ME 323: Mechanics of Materials **Homework 6**

Spring 2024 **Due: February 23, 2024**

Problem 1 (10 points):

A wide-flange beam with an I-shaped cross section is subjected to three concentrated forces as shown in the *figure 1*.

(a) Construct the shear force V(x) and bending moment M(x) diagrams. Mark the critical values on the diagrams.

(b) Determine the flexural stresses at points A and B.

(c) Determine the shear stress at points A and B.

(d) Draw the stress elements to represent the stress states at points A and B.

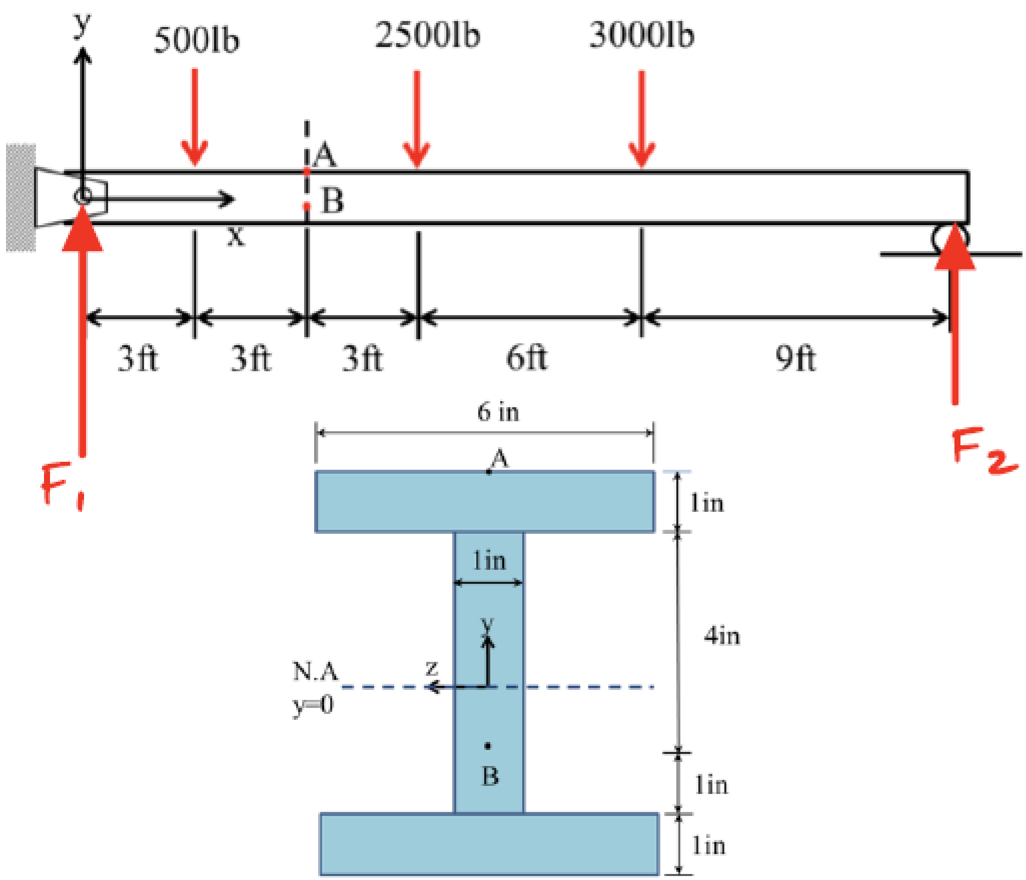
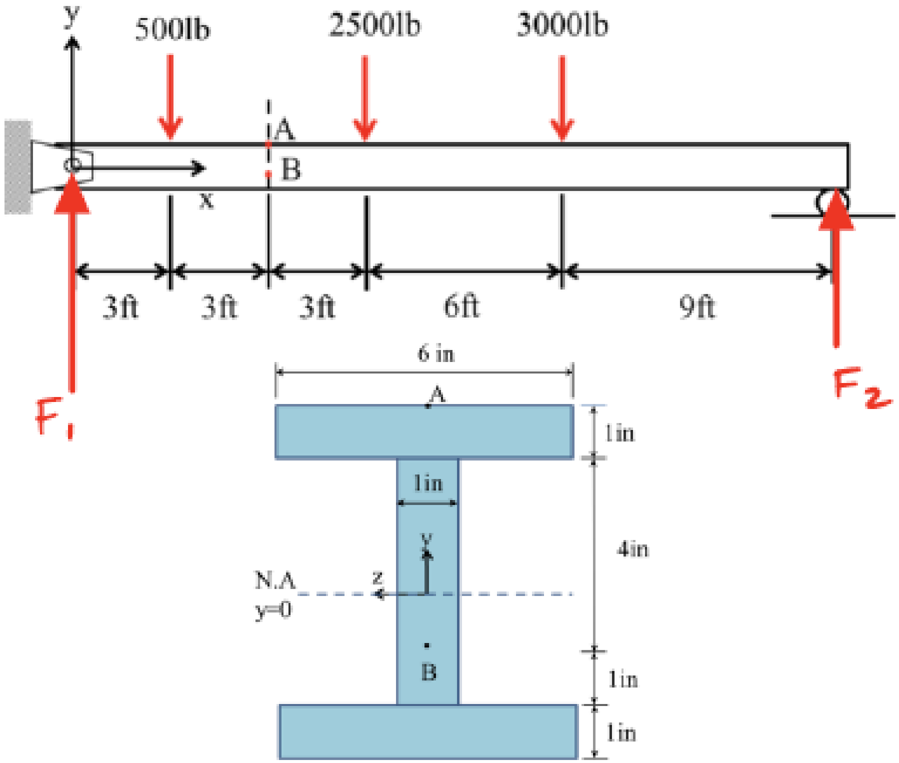


