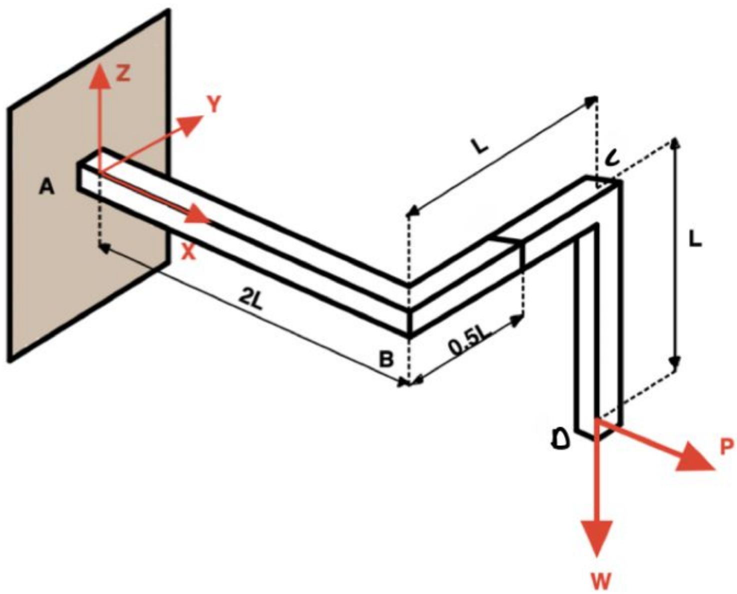
Problem 1.1 *(10 points)*

ABCDE represents a frame attached to the wall at A, with the free end (E) subjected to two forces: 1) W acting in the negative Z direction and 2) P acting in the positive X direction.

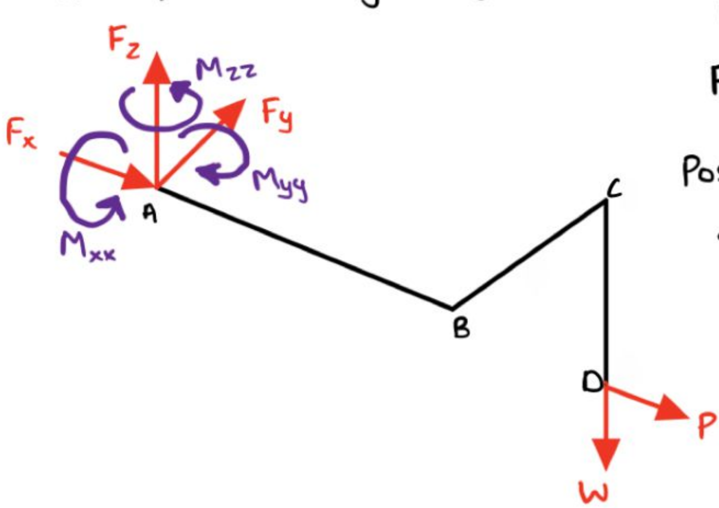
(a) Calculate the internal resultants, i.e, forces and moments developed at the centroid of the cross sections-

- A, and
- B

(b) Does the internal resultant force vary as you along the line segment AB? What about along line segment BC?



ⓐ • At A, Free body diagram:



Force vector,

$F = P\hat{i} + 0\hat{j} - W\hat{k}$

Position vector,

$r_{AD} = 2L\hat{i} + L\hat{j} - L\hat{k}$

Force equilibrium eq, $\Sigma F = 0$

i.e,

$$\Rightarrow (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) + F = 0$$

$$\Rightarrow (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) + P\hat{i} + 0\hat{j} - W\hat{k} = 0$$

$\Rightarrow$

$$F_x = -P$$

$$F_y = 0$$

$$F_z = W$$

Moment Equilibrium eq, $\sum M_A = 0$

i.e,

$$\Rightarrow (M_{xx} \hat{i} + M_{yy} \hat{j} + M_{zz} \hat{k}) + r_{AD} \times F = 0$$

$$\Rightarrow (M_{xx} \hat{i} + M_{yy} \hat{j} + M_{zz} \hat{k}) + (2L\hat{i} + L\hat{j} - L\hat{k}) \times (P\hat{i} - W\hat{k}) = 0$$

