ME 323: Mechanics of Materials **Homework 2**

Spring 2024 **Due: January 26, 2024**

Problem 1 (10 points):

Dwight Schrute from *The Office US* is attempting his tightrope stunt again and wants to present a spectacle (*Figure 1.1 left*). He has modified the high-wire design in a Mechanics textbook and plans to make it taller. His design is as described in *figure 1.2*. The high-wire is attached to a rigid vertical beam AC and is kept taut by a tensioner cable BD. At C, the beam is attached by a 20 mm diameter bolt to the bracket (also shown in the figure). The tension in the wire is 7 kN. The maximum allowable shear stress in the bolt C is 31.25 MPa. As his friend, you are asked to help him figure out the maximum length L that he can consider. Assume the high-wire to be horizontal and neglect the weight of AC.

[**Note**: Please do not try this at home]

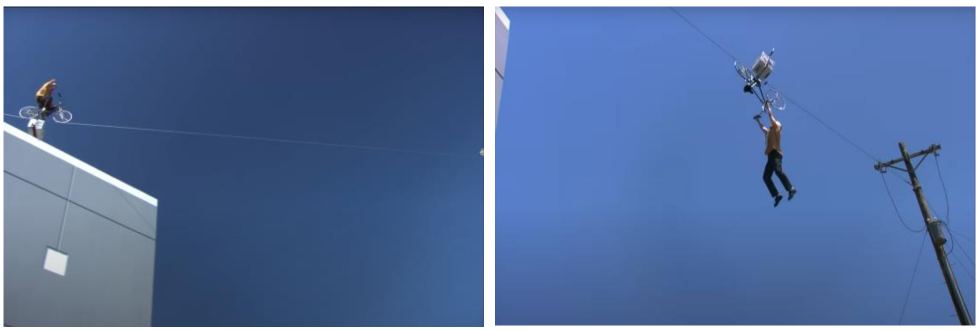


Figure 1.1: *Left:* Dwight's tightrope stunt; *Right:* His failed attempt.

[Image courtesy: PeacockTV]

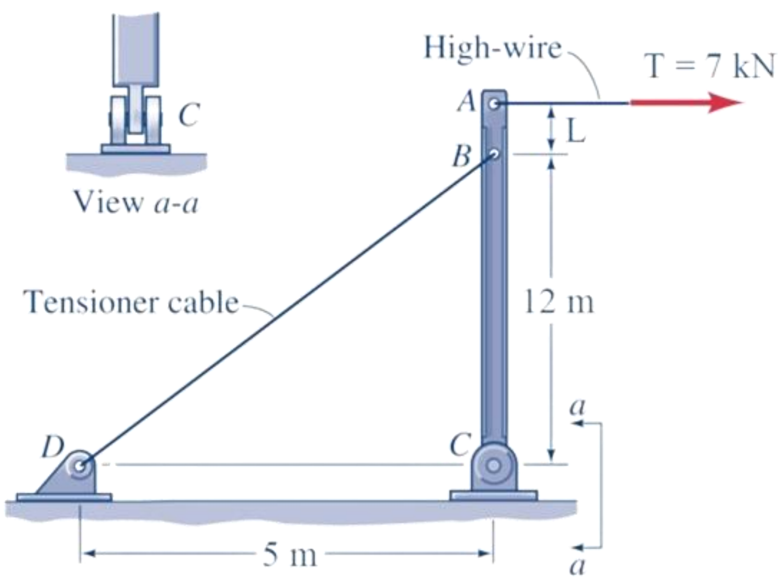
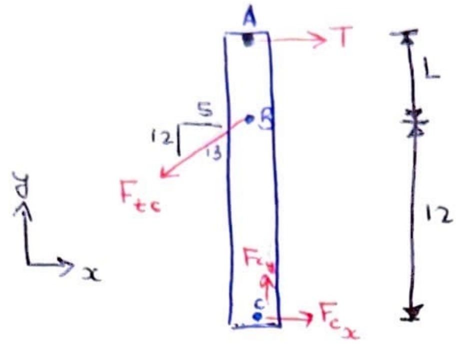


Figure 1.2: Design of the high-wire setup.