Figure 1: (Top): Wide-flange beam for Problem 1; Bottom: Cross-section of the beam.



(a)

Figure 2: Top: Pinned-Roller support beam Bottom: Cross section of the beam

$\uparrow^{+} \sum F_y = 0$: $F_1 - 500 - 2500 - 3000 + F_2 = 0$

$\circlearrowleft^{+} \sum M_1 = 0$: $-500(3) - 2500(9) - 3000(15) + F_2(24) = 0$

$F_2 = 2875$ lbs

$F_1 = 3125$ lbs

Draw SFD & BMD: [graphically / numerically]

0:

$V(0^+) = F_1 = 3125 \text{ lbs}$

$0 < x < 3$:

$$V(x) = V(0) + \int_0^x p(x)\,dx \quad (p(x) = 0)$$

$$= 3125 \text{ lbs}$$

$x = 3$:

$V(3^-) = 3125$

$V(3^+) = 3125 - 500$

$= 2625 \text{ lbs}$

$3 < x < 9$:

$$V(x) = V(3) + \int_3^x p(x)\,dx \quad (p(x) = 0)$$

$$= 2625 \text{ lbs}$$

$x = 9$:

$V(9^-) = 2625 \text{ lbs}$

$V(9^+) = 2625 - 2500$

$= 125 \text{ lbs}$

$9 < x < 15$:

$M(0^+) = 0 \text{ lb.ft}$.

$$M(x) = M(0) + \int_0^x V(x)\,dx$$

$$= 3125x\Big|_0^x$$

$M(3^-) = M(3) = 9375 \text{ lb.ft}$

$M(3^+) = 9375 \text{ lb.ft}$

$$M(x) = M(3) + \int_3^x V(x)\,dx$$

$$= 9375 + 2625x\Big|_3^x$$

$$= 1500 + 2625x$$

$M(9^-) = M(9^+) = 25125 \text{ lb.ft}$

$$V(x) = V(9) + \int_9^x \cancel{P(x)}^{\,0}\,dx$$

$$= 125 \text{ lbs}$$

$\underline{x = 5}$:

$$V(15^+) = V(15^-) - 3000$$

$$= 2875 \text{ lbs}$$

$\underline{15 < x < 24}$:

$$V(x) = V(15) + \int_{15}^x \cancel{P(x)}^{\,0}\,dx$$

$$= -2875 \text{ lbs}$$

$\underline{x = 24}$:

$$V(24) = -2875 \text{ lbs}$$

$$V(24^+) = -2875 + F_2$$

$$= 0$$

[checked!]

