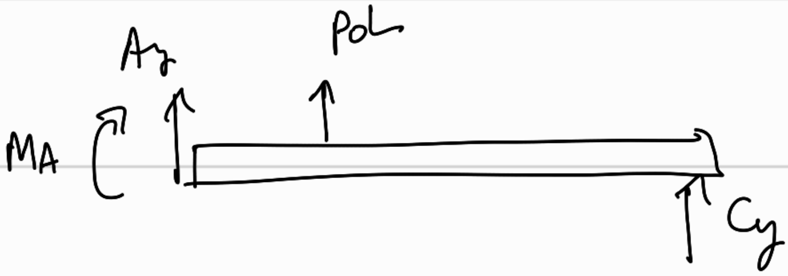
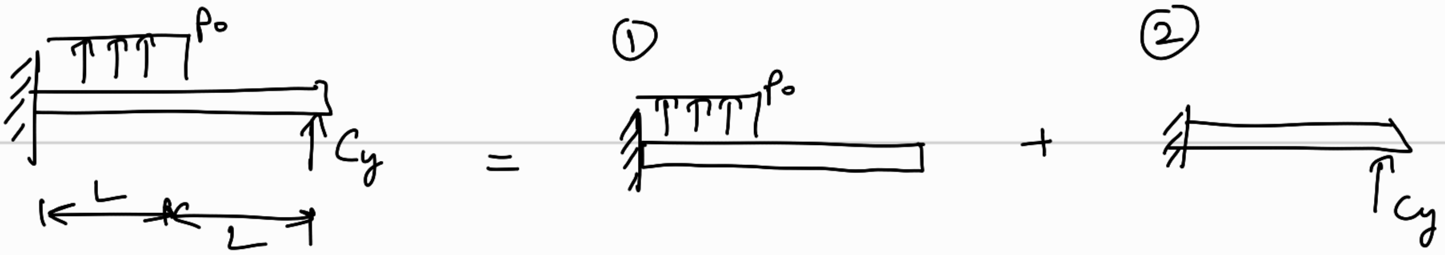
9.1 (a)



(b) ΣF_y : $A_y + p_o L + C_y = 0$

ΣM_A : $-M_A + \frac{p_o L^2}{2} + 2 C_y L = 0$

(c) Given loading is split as



Problem ① :

$$v_1(x) = \begin{cases} \frac{p_o x^2}{24EI}(6L^2 - 4Lx + x^2) & 0 \leq x \leq L \\ \frac{p_o L^3}{24EI}(4x - L) & L \leq x \leq 2L \end{cases}$$

Problem ② :

$$v_2(x) = \frac{C_y x^3}{6EI}(6L - x)$$

Total displacement :

$$v(x) = \begin{cases} \frac{p_o x^2}{24EI}(6L^2 - 4Lx + x^2) + \frac{C_y x^3}{6EI}(6L - x), & 0 \leq x \leq L \\ \frac{p_o L^3}{24EI}(4x - L) + \frac{C_y x^3}{6EI}(6L - x), & L \leq x \leq 2L \end{cases}$$

$$V(2L) = 0 = \frac{p_0 \cancel{L^3}}{24\cancel{EI}}\cdot 7L + \frac{8\, C_y \cancel{L^3}}{3\cancel{EI}}$$

$$\boxed{C_y = -\frac{7}{64}\, p_0 L}$$

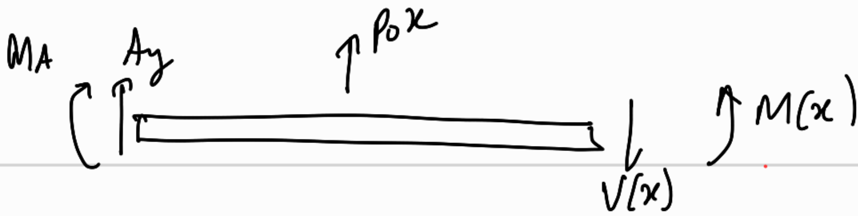
Substituting into equilibrium,

$$\boxed{A_y = -\frac{57}{64}\, p_0 L}$$

$$M_A = \frac{p_0 L^2}{2} - \frac{7}{32}\, p_0 L^2$$

$$\boxed{M_A = \frac{9}{32}\, p_0 L^2}$$

(d,e) $0 \leq x \leq L$:

