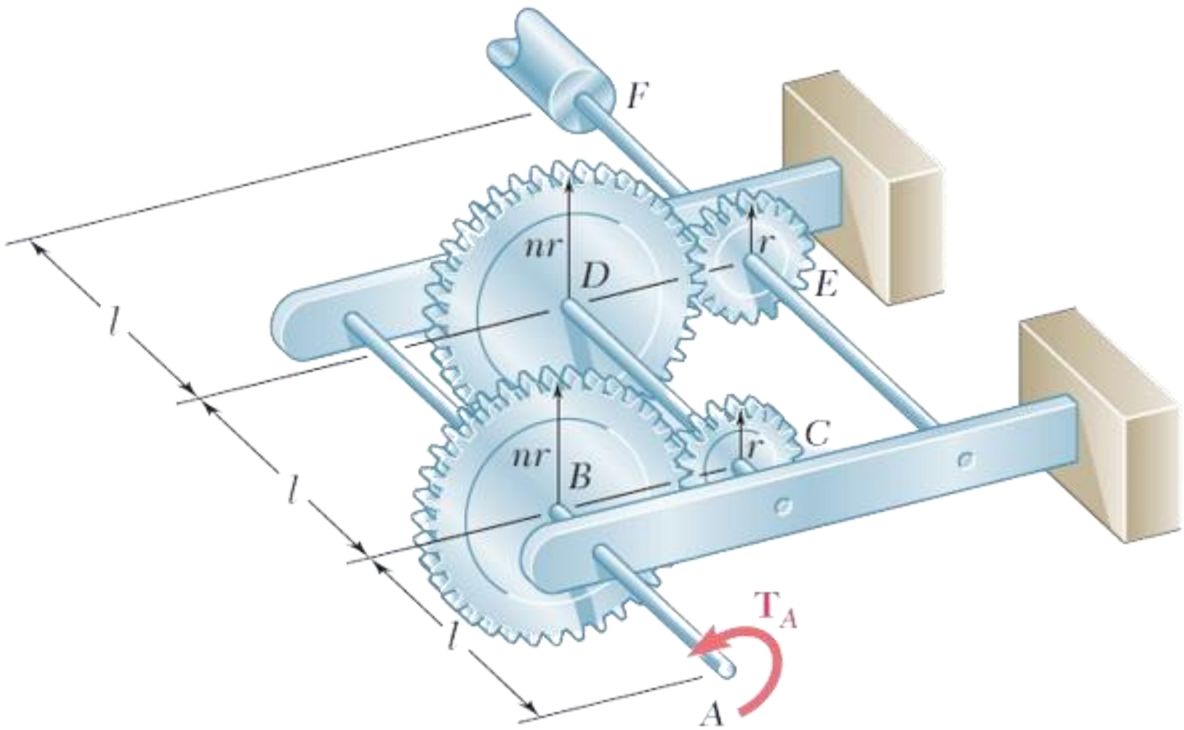
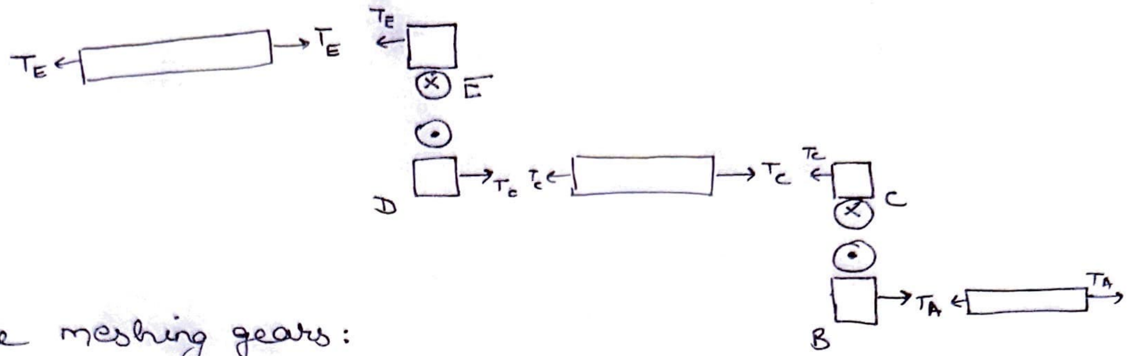


Figure 3: Coder assembly for problem 3.