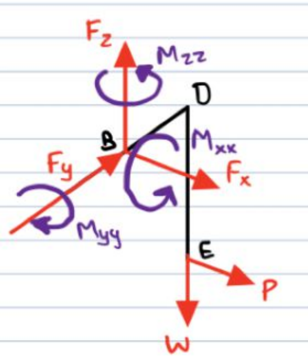
$$\Rightarrow (M_{xx} \hat{i} + M_{yy} \hat{j} + M_{zz} \hat{k}) + (-WL\hat{i} + (2WL - PL)\hat{j} - PL\hat{k}) = 0$$

$$M_{xx} = WL$$

$$M_{yy} = PL - 2WL$$

$$M_{zz} = PL$$

• At B, <u>Free body diagram</u>:



FORCE VECTOR,

$$F = P\hat{i} + 0\hat{j} - W\hat{k}$$

POSITION VECTOR,

$$r_{BD} = 0\hat{i} + L\hat{j} - L\hat{k}$$

Force equilibrium eq, $\Sigma F = 0$

i.e,

$$\Rightarrow (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) + F = 0$$

$$\Rightarrow (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}) + P\hat{i} + 0\hat{j} - W\hat{k} = 0$$

$$\Rightarrow \quad F_x = -P$$

$$F_y = 0$$

$$F_z = W$$

Moment Equilibrium eq, $\sum M_c = 0$

i.e,

$$\Rightarrow (M_{xx}\hat{i} + M_{yy}\hat{j} + M_{zz}\hat{k}) + r_{BD} \times F = 0$$

$$\Rightarrow (M_{xx}\hat{i} + M_{yy}\hat{j} + M_{zz}\hat{k}) + (0\hat{i} + L\hat{j} - L\hat{k}) \times (P\hat{i} - W\hat{k}) = 0$$

$$\Rightarrow (M_{xx}\hat{i} + M_{yy}\hat{j} + M_{zz}\hat{k}) + (-WL\hat{i} + (-PL)\hat{j} - PL\hat{k}) = 0$$

$$M_{xx} = WL$$

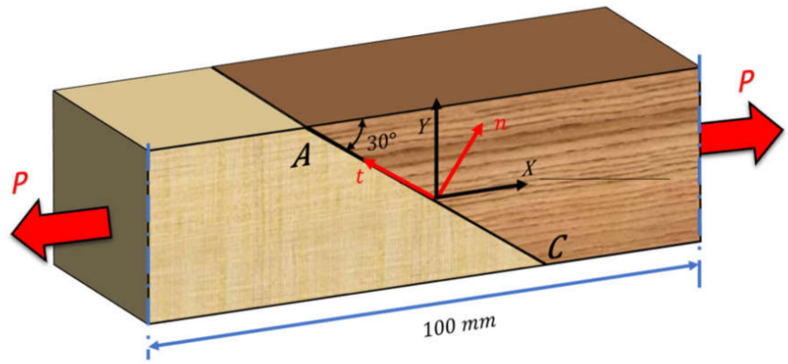
$$M_{yy} = PL$$

$$M_{zz} = PL$$

(b) No, it does not vary along line segment AB or along BC

Problem 1.2 *(10 points)*

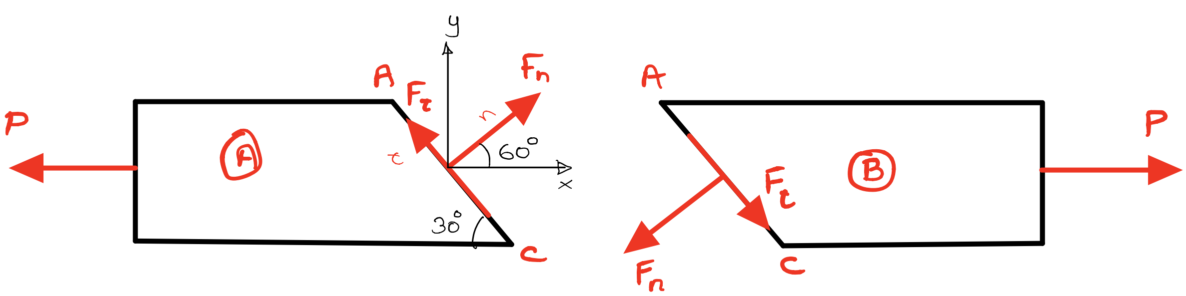
Two wooden blocks of identical square cross section (10 mm x 10 mm) are glued together along the plane AC as shown in the figure below. The dimension of the entire setup is 10 mm x 10 mm x 100 mm.

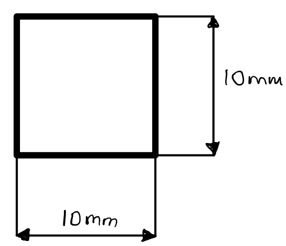


(a) Assuming that the glue can withstand a maximum shear stress of 10,000 Pa, calculate the maximum force P that can be applied at the end of the wooden blocks.

