Problem 11.1 (10 points)

Given the stress state shown.

a) Draw the in-plane Mohr's circle.
b) Find the principal stresses σ_{p1} and σ_{p2}, and label them in Mohr's circle.
c) Draw the out-of-plane Mohr's circles.
d) Find the maximum in-plane shear stress and label it in Mohr's circle.
e) Find the absolute maximum shear stress and label it in Mohr's circle.

Write your answers in decimal form and box in your answers at each step.

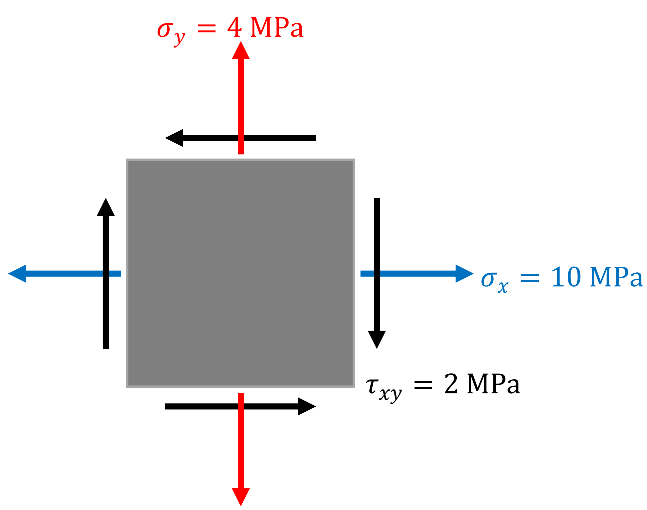


Fig. 11.1

Solution:

Part a)

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{4+10}{2} = 7 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{4-10}{2}\right)^2 + (-2)^2} = 3.61 \text{ MPa}$$

See the plot at the end of the solution.

Part b)

$$\sigma_{p1} = \sigma_{ave} + R = 7 + 3.61 \Longrightarrow \sigma_{p1} = 10.61 \text{ MPa}$$

$$\sigma_{p2} = \sigma_{ave} - R = 7 - 3.61 \Longrightarrow \sigma_{p2} = 3.39 \text{ MPa}$$

σ_{p1} and σ_{p2} need to be labeled in Mohr's circle.

Part c) See the plot at the end of the solution.

Part d)

$$\tau_{max} = R = 3.61 \text{ MPa}$$

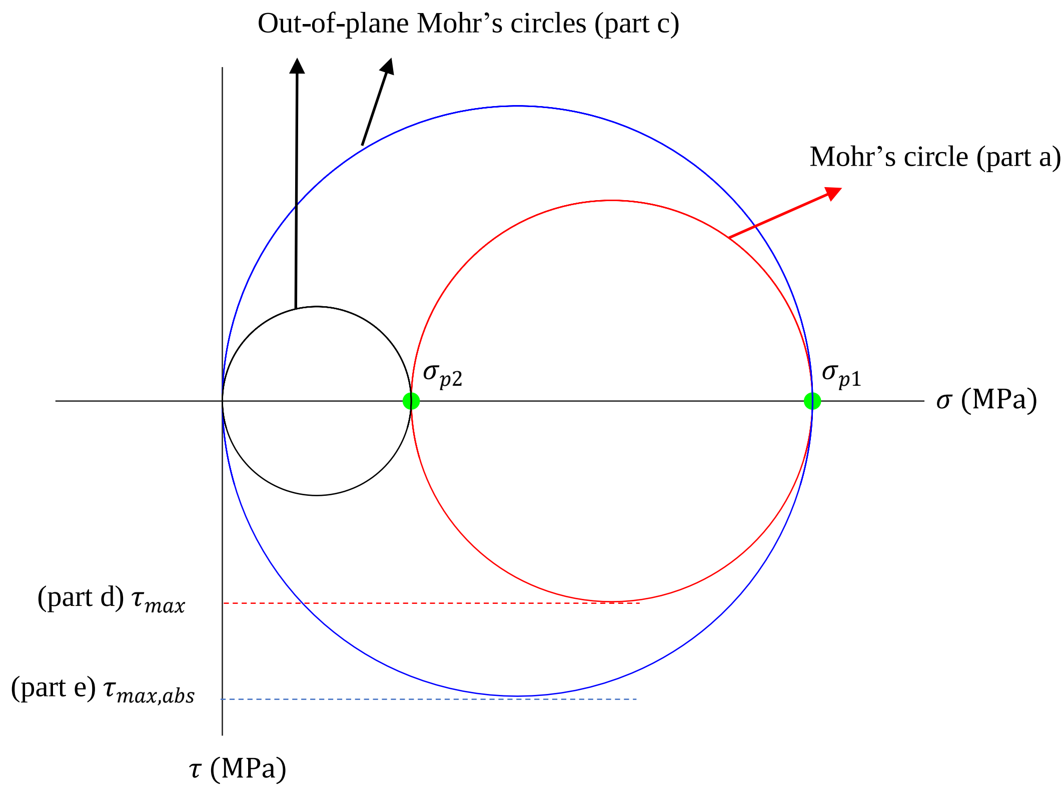
needs to be labeled in Mohr's circle.