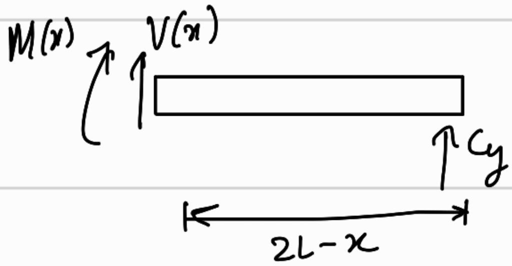


$$V(x) = p_0 x + A_y = p_0 x - \frac{57}{64}\, p_0 L$$

$$M(x) = M_A + A_y \cdot x + \frac{p_0 x^2}{2}$$

$$M(x) = \frac{9}{32}\, p_0 L^2 - \frac{57}{64}\, p_0 L x + \frac{p_0 x^2}{2}$$

$L \leq x \leq 2L$:



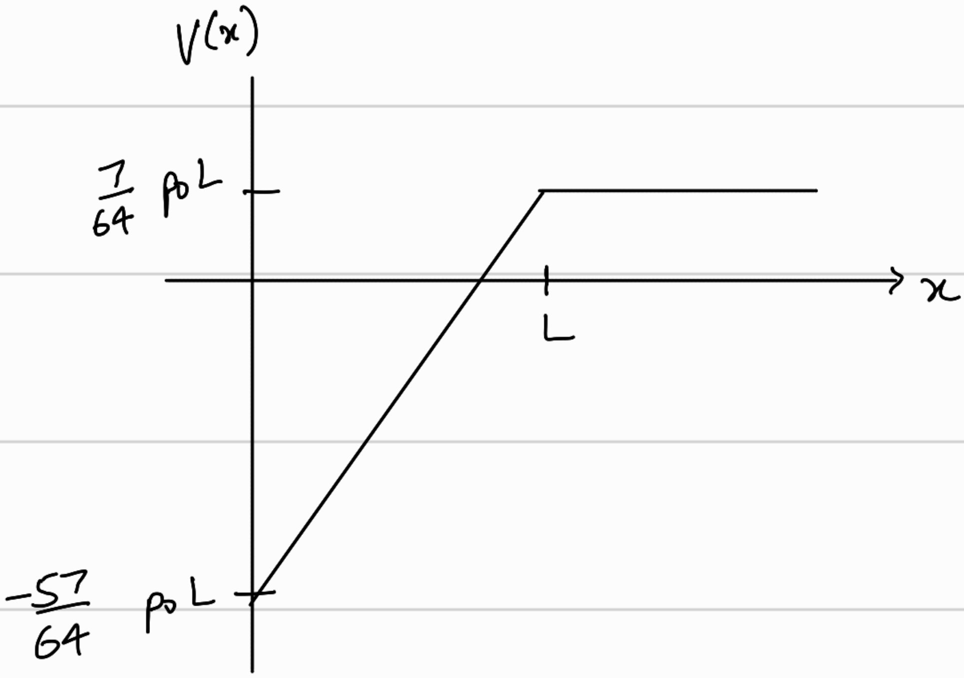
$$V(x) = -C_y = \frac{7}{64}\, p_0 L$$

$$M(x) = C_y (2L - x) = +\frac{7}{64}\, p_0 x L - \frac{7}{32}\, p_0 L^2$$

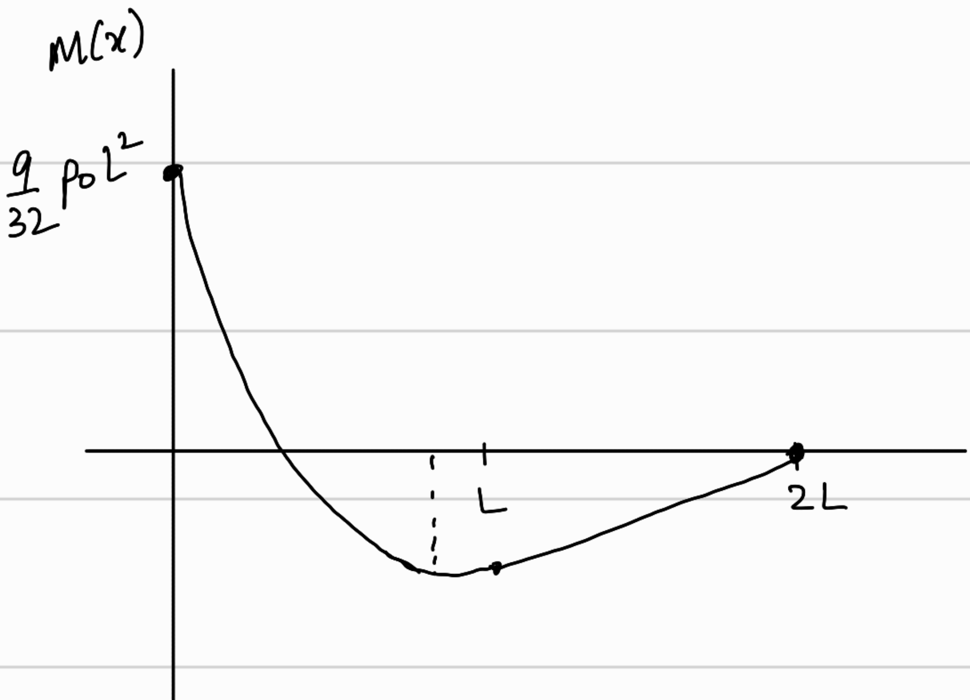
$$V(x) = \begin{cases} p_0 x - \frac{57}{64} p_0 L & 0 \leq x \leq L \\ \frac{7}{64} p_0 L & L \leq x \leq 2L \end{cases}$$