Solution 1 :

FBD :



$T = 7\,kN$

$\tau_{all_c} = 31.25 MPa$

- Sum of moments about B :

$$\Sigma M_B = 0 = -T(L) + F_{cx}(12) \qquad (1)$$

- Sum of forces in x and y direction :

$$\Sigma F_x = 0 = T + F_{cx} - \frac{5}{13} F_{tc} \qquad (2)$$

$$\Sigma F_y = 0 = F_{cy} - \frac{12}{13} F_{tc} \qquad (3)$$

- Now, the bolt C is in double shear

$$\therefore A = 2\left(\pi \frac{[0.02]^2}{4}\right) m^2 = 6.283 \times 10^{-4} m^2$$

- $$\tau_{all_c} = \frac{F_c}{A} = \frac{\sqrt{(F_{cx})^2 + (F_{cy})^2}}{A}$$

Using (2) and (3)

$$31.25 \text{ MPa} = \frac{\sqrt{\left(\frac{5}{13}F_{tc} - 7000\right)^2 + \left(\frac{12}{13}F_{tc}\right)^2}}{6.283 \times 10^{-4}}$$

$\boxed{F_{tc} = 21233 \text{ N}}$ $\qquad F_{tc} = -15848 \text{ N}$

neglected because cable comes in Compression

Using (2) $\Rightarrow$ $\boxed{F_{cx} \cong 1.17 \text{ kPa}}$

using (1) $\Rightarrow$ $\boxed{L \cong 2m}$

Problem 2 (10 points):

Links BC and DE are connected to a rigid member ACD as shown in *figure 2*. A load P is applied to the structure at A. Each pin has a diameter of 10 mm and experiences single shear. The material strength of the pins is $\tau = 250$ MPa and the material strength in tension of the links is $\sigma = 500$ MPa. Both BC and DE have a cross-sectional area of 125 mm^2. Calculate the maximum P so that the structure does not fail.

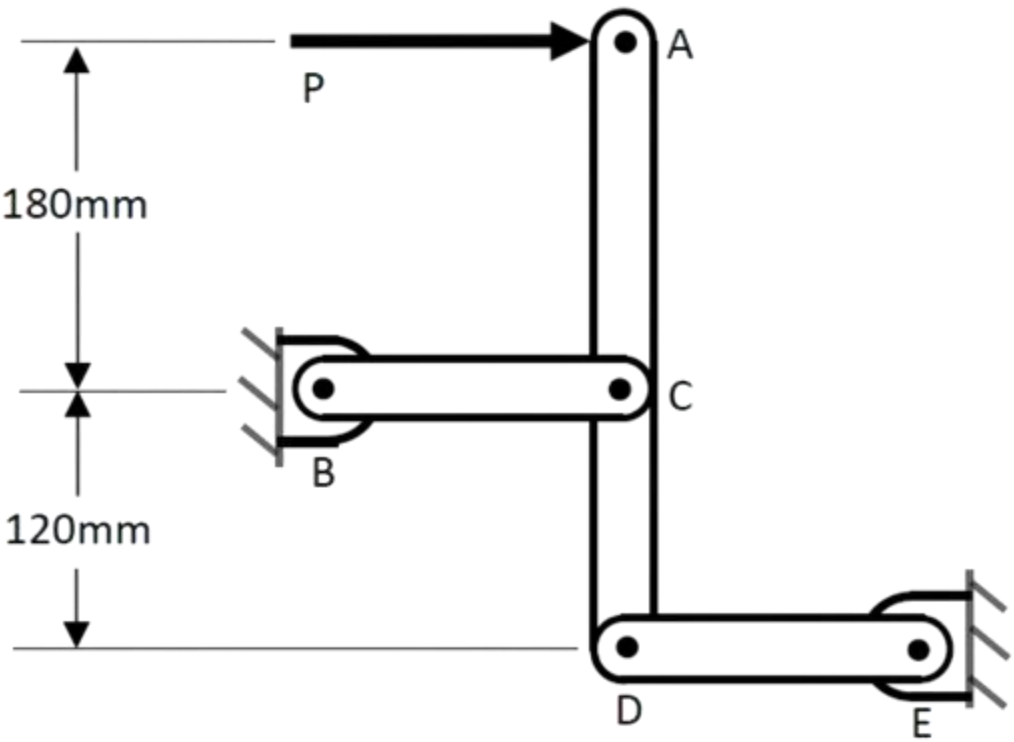
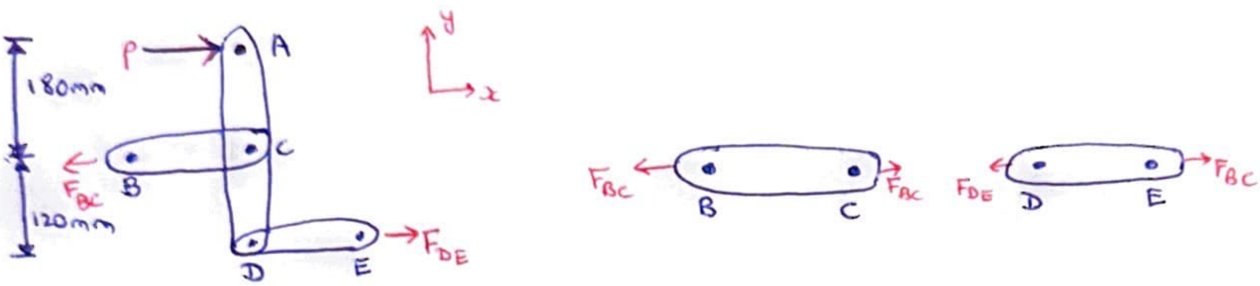