Part e)

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{p1}}{2} \Longrightarrow \tau_{max,abs} = 5.31 \text{ MPa}$$

needs to be labeled in Mohr's circle.

Plot


Out-of-plane Mohr's circles (part c)
Mohr's circle (part a)
σ_{p2}
σ_{p1}
σ (MPa)
(part d) τ_{max}
(part e) $\tau_{max,abs}$
τ (MPa)

Problem 11.2 (10 points)

A cantilevered beam AC of length $L = 1.5$ m is subjected to a point load $P = 20$ kN and torque $T = 15$ kN.m at C, as shown in Fig. 11.2 (a). The beam has a circular cross section with radius $R = 0.3$ m as shown in Fig. 11.2 (b).

a) Find the reactions at A.
b) Construct the shear force and bending moment diagrams for the beam.
c) Determine the stresses induced at points “a” and “b” at section B as shown in Fig 11.2 (b).
d) Draw a three-dimensional stress element for each point. On your stress elements, show the directions of the components of stress and their magnitudes.

Write your answers in decimal form and box in your answers at each step.

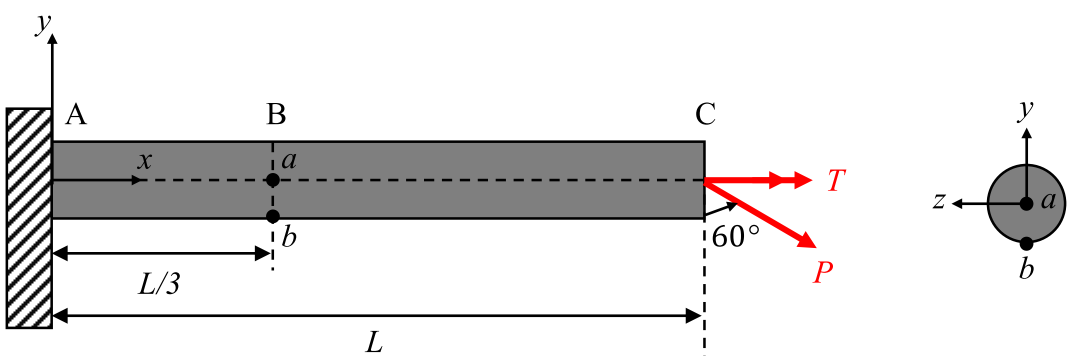
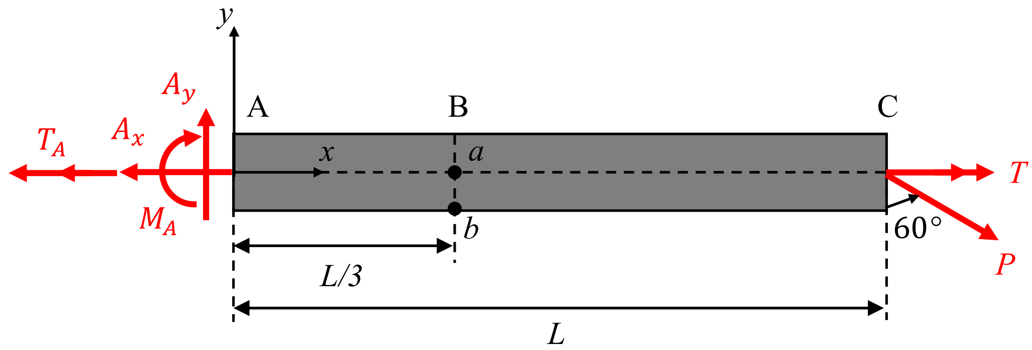