$$M(x) = M(9) + \int_9^x V(x)\,dx$$

$$= 25125 + 125x \Big|_9^x$$

$$= 2400 + 125x$$

$$M(15^-) = M(15^+) = 25875 \text{ lb.ft}$$

$$M(x) = 25875 + \int_{15}^x V(x)\,dx$$

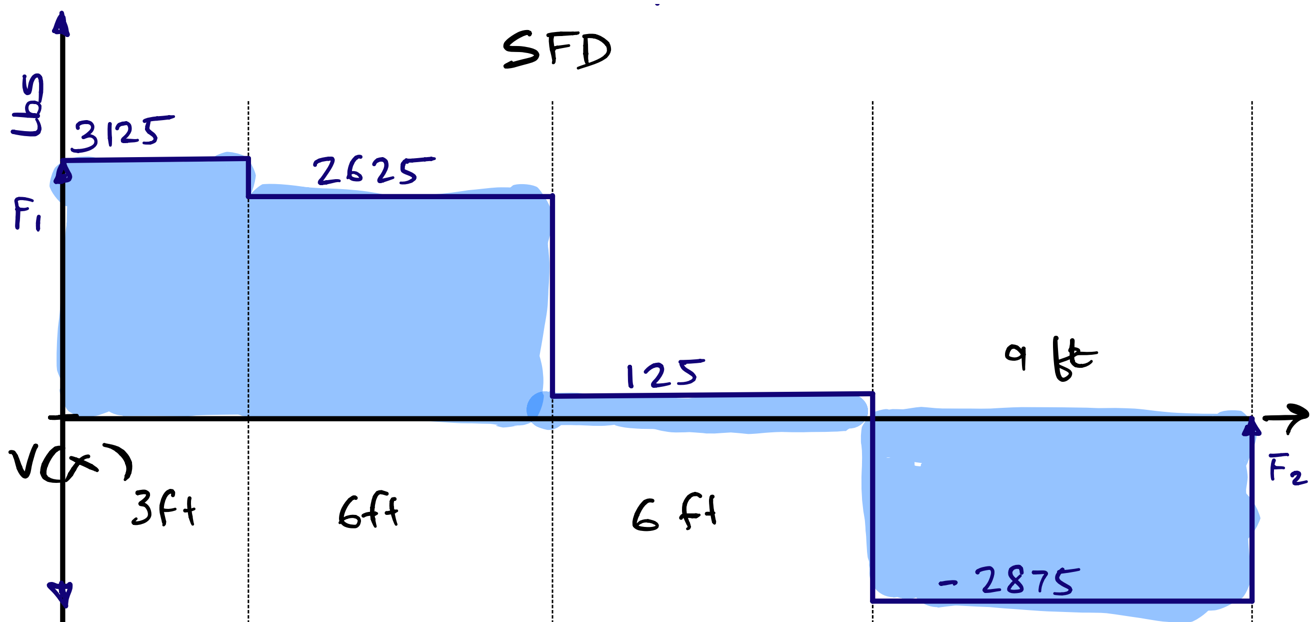
$$= 25875 - 2875 \Big|_{15}^x$$

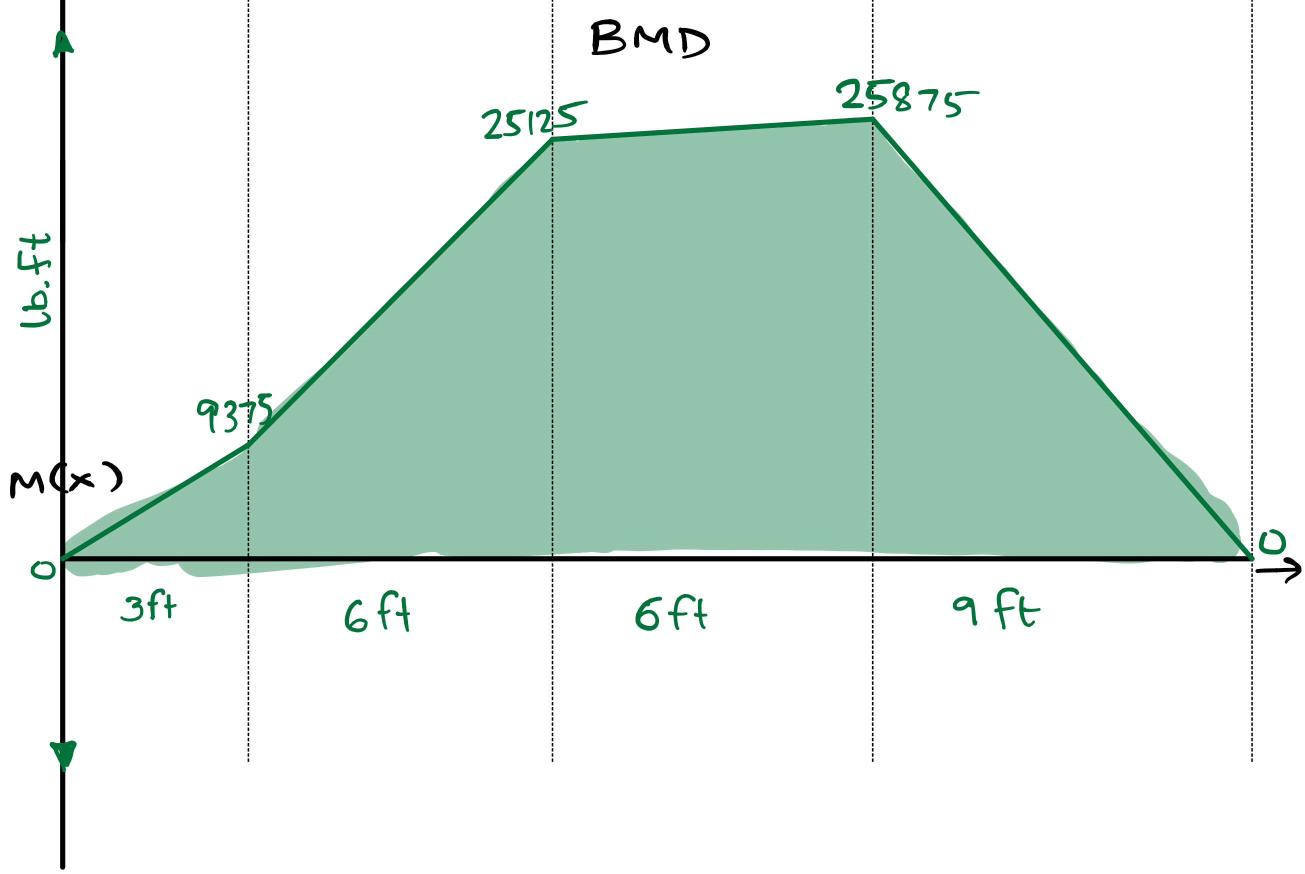
$$= 69000 - 2875x$$

$$M(24^-) = M(24^+)$$

$$= 0 \text{ lb.ft}$$

[checked!]