Problem 3

FBD:



Using the meshing gears:

$$T_C = -\frac{r_C}{r_A} T_A = -\frac{T_A}{n} \qquad (1)$$

$$T_E = -\frac{r_E}{r_C} T_C = +\frac{T_A}{n^2} \qquad (2)$$

$$\phi_B \times n r = -\phi_C \times r$$

$$\phi_B = -\frac{\phi_C}{n} \qquad (3)$$

$$\phi_D \times n r = -\phi_E \times r$$

$$\phi_D = -\frac{\phi_E}{n} \qquad (4)$$

Also,

$$\Delta\phi_{AB} = \phi_A - \phi_B = \frac{T_A l}{G I_p}$$

$$\Delta\phi_{CD} = \phi_C - \phi_D = \frac{T_C l}{G I_p} = -\frac{T_A l}{n G I_p} \quad (\text{using } (1))$$

$$\Delta\phi_{EF} = \phi_E - \phi_F = \frac{T_E l}{G I_p} = \frac{T_A l}{n^2 G I_p} \quad (\text{using } (2))$$

But $\phi_F = 0$ (Given)

$$\Rightarrow \phi_E = \frac{T_A l}{n^2 G I_p}$$

$$\phi_D = -\frac{T_A l}{n^3 G J_p} \quad (\text{using } (4))$$

$$\phi_C = -\frac{T_A l}{n^3 G I_p} - \frac{T_A l}{n G I_p}$$

$$\phi_B = \frac{T_A l}{n^4 G I_p} + \frac{T_A l}{n^2 G I_p} \quad (\text{using } (3))$$

$$\boxed{\phi_A = \frac{T_A l}{G I_p} + \frac{T_A l}{n^2 G I_p} + \frac{T_A l}{n^4 G I_p}}$$

Problem 4 (2.5 + 2.5 points):

For the setup shown in *figure 4*, the rod has $\alpha = 12.5 \times 10^{-6}$ /°C. E = 200 GPa and is strongly fitted between the wall and the spring. If the rod is heated by 20°C,