Fig. 11.2 (a) **Fig. 11.2 (b)**

Solution:

Part a)

FBD



$$\sum F_x = 0 \Longrightarrow P\sin(60) - A_x = 0 \Longrightarrow A_x = P\sin(60) \Longrightarrow \boxed{A_x = 17.32\text{ kN}}$$

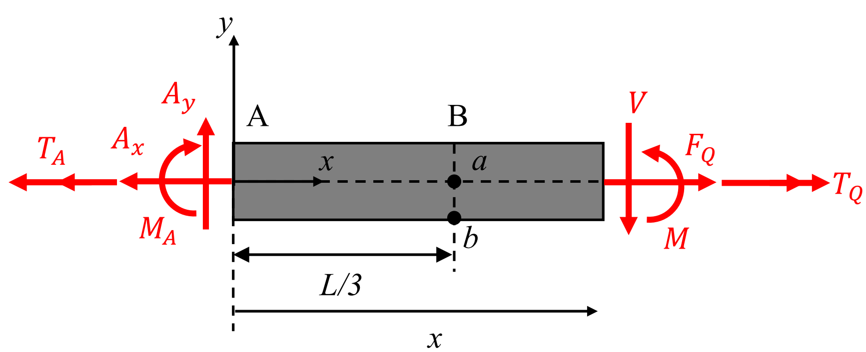
$$\sum F_y = 0 \Longrightarrow -P\cos(60) + A_y = 0 \Longrightarrow A_y = P\cos(60) \Longrightarrow \boxed{A_y = 10\text{ kN}}$$

$$\sum M_A = 0 \Longrightarrow -M_A - P\cos(60)\,L = 0 \Longrightarrow M_A = -P\cos(60)\,L \Longrightarrow \boxed{M_A = -15\text{ kN.m}}$$

$$\sum T = 0 \Longrightarrow -T_A + T = 0 \Longrightarrow T_A = T \Longrightarrow \boxed{T_A = 15\text{ kN.m}}$$

Part b)

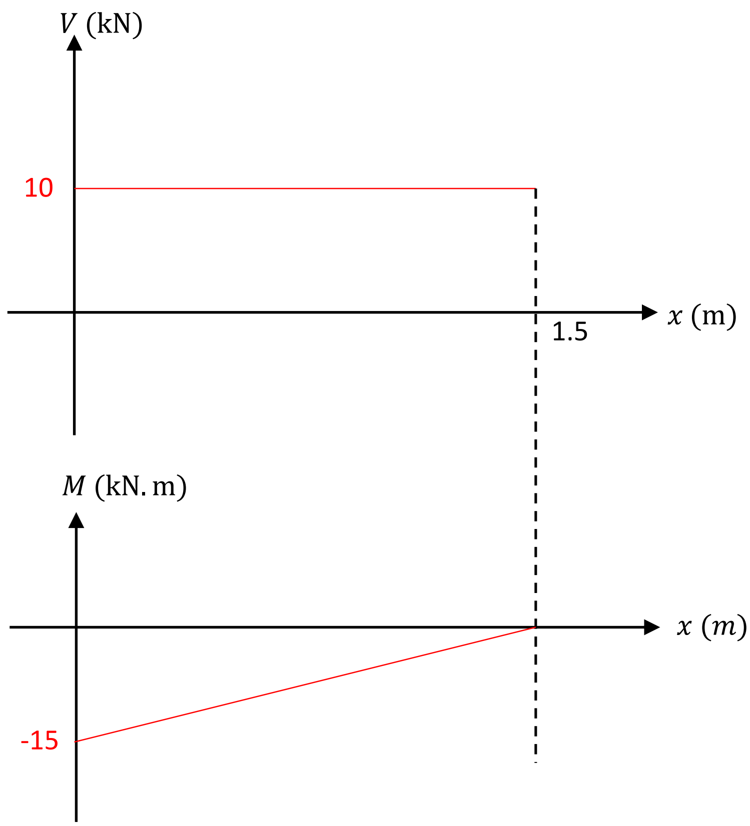
FBD at cut section



$$\sum F_y = 0 \Longrightarrow -V + A_y = 0 \Longrightarrow V = A_y = P\cos(60) \Longrightarrow V = 10 \text{ kN}$$