(b) A new glue was tested which can withstand a maximum normal stress of 5,000 Pa and shear stress of 3500 Pa. What is the maximum load that can be applied?

Free body diagram:





C.S Area = 100 mm^2

Considering FBD of A:

$$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow F_n \cos 60^\circ - F_t \sin 60^\circ - P = 0 \longrightarrow (1)$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow F_n \sin 60^\circ + F_t \cos 60^\circ = 0 \longrightarrow (2)$$

$$F_n = P \cos 60^\circ = 0.5P$$

$$F_t = -P \sin 60^\circ = -0.866P \quad \text{[opp. direction]}$$

C.S Area at the Intersection [at AC] => AC * 10mm

Now, $\sin 30° = \frac{10mm}{AC}$ $\Rightarrow AC = \frac{10mm}{1/2} = 20mm$

$\therefore$ C.S Area at AC = $20mm * 10mm$

$A_{AC} = 200mm^2$.

Normal Strength [Maximum]: $\sigma_n = \frac{F_n}{A_{AC}} = \frac{0.5P}{200}$

$\sigma_n = 0.0025P \; N/mm^2$

Shear Strength [Maximum]: $\tau = \frac{|F_t|}{A_{AC}} = \frac{0.866P}{200}$

$\tau = 0.00433P \; N/mm^2$

(a) $\tau_{max} = 10{,}000 \; Pa \; [10^{-2} N/mm^2]$

then

$0.00433P = 10^{-2}$

$P = 2.3095 \; N$

(b) $\sigma_{max} = 5 \times 10^{3} \; N/mm^2$

$0.0025P = 5 \times 10^{-3}$

$P = 2N$

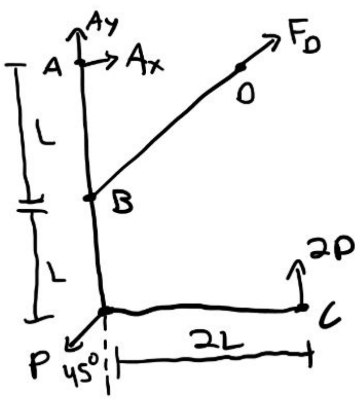
$\tau_{max} = 3.5 \times 10^{-3} \; N/mm^2$

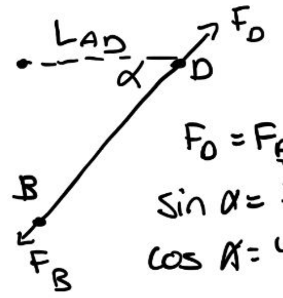
$0.00433P = 3.5 \times 10^{-3}$

$P = 0.8083 \; N$

The maximum value of P that allows for a conservative design would be $P = 0.8083 \; N$. Any load greater than that would result in failure due to shear.

Problem 3:

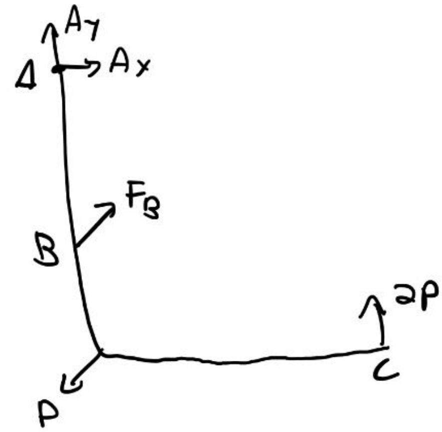




$F_D = F_B$

$\sin \alpha = 3/5$

$\cos \alpha = 4/5$



$\tan \alpha = \frac{3}{4} = \frac{L}{L_{AD}}$

$L_{AD} = \frac{4}{3} L$

FBD 1: $\Sigma M_A \circlearrowleft^{+} = 0 = 2P(2L) - P\sin 45 (2L) + F_D \sin \alpha \, L_{AD} = 0$

$$F_D \sin\alpha \left(\tfrac{4}{3}\cancel{L}\right) = P\sin(45)(2\cancel{L}) - 2P(2\cancel{L})$$

$$F_D = \tfrac{5}{4}\left(\tfrac{1}{\sqrt{2}} \cdot 2 \cdot P - 4P\right) = -3.232P$$