4.1 The compression in the spring is:

a) 0.125 mm
b) 0.250 mm
c) 0.750 mm
d) 0.600 mm

4.2 The stress induced in the rod is:

a) -0.0992 MPa
b) -0.0796 MPa
c) -0.0397 MPa
d) -0.0636 MPa

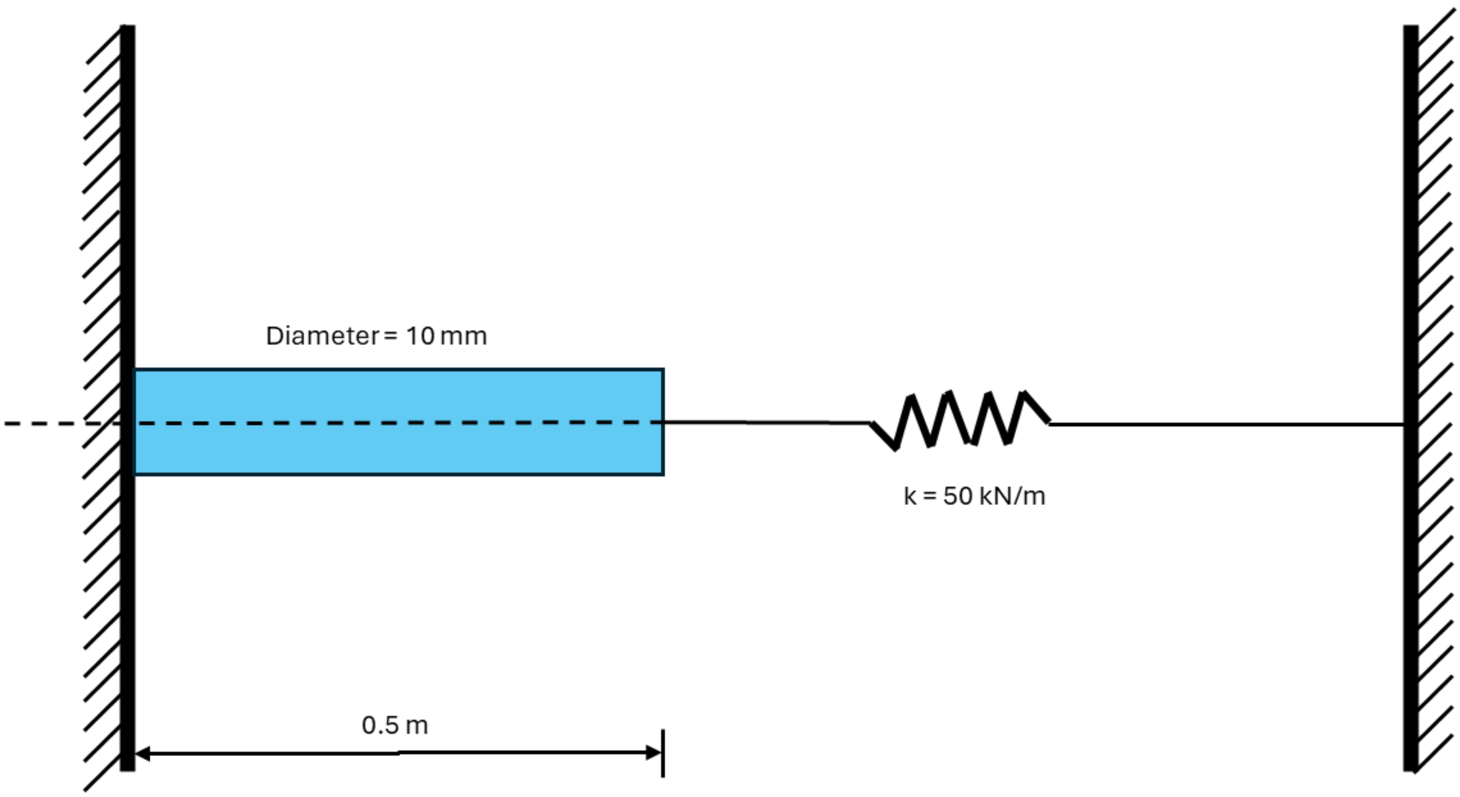


Figure 4: Side view of the setup for Problem 4

Problem4 :

Let the compression of spring be x m

$\therefore$ Spring Force $= F_{sp} = kx$

Compression in length of rod due to compressive Force

$$\delta_1 = \frac{(kx) \times L}{AE}$$

Elongation in length of rod due to temperature change

$$\delta_2 = L \alpha \Delta T$$

$$\delta_2 - \delta_1 = x$$

$$L \alpha \Delta T - \frac{kxL}{AE} = x$$

$$x = \frac{L \alpha \Delta T}{1 + \frac{kL}{AE}} = \frac{0.5 \times 12.5 \times 10^{-6} \times 20}{1 + \frac{50 \times 0.5}{\frac{\pi}{4} \times (0.01)^2 \times 200 \times 10^6}}$$

$$\boxed{x \cong 0.125 \text{ mm}}$$

$$\text{Compressive stress} = -\frac{kx}{A} = -\frac{50 \times 0.125}{\frac{\pi}{4} \times (0.01)^2}$$

$$\sigma = 79577.47 \text{ Pa} \cong \boxed{0.0796 \text{ MPa}}$$