Figure 2: Structure for Problem 2

Solution 2 :

FBDs →



Consider the entire structure

- Summing moment about B

$$\Sigma M_B = 0 = -P_{all}(0.18) + (0.12)F_{DE}$$

$$\boxed{F_{DE} = 1.5\,P_{all}} \quad (1)$$

- Summing moment about E

$$\Sigma M_E = 0 = -P_{all}(0.3) + (0.12)F_{BC}$$

$$\boxed{F_{BC} = 2.5\,P_{all}} \quad (2)$$

- For link BC,

$$\sigma_{all} = 500\,MPa = \frac{F_{BC}}{A} = \frac{2.5\,P_{all}}{125\,mm^2}$$

$$\boxed{P_{all} = 25\,kN \text{ (link BC)}} \quad (3)$$

- Pins B and C have shear force F_{BC} acting on them

$$\tau_{all} = 250\,MPa = \frac{F_{BC}}{A_{pin}} = \frac{2.5\,P_{all}}{\frac{\pi}{4}(10)^2\,mm^2}$$

$$\boxed{P_{all} \approx 7854\,N \text{ (pins B and c)}} \quad (4)$$

- For link DE,

$$\sigma_{all} = 500\,MPa = \frac{F_{DE}}{A} = \frac{1.5\,P_{all}}{125\,mm^2}$$

$$\boxed{P_{all} \approx 41.67\,kN \text{ (link DE)}} \quad (5)$$

- Pins D and E have shear force F_{DE} acting on them

$$\tau_{all} = 250\,MPa = \frac{F_{DE}}{A_{pin}} = \frac{1.5\,P_{all}}{\frac{\pi}{4}(10mm)^2}$$

$$\boxed{P_{all} \approx 13090\,N \text{ (pins D and E)}} \quad (6)$$

★ In order to satisfy all failure criterion, we take the minimum among (3), (4), (5), (6)

$$\therefore \boxed{P_{all} \approx 7854\,N \text{ (at Pins B and c)}}$$

Problem 3 (10 points):

A biaxial loading, as shown in *figure 3*, acts on a homogeneous plate ABCD. Let $\sigma_z = \sigma_0$. The plate must not have a change in length in the x-direction (or $\varepsilon_x = 0$). The plate has a modulus of elasticity E and Poisson's ratio ν. Determine:

a) The required magnitude of σ_x in terms of the given parameters,
b) The ratio σ_0/ε_z.