SFD
Lbs
3125
2625
F1
125
9 ft
F2
V(x)
3ft
6ft
6 ft
-2875


BMD
25125
25875
Lb.ft
9375
M(x)
0
0
3ft
6ft
6ft
9 ft

(b) Flexural stresses at A & B:

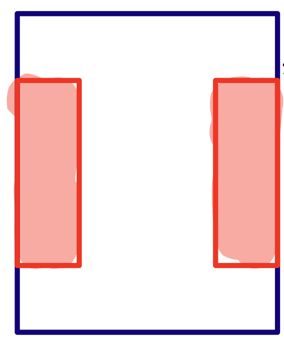
$y(A) = +3.5$ in

$y(B) = -1.5$ in

$$|\sigma_{flex}| = \left|\frac{M y}{I}\right|$$

$$\therefore M(6 \text{ feet}) = M(3) + \int_3^x V(x)\,dx$$

$M(A) = M(B) = 17250$ lb.feet $= 17250 \times 12$ lb.in

$I =$ 

$\Rightarrow I[\text{rectangle}] - 2I[\text{red block}]$

$$\Rightarrow \frac{1}{12}(6)(7)^3 - 2\left[\frac{1}{12}(2.5)(5^3)\right]$$

$\Rightarrow 171.5 - 52.083$

$\Rightarrow 119.42 \text{ in}^4$

$$\therefore \sigma_A = \frac{-M(A)\,y(A)}{I} = -6.06 \text{ ksi}$$

$$\sigma_B = \frac{-M(B)\,y(B)}{I} = 2.6 \text{ ksi}$$

(c) Shear stresses at A & B.

At A, shear stress = 0 [outer most from neutral axis]

B: $\tau_B = \dfrac{V Q_B}{It}$ $= 0.44$ ksi

$V = 2625$ lbs

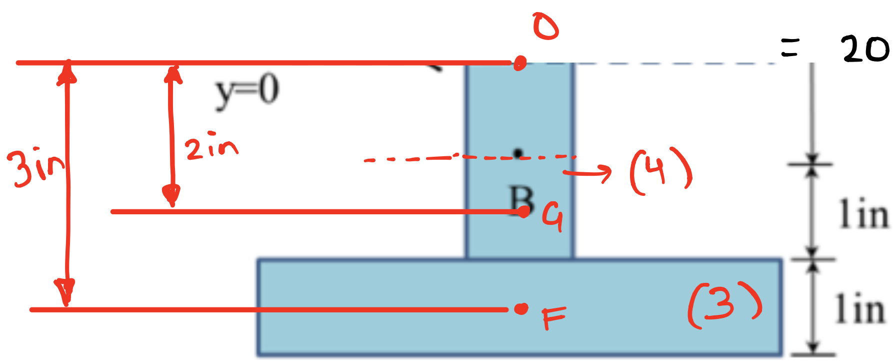
$I = 119.42 \text{ in}^4$

$t = 1$ in

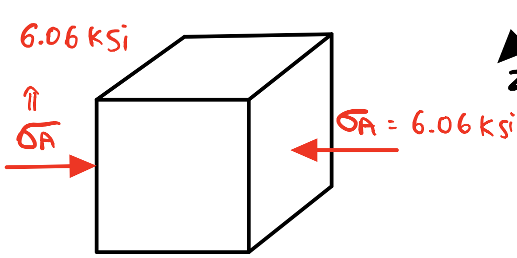
$Q_B = \sum \bar{y}^* A^*$

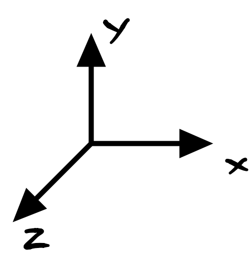
$= A_3 d_{OF} + A_4 d_{OG}$

$= 20 \text{ in}^3$

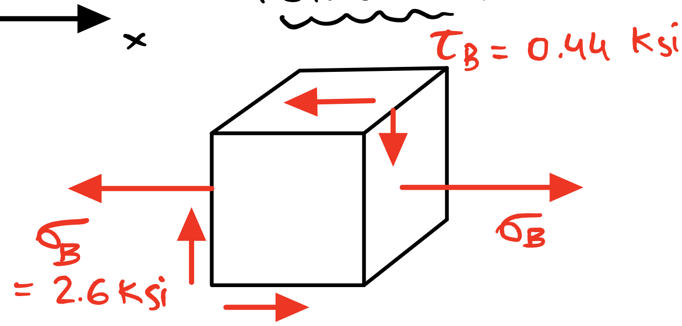


(d) Point A :





Point B :



Problem 2 (10 points):

Shown in *figure 2* is a beam supported by pin joints at A and D. It is acted upon by a line load that increases uniformly from zero at A to B and maintains a constant value of 600 N/m between B and C. It is also acted upon by a concentrated couple at E.

a) Using the integral approach, determine expressions for shear force V(x) and bending moments M(x) between x = 0m to x = 1 m.

b) Construct shear force and bending moment diagrams using expressions determined in (a).