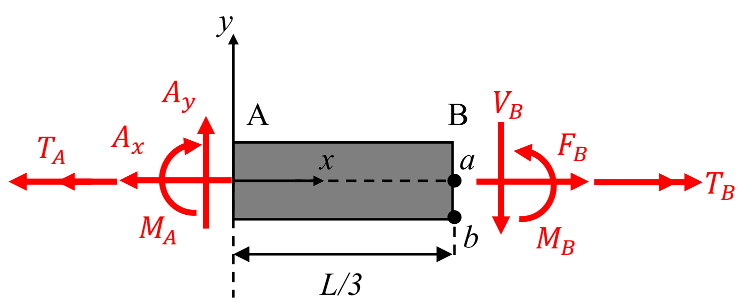
$$\sum M_A = 0 \Longrightarrow -M_A - Vx + M = 0 \Longrightarrow M = M_A + Vx = -P\cos(60)\,L + P\cos(60)\,x \Longrightarrow$$

$$M = P\cos(60)\,(x - L) \Longrightarrow M = 10x - 15 \ \text{kN.m}$$



Part c)

FBD at cut section B



From equilibrium equations:

$$\sum F_x = 0 \Longrightarrow F_B - A_x = 0 \Longrightarrow F_B = A_x \Longrightarrow F_B = 17.32 \text{ kN}$$

$$\sum F_y = 0 \Longrightarrow -V_B + A_y = 0 \Longrightarrow V_B = A_y \Longrightarrow V_B = 10 \text{ kN}$$

$$\sum M_A = 0 \Longrightarrow -M_A - V_B L + M_B = 0 \Longrightarrow M_B = M_A + V_B L \Longrightarrow M_B = -10 \text{ kN.m}$$

$$\sum T = 0 \Longrightarrow -T_A + T_B = 0 \Longrightarrow T_B = T_A \Longrightarrow T_B = 15 \text{ kN.m}$$

Loads	Stress	Point "a"	Point "b"
Torque: $T_B = 15$ kN.m	$\tau = \frac{Tr}{I_P}$	$0 \ (r_A = 0)$	$\tau_{xz} = -353.68$ kPa
Axial force: $F_B = 17.32$ kN	$\sigma = \frac{F}{A}$	$\sigma_x = 61.26$ kPa	$\sigma_x = 61.26$ kPa
Shear force: $V_B = 10$ kN	$\tau = \frac{VA^* y^*}{It}$	$\tau_{xy} = -47.16$ kPa	0
Bending moment: $M_B = -10$ kN.m	$\sigma = \frac{My}{I}$	$0 \ (y_A = 0)$	$\sigma_x = -471.57$ kPa

At point "a":

$$\sigma_x = 61.26 \text{ kPa}$$

$$\tau_{xy} = -47.16 \text{ kPa}$$

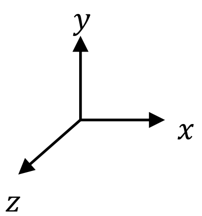
At point "b":

$$\sigma_x = 61.26 - 471.57 = -410.31 \text{kPa}$$

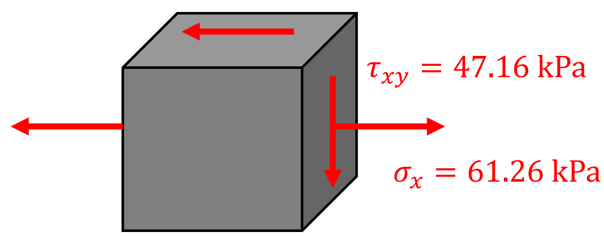
$$\tau_{xz} = -353.68 \text{ kPa}$$

Part d)