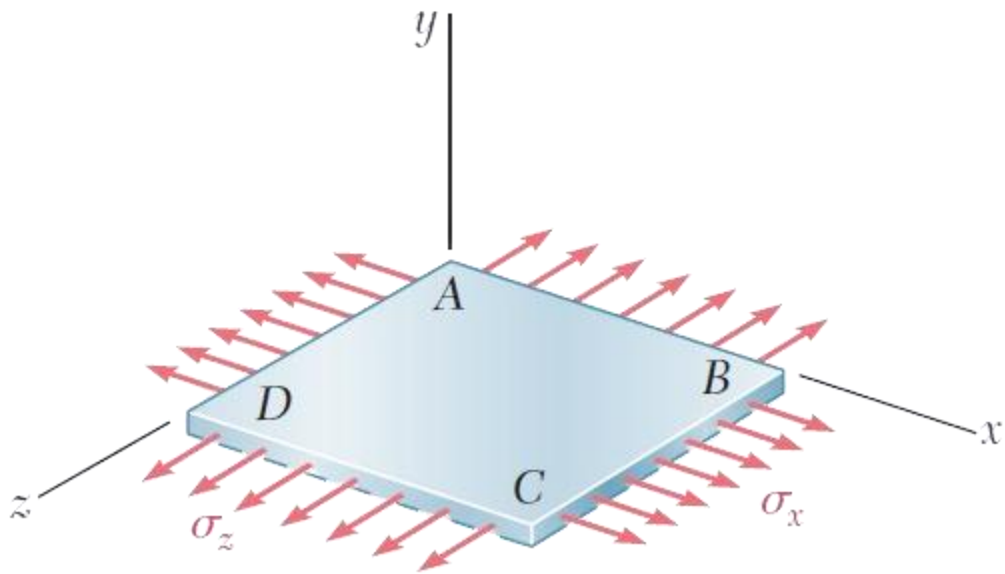
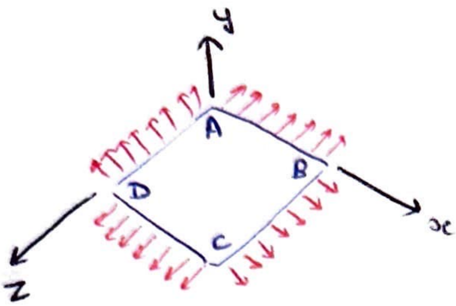


Figure 3: Loading conditions for plate ABCD

Solution 3 :



Given,

$\sigma_z = \sigma_0$

$\sigma_y = 0$ (Biaxial loading)

$\varepsilon_x = 0$

Now,

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\cancelto{0}{\sigma_y} + \sigma_z\right)\right)$$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_z\right)$$

a.) $\varepsilon_x = 0 \Rightarrow \sigma_x - \nu\sigma_z = 0$

$$\boxed{\sigma_x = \nu\sigma_z = \nu\sigma_0}$$

b.) $$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_x + \cancelto{0}{\sigma_y}\right)\right)$$

$$= \frac{1}{E}\left(\sigma_0 - \nu\left(\nu\sigma_0\right)\right)$$

$$= \frac{(1-\nu^2)\sigma_0}{E}$$

$$\boxed{\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}}$$

Problem 4 (2.5 points + 2.5 points):

I. You are given two bars of the same length. One of the bars has a uniform circular cross-section with diameter d and the other has a tapered circular cross-section with end diameters d_1 and d_2 as shown in *figure 4.1*. If they are subjected to the same axial pull, determine the criteria for them to have the same elongation.

a. $d = \frac{d1+d2}{2}$

b. $d = \sqrt{(d1 \times d2)}$

c. $d = \sqrt{\frac{(d1 \times d2)}{2}}$

d. $d = \sqrt{\frac{(d1+d2)}{2}}$

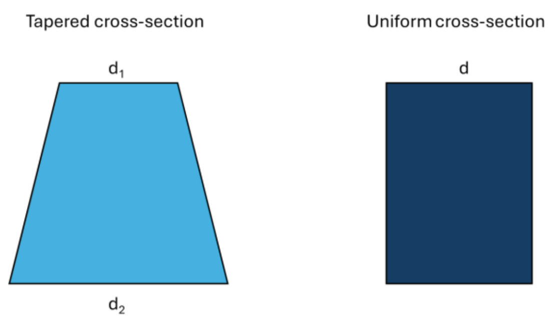


Figure 4.1: Tapered and uniform cross-section

II. Consider a rigid beam supported in a horizontal position by two rods as shown in *figure 4.2*. The steel rod has a cross-sectional area of 1 cm^2 and Elasticity modulus of 200 GPa, while those of aluminum rod are 2 cm^2 and 100 GPa respectively. For the beam to remain horizontal,

a. The forces on both sides should be equal.