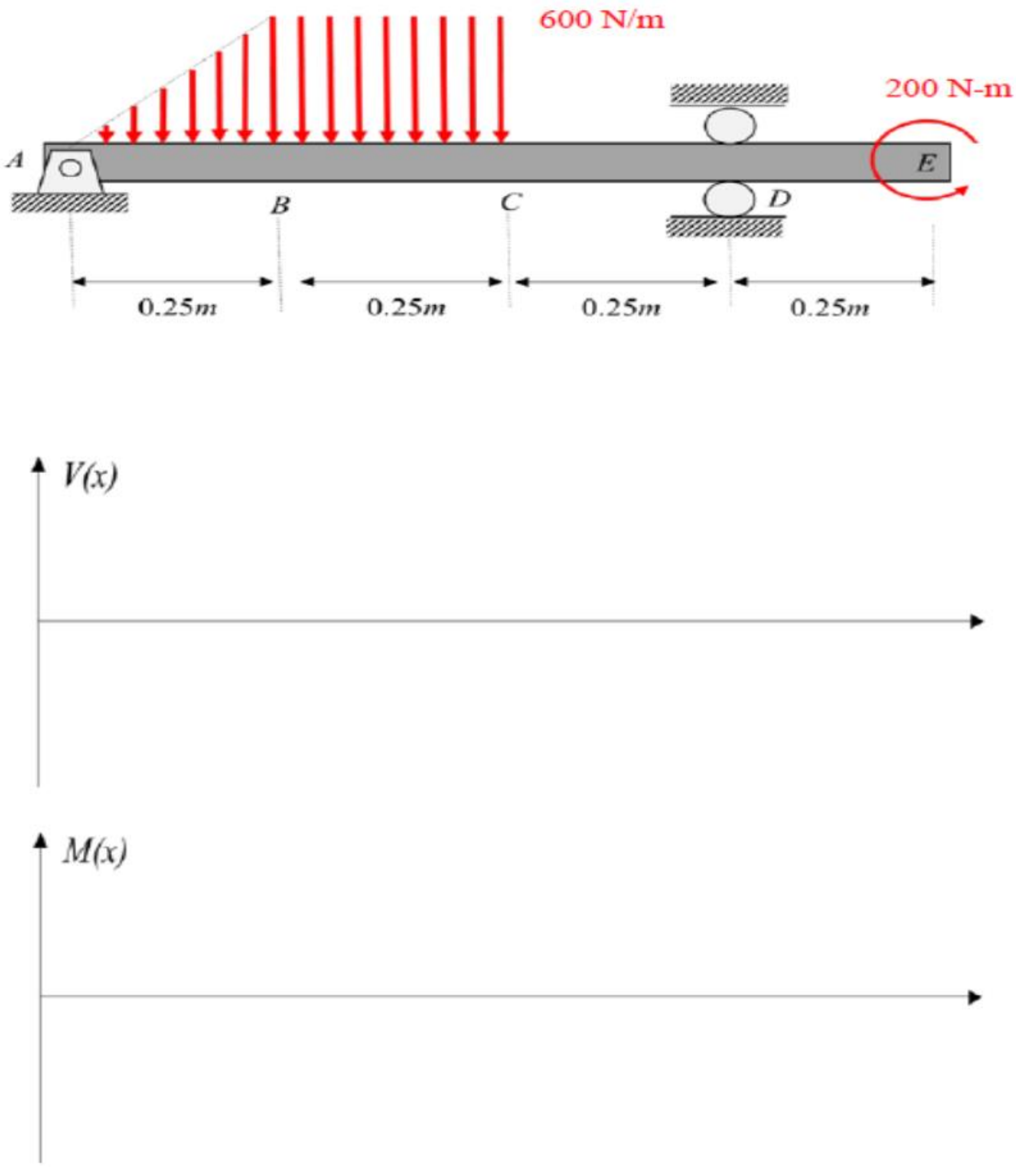
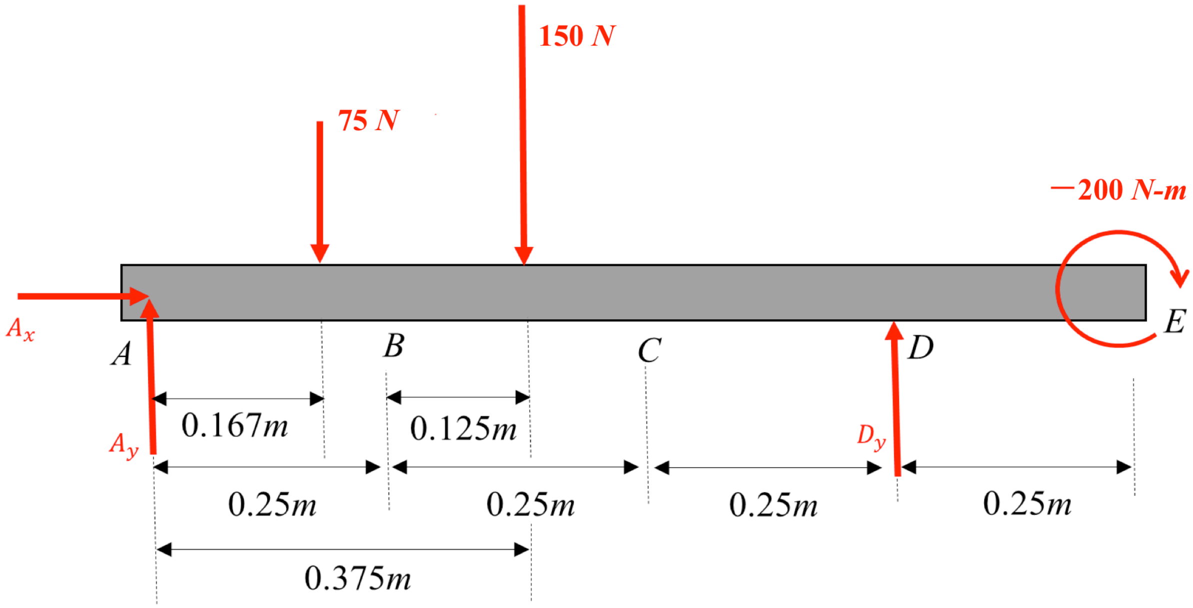