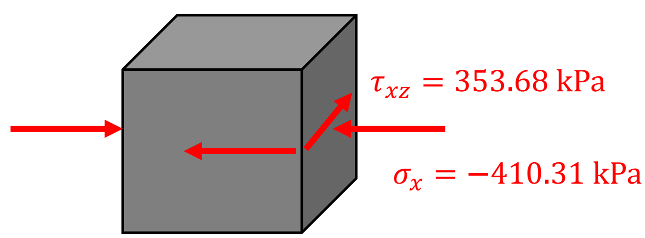
Three-dimensional stress element



Point "a"



Point "b"



Problem 11.3 (10 points)

A structure is attached to a fixed surface at its bottom C, as shown in the figure below. The weight of the structure is negligible. The length of sections BO and OH is L. Both BO and OH have radii of r, with $L = 10r$. The end H is acted upon by a force $\vec{F}_H = 3P\hat{\imath} - P\hat{\jmath} + 0\hat{k}$. It is desired to understand the states of stress at points "a" and "b" at B.

a) Determine the internal resultants and their types at cross section B. (i.e., axial force, two shear forces, torque, and two bending moments).
b) Identify the components of stress for each of the internal resultants at locations "a" and "b" on the cross section of the cut.
c) Draw the stress distributions at location B.
d) Determine the states of stress at locations "a" and "b".
e) Draw a corresponding three-dimensional stress element. On your stress elements, show the directions of the components of stress and their magnitudes.

Report the stress components, stress distributions, and stress elements in the worksheet provided in the next page. Write your answers in terms of P and r, and in decimal form.

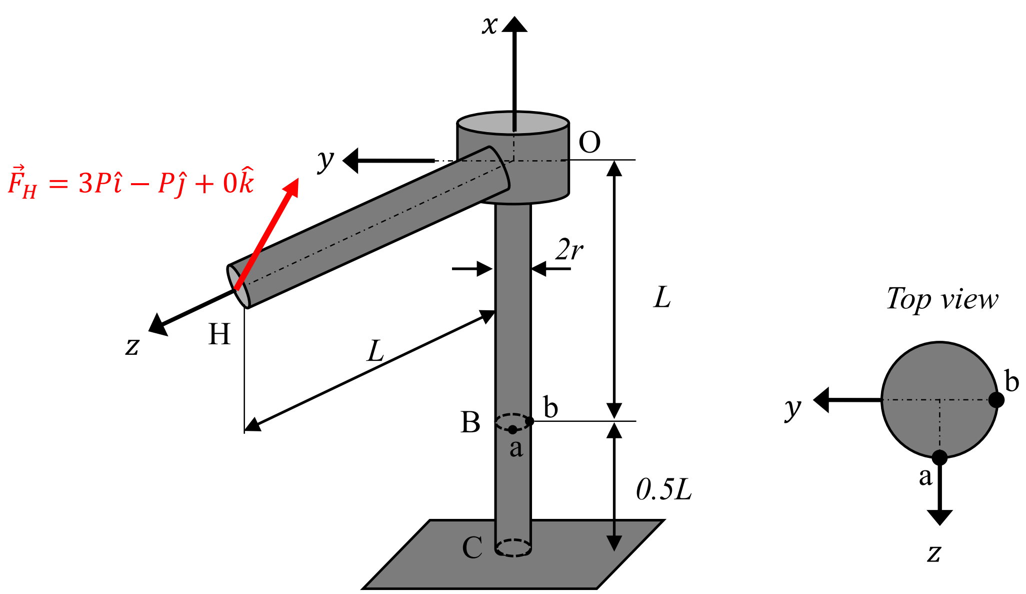
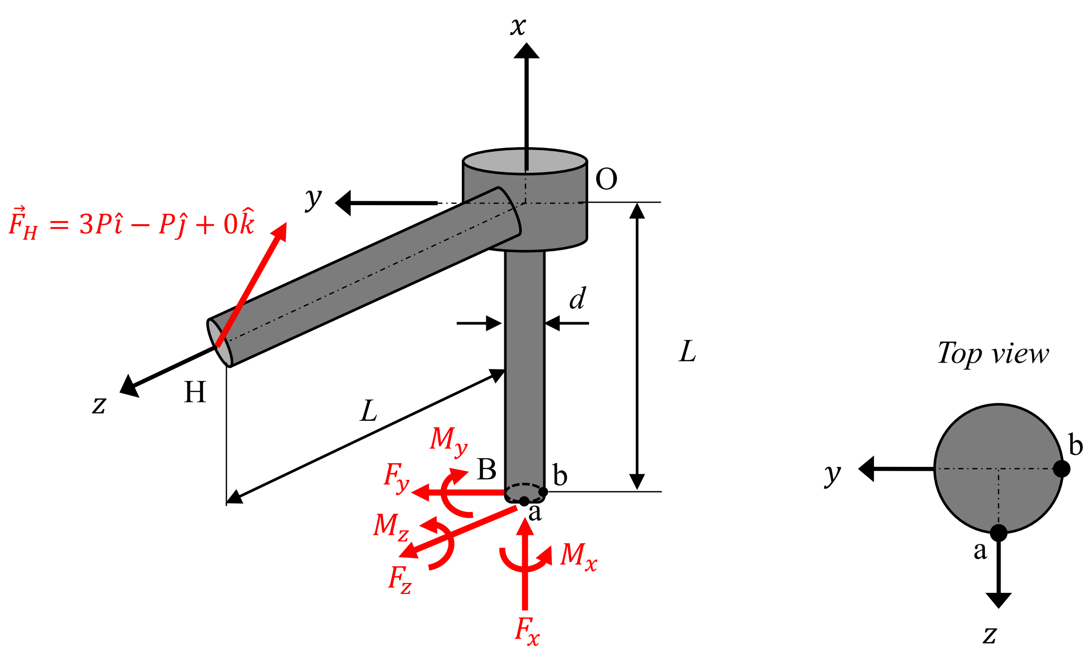