FBD 2: $|F_B| = |F_D| = 3.232P$ $\quad F_B = 3.232P$

FBD 3: $\Sigma F_x = 0 = A_x - F_B \cos\alpha - P\sin(45°)$

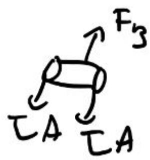
$$A_x = P\left(\tfrac{1}{\sqrt{2}}\right) + 3.232P\left(\tfrac{4}{5}\right) = 3.293P$$

$$\Sigma F_y = 0 = A_y + 2P - F_B \sin\alpha - P\cos(45°)$$

$$A_y = 3.232P\left(\tfrac{3}{5}\right) + P\left(\tfrac{1}{\sqrt{2}}\right) - 2P = 0.646P$$

Shear forces on pins
- B: $V_B = |F_B| = 3.232P$
- D: $V_D = \uparrow = 3.232P$
- A: $V_A = \sqrt{(3.293)^2 + (.646)^2}\, P = 3.356P$

Pins B/D:



$\Sigma F = 0 = F_B = 2\tau A = 3.232P = 2\tau A$

$P_{B,D} = 0.619\,\tau A$

Pin A:

$\Sigma F = 0 = F_A = \tau A = 3.356P = \tau A$

$P_A = 0.298\,\tau A$

max P w/o failure: $0.298\,\tau A$

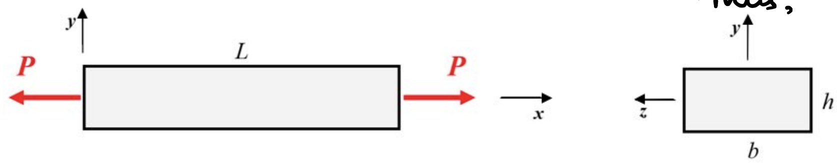
Problem 1.4 Conceptual *(2.5+2.5 points)*

From def. of strain $\boxed{\begin{matrix}\varepsilon_y = \nu\,\varepsilon_x \\ \varepsilon_z = \nu\,\varepsilon_x\end{matrix}}$

(a) The rod shown below has a rectangular cross section and is made of a material with Young's modulus E and positive Poisson's ratio ν. It has dimensions $\boxed{L > b > h}$ and it is subjected to a tensile load P in the x direction, as shown.

thus, $\varepsilon_y = \varepsilon_z$

$\frac{\Delta h}{h} = \frac{\Delta b}{b}$, then



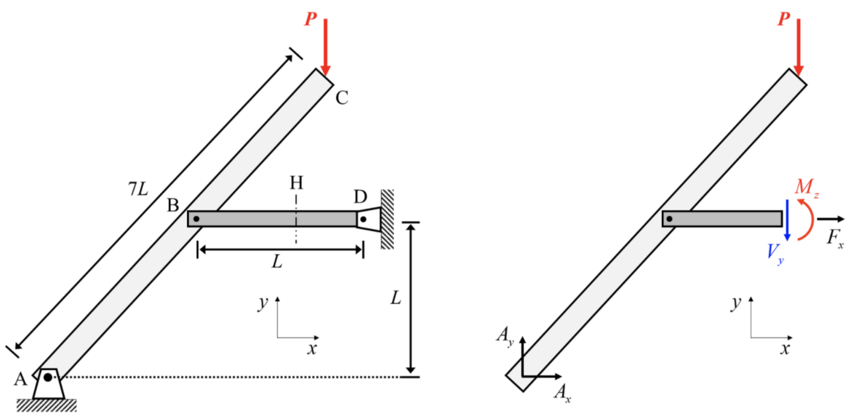
Select the correct statement. And justify your stance.
(a) $|\Delta h| > |\Delta b|$
(b) $|\Delta h| < |\Delta b|$
(c) $|\Delta h| = |\Delta b|$
(d) Insufficient information. Depends on the value of Poisson's ratio.

$\Delta h = \frac{h}{b}\Delta b$ if $h < b$

$\frac{h}{b} < 1$ then

$\Rightarrow \boxed{\Delta h < \Delta b}$

(b) Bar ABC is supported by a pin joint at point A and loaded by a vertical force P at point C. Bar ABC is also connected to bar BD via a pin joint at B which in turn is supported by a pin joint at D. Bar BD is oriented along the x axis.



Select the zero internal resultant(s). And justify your stance.
(a) Normal force F_x
(b) Shear force V_y
(c) Bending moment M_z
(d) All internal resultants are non zero

BD is a two force member, pinned at B and D, but free to rotate.

[i.e, The only non zero internal resultant → F_x]

Thus, (b) and (c) are zero