Figure 2: Beam system for Problem 2

Solution:



Equilibrium: (1 *point*)

$\sum M_A = 0 \qquad -(75)(0.167) - (150)(0.375) + D_y(0.75) + 200 = 0$

$$D_y = -175\ N$$

$\sum F_y = 0 \qquad D_y + A_y - 75 - 150 = 0$

$$A_y = 400\ N$$

$\sum F_x = 0 \qquad A_x = 0$

Shear force:

$p(x) = -2400x\ N/m \ \ (0 < x < 0.25m)$

$p(x) = -600\ N/m \ \ \ (0.25 < x < 0.5m)$

$p(x) = 0 \qquad\qquad (0.5m < x < 1m)$

$V(0) = A_y = 400\ N$

$0 < x < 0.25m$: (1 *point*)

$$V(x) = V(0) + \int_0^x p(x)\,dx = 400 + \int_0^x -2400x\,dx = 400 - 1200x^2$$

$V(0.25) = 325\ N$

$0.25\ m < x < 0.5\ m$: $(1\ point)$

$$V(x) = V(0.25) + \int_{0.25}^{x} p(x)\,dx = 325\ + \int_{0.25}^{x} -600\,dx = 475 - 600x$$

$V(0.5) = 175\ N$

$0.5\ m < x < 0.75\ m$: $(1\ point)$

$$V(x) = V(0.5) + \int_{0.5}^{x} p(x)\,dx = 175\ + \int_{0.5}^{x} 0\,dx = 175\ N$$

$V(0.75^{-}) = 175\ N$

$0.75\ m < x < 1\ m$: $(1\ point)$

$V(0.75^{+}) = V(0.75^{+}) + D_y = 175 - 175 = 0\ N$

$$V(x) = V(0.75^{+}) + \int_{0.75}^{x} p(x)\,dx = 0\ + \int_{0.5}^{x} 0\,dx = 0\ N$$

$V(1) = 0\ N$

Bending moment:

$M(0) = 0\ Nm$

$0 < x < 0.25m$: $(1\ point)$

$$M(x) = M(0) + \int_{0}^{x} V(x)\,dx = 0\ + \int_{0}^{x} 400 - 1200x^2\,dx = -400x^3 + 400x$$

$M(0.25) = 93.8Nm$

$0.25m < x < 0.5m$: $(1\ point)$

$$M(x) = M(0.25) + \int_{0.25}^{x} V(x)\,dx = 93.8 + \int_{0.25}^{x} 475 - 600x\,dx = -300x^2 + 475x - 6.2$$