Fig. 11.3

Solution:

Part a)

FBD (Cut section at B)



At cut section:

Resultant force: $\vec{F}_B = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$

Resultant moment: $\vec{M}_B = M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k}$

Force and moment equilibrium equations:

$$\sum \vec{F} = \vec{0} \Longrightarrow \vec{F}_H + \vec{F}_B = \vec{0} \Longrightarrow 3P\hat{\imath} - P\hat{\jmath} + 0\hat{k} + F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k} = \vec{0} \Longrightarrow \begin{cases} F_x = -3P \\ F_y = P \\ F_z = 0 \end{cases}$$

$$\sum \vec{M}_B = \vec{0} \Longrightarrow \vec{M}_B + \vec{r}_{BH} \times \vec{F}_H = \vec{0}$$

$$\Longrightarrow M_x\hat{\imath} + M_y\hat{\jmath} + M_z\hat{k} + (L\hat{\imath} + 0\hat{\jmath} + L\hat{k}) \times (3P\hat{\imath} - P\hat{\jmath} + 0\hat{k}) = \vec{0}$$

$$\Longrightarrow \begin{cases} M_x = -LP = -10Pr \\ M_y = -3LP = -30Pr \\ M_z = LP = 10Pr \end{cases}$$

Note: These results are based on the assumed positive internal results in the FBD. The sign of results will change if opposite directions are assumed.

Resultants and their types are as follow:

Torque: $M_x = -10Pr \quad (M_x = 10Pr \;\; on\ BC\ cut\ section)$

Axial force: $F_x = -3P \quad (F_x = 3P \;\; on\ BC\ cut\ section)$

Shear force 1: $F_y = P \quad (F_y = -P \;\; on\ BC\ cut\ section)$

Shear force 2: $F_z = 0$

Bending moment 1 (about y axis): $M_y = -30Pr \;\; (M_y = 30Pr \;\; on\ BC\ cut\ section)$

Bending moment 2 (about z axis): $M_z = 10Pr \;\; (M_z = -10Pr \;\; on\ BC\ cut\ section)$

$$A = \pi r^2, \qquad I = \frac{\pi}{4} r^4, \qquad I_P = \frac{\pi}{2} r^4,$$

$$A^* = \frac{\pi}{2} r^2, \qquad y^* = \frac{4r}{3\pi}, \qquad t = 2r \quad \Longrightarrow \tau = \frac{VA^* y^*}{It} = \frac{4}{3}\frac{V}{A} \quad (for\ point\ a)$$