$$M(x) = \begin{cases} \frac{9}{32} p_0 L^2 - \frac{57}{64} p_0 L x + \frac{p_0 x^2}{2} & 0 \leq x \leq L \\ + \frac{7}{64} p_0 x L - \frac{7}{32} p_0 L^2 & L \leq x \leq 2L \end{cases}$$

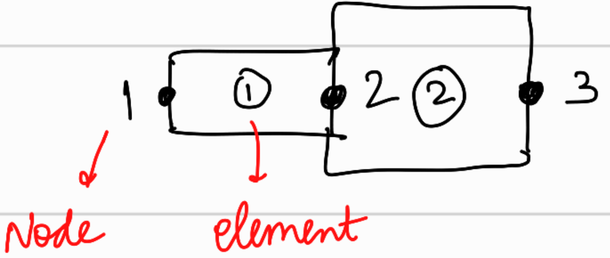
(f)



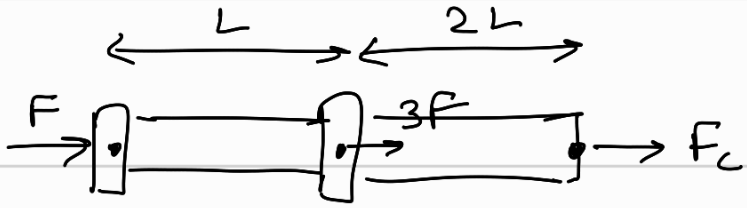
(g)



9.2 (a)



(b)



(c) $k_1 = \frac{EA}{L}$, $k_2 = \frac{EA}{2L}$

$$K_{total} = \begin{pmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{pmatrix}$$

$$\boxed{[K_{total}] = \frac{EA}{L}\begin{pmatrix} 1 & -1 & 0 \\ -1 & 3/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}}$$

(d) $$\boxed{\{F_{total}\} = \begin{pmatrix} F \\ 3F \\ F_c \end{pmatrix}}$$

(e) $$\boxed{[K] = \frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 3/2 \end{pmatrix}}$$

(f) $$\boxed{\{F\} = \begin{pmatrix} F \\ 3F \end{pmatrix}}$$

(g)

$$\frac{EA}{L}\begin{pmatrix} 1 & -1 \\ -1 & 3/2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} F \\ 3F \end{pmatrix}$$

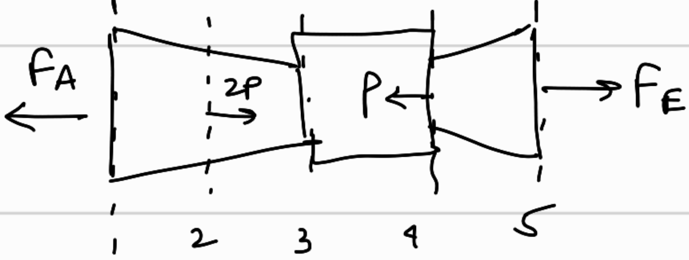
$$\begin{pmatrix} 1 & -1 \\ -1 & 3/2 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \frac{FL}{EA}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$u_1 - u_2 = FL/EA$$

$$-u_1 + \frac{3}{2}u_2 = 3FL/EA$$

$$\boxed{u_1 = u_A = \frac{9FL}{EA}} \quad \boxed{u_2 = u_B = \frac{8FL}{EA}}$$

9.3 (a)



(b) Stiffness of each element :

$$k_1 = \frac{5EA}{L}, \quad k_2 = \frac{3EA}{L}, \quad k_3 = \frac{8EA}{L}, \quad k_4 = \frac{6EA}{L}$$

$$[K_{total}] = \begin{pmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & 0 & -k_4 & k_4 \end{pmatrix}$$

$$\boxed{[K_{total}] = \frac{EA}{L} \begin{pmatrix} 5 & -5 & 0 & 0 & 0 \\ -5 & 8 & -3 & 0 & 0 \\ 0 & -3 & 11 & -8 & 0 \\ 0 & 0 & -8 & 14 & -6 \\ 0 & 0 & 