$M(0.5) = 156.3\ Nm$

$0.5m < x < 0.75m$: $(1\ point)$

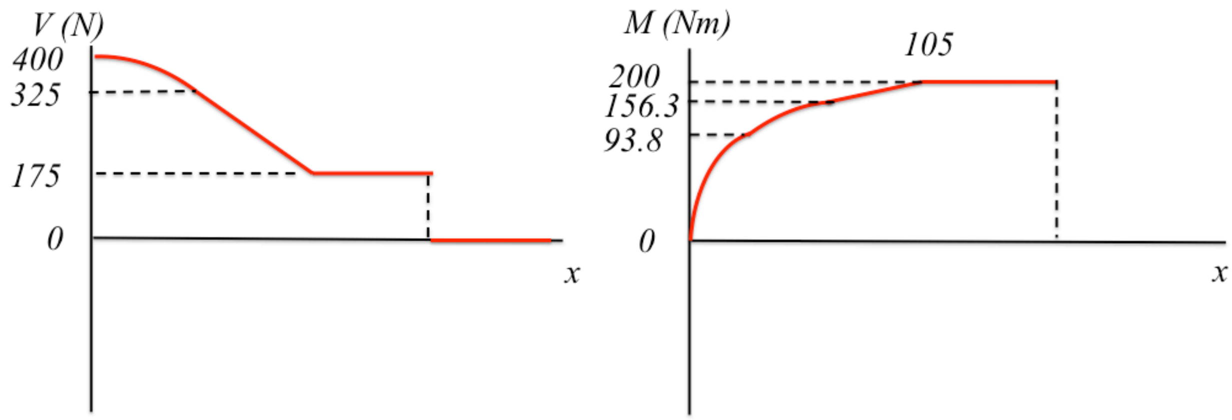
$$M(x) = M(0.5) + \int_{0.5}^{x} V(x)\,dx = 156.3\ + \int_{0.5}^{x} 175\ \ dx = 68.8 + 175x$$

$M(0.75) = 200\ Nm$

$0.75m < x < 1m$: (1 *point*)

$$M(x) = M(0.75) + \int_{0.75}^{x} V(x)\,dx = 200 + \int_{0.75}^{x} 0\;dx = 200$$

$M(1) = 200\;Nm$



(1 *point*)

Problem 3 (10 points):

For the loading shown in *figure 3*, determine the shearing stress on cross-section n-n at i) point a, ii) point b.

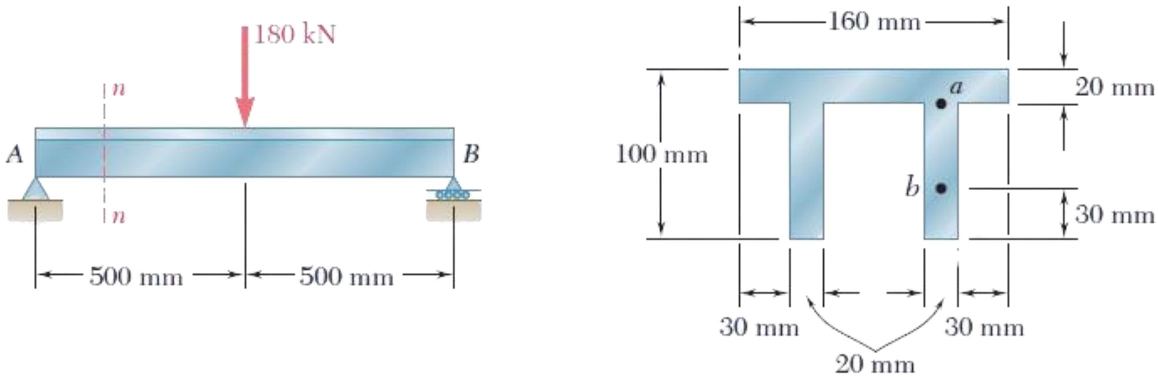
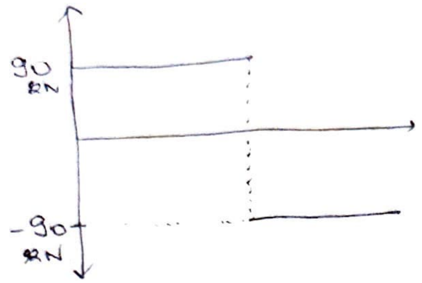


Figure 3: Left: Loading on the beam; Right: Cross section view n-n

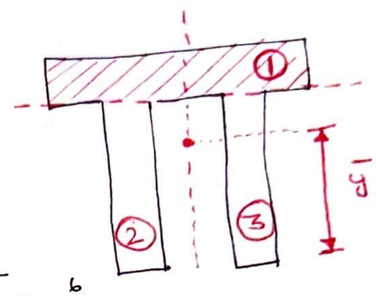
Problem 3

Shear force diagram:



$R_A = R_B = 90$ kN [Reactions at A and B]

Now consider

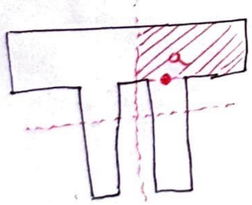


Part	A	$\bar{y}$	$A\bar{y}$	d	Ad^2	$\bar{I}$
1	3200 mm^2	90 mm	288000 mm^3	25 mm	2×10^6	0.1067×10^6
2	1600 mm^2	40 mm	64000 mm^3	25 mm	1×10^6	0.8533×10^6
3	1600 mm^2	40 mm	64000 mm^3	25 mm	1×10^6	0.8533×10^6

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{416\times10^3}{6400}\text{ mm} = 65\text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma\bar{I} = (4 + 1.8133)\times10^6\text{ mm}^4$$
$$= 5.8133\times10^{-6}\text{ m}^4$$

a.)