Part d)

At point "a":

$$\sigma_x = 3\frac{P}{\pi r^2} + 120\frac{P}{\pi r^2} \Longrightarrow \sigma_x = 123\frac{P}{\pi r^2} = 39.15\frac{P}{r^2}$$

$$\tau_{xy} = -20\frac{P}{\pi r^2} - \frac{4}{3}\frac{P}{\pi r^2} \Longrightarrow \tau_{xy} = -\frac{64}{3}\frac{P}{\pi r^2} = -6.79\frac{P}{r^2}$$

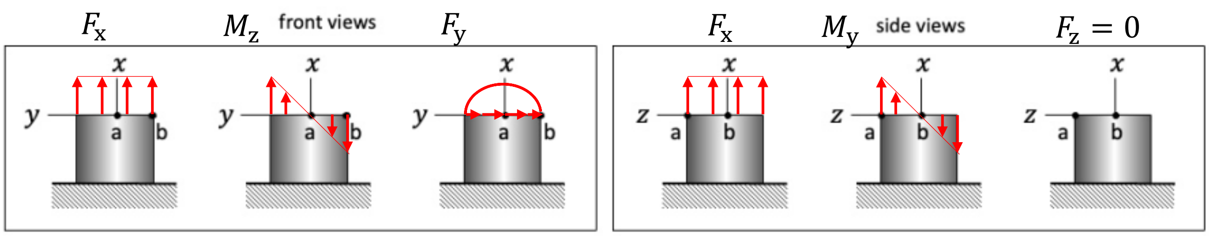
At point "b":

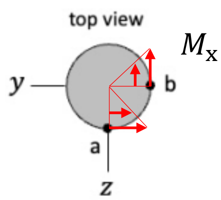
$$\sigma_x = 3\frac{P}{\pi r^2} - 40\frac{P}{\pi r^2} \Longrightarrow \sigma_x = -37\frac{P}{\pi r^2} = -11.78\frac{P}{r^2}$$

$$\tau_{xz} = -20\frac{P}{\pi r^2} = -6.37\frac{P}{r^2}$$

Problem 11.3 worksheet

Stress distributions at location B

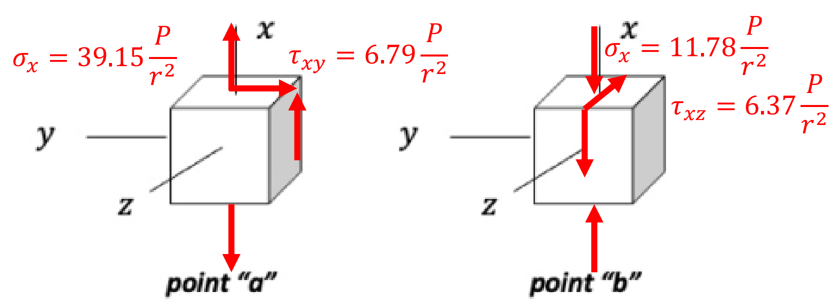




Stress components at location B (note: based on the cut section BC)