b. The force on aluminum rod should be twice the force on steel rod.

c. The force on steel rod should be twice the force on aluminum rod.

d. The force P must be at the center of the beam.

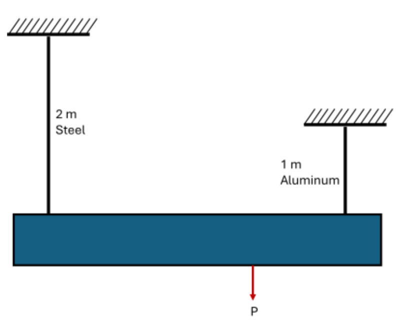


Figure 4.2: Rigid Beam for problem 4.II

Solution 4I

We know that

$$\delta = \frac{FL}{AE}$$

where F = Force or Load

δ = elongation of rod

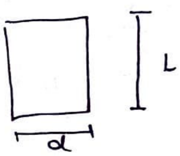
L = Length of rod

A = Area of cross-section

E = Modulus of Elasticity

For uniform cross-section

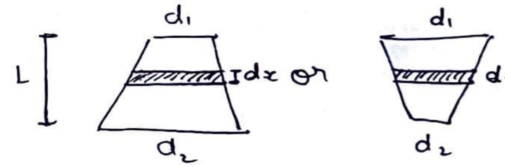
$$\delta_1 = \frac{4FL}{\pi d^2 E}$$



For tapered cross-section

Consider an intermediate strip with diameter

$$d(x) = d_1 - kx$$



$$\delta_2 = \int_0^L \frac{4P\,dx}{\pi E (d_1 - kx)^2}$$

$$= \frac{4P}{\pi E}\left[\frac{-1}{-k(d_1 - kx)}\right]_0^L = \frac{4P}{\pi E k}\left[\frac{1}{d_1 - kL} - \frac{1}{d_1}\right]$$

But $d_1 - kL = d_2$

$$\Rightarrow \delta_2 = \frac{4P}{\pi E k}\left(\frac{1}{d_2} - \frac{1}{d_1}\right) = \frac{4P}{\pi E k}\left(\frac{d_1 - d_2}{d_1 d_2}\right) = \frac{4PkL}{\pi E k\, d_1 d_2}$$

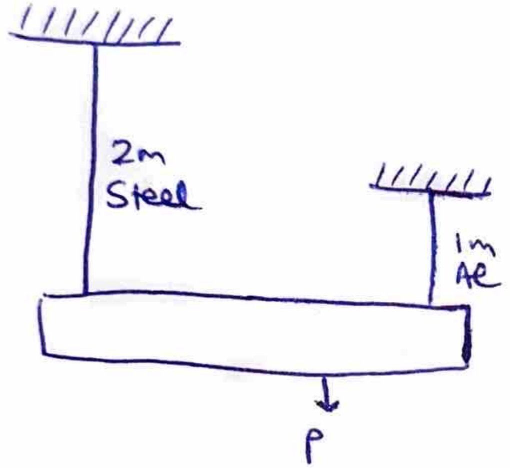
$$\delta_2 = \frac{4PL}{\pi d_1 d_2 E}$$

$\delta_1 = \delta_2$, $P = F$, E is same, L is same

$$\Rightarrow \boxed{d = \sqrt{d_1 d_2}}$$

[Option B]

II



For the beam to remain horizontal,

$$\delta_{Al} = \delta_{Steel}$$

$$\frac{F_{Al} \times 1}{2 \times 100} = \frac{F_{St} \times 2}{1 \times 200}$$

$$\boxed{F_{Al} = 2 F_{St}}$$ Option b.)