$A = 80\times20 = 1600\text{ mm}^2$

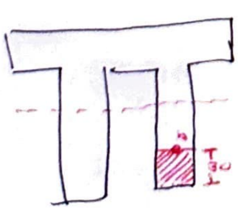
$y^* = 25\text{ mm}$

$Q_a = Ay^* = 40\times10^{-6}\text{ m}^3$

$$\tau_a = \frac{VQ_a}{It} = \frac{(90\times10^3)(40\times10^{-6})}{5.8133\times10^{-6}\times20\times10^{-3}}$$

$\boxed{\tau_a = 31\text{ MPa}}$

b.)



$A = 30\times20 = 600\text{ mm}^2$

$y^* = 65-15 = 50\text{ mm}$

$Q_b = Ay^* = 30\times10^{-6}\text{ m}^2$

$$\tau_b = \frac{VQ_b}{It} = \frac{90\times10^3\times30\times10^{-6}}{5.8133\times10^{-6}\times20\times10^{-3}}$$

$\boxed{\tau_b = 23.2\text{ MPa}}$

Problem 4 (2.5 + 2.5 points):

Identify the schematic that represents the deflection curve of the following beams:

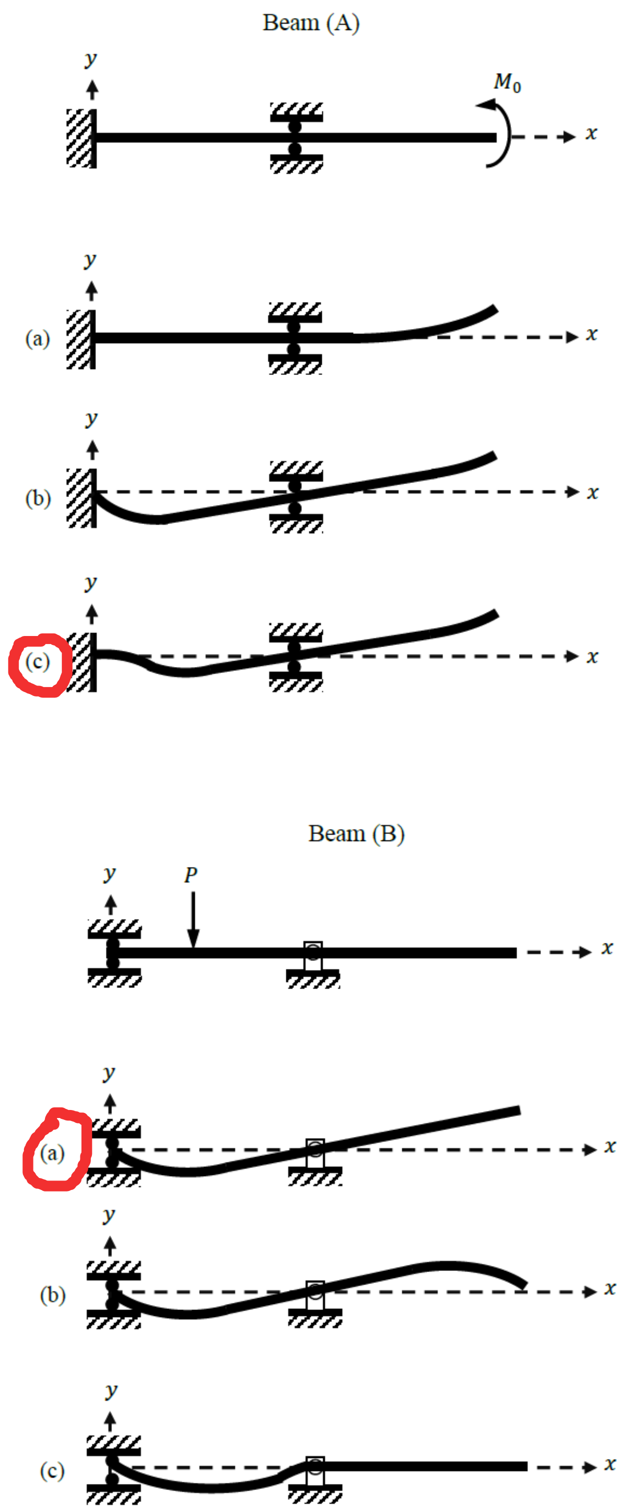


Figure 4: Top: Beam A; Bottom: Beam B.