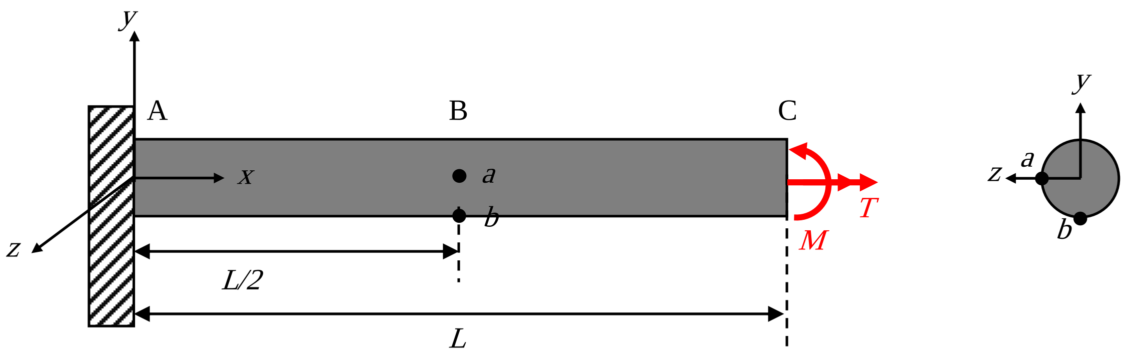
Internal resultant	Stress @ point "a"	Stress @ point "b"
$M_x = T = 10Pr$	$\tau_{xy} = -20\dfrac{P}{\pi r^2} = -6.37\dfrac{P}{r^2}$	$\tau_{xz} = -20\dfrac{P}{\pi r^2} = -6.37\dfrac{P}{r^2}$
$F_x = 3P$	$\sigma_x = 3\dfrac{P}{\pi r^2} = 0.95\dfrac{P}{r^2}$	$\sigma_x = 3\dfrac{P}{\pi r^2} = 0.95\dfrac{P}{r^2}$
$F_y = V = -P$	$\tau_{xy} = -\dfrac{4}{3}\dfrac{P}{\pi r^2} = -0.42\dfrac{P}{r^2}$	0
$F_z = 0$	0	0
$M_y = 30Pr$	$\sigma_x = 120\dfrac{P}{\pi r^2} = 38.20\dfrac{P}{r^2}$	0
$M_z = -10Pr$	0	$\sigma_x = -40\dfrac{P}{\pi r^2} = -12.73\dfrac{P}{r^2}$

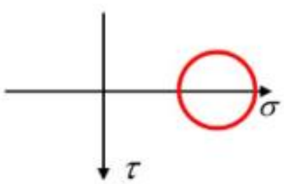
Stress elements at location B



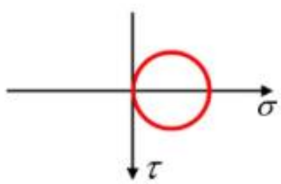
Problem 11.4 (Conceptual) (5 points)

A moment **M** (about positive z) and torque **T** (about positive x) are applied to a circular rod as shown below. Choose the correct in-plane Mohr's circles, from the given options, for the stress states at *Point a* and *Point b*. (Note that the location of Point a is at ($L/2$, 0, R) where R is the radius of the cross section.)

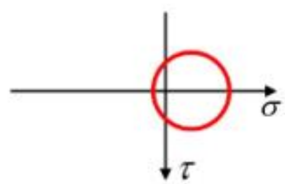




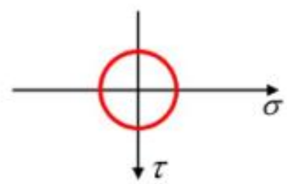
Mohr's circle #1



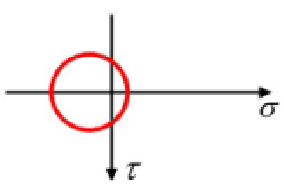
Mohr's circle #2



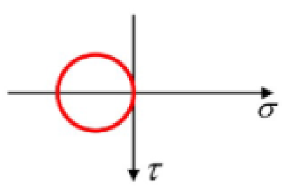
Mohr's circle #3



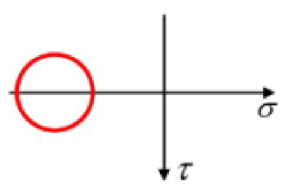
Mohr's circle #4



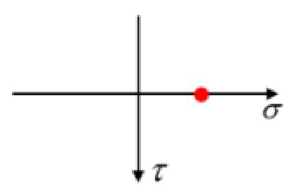
Mohr's circle #5



Mohr's circle #6



Mohr's circle #7



Mohr's circle #8
(Mohr's circle is a point!)

Solution:

At point a:

$$\sigma_x = \sigma_y = 0$$

$$\tau_{xy} < 0$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Mohr's circle #4

At point b:

$$\sigma_x > 0, \qquad \sigma_z = 0$$

$$\tau_{xz} < 0$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_z}{2} = \frac{\sigma_x}{2} > 0$$

Mohr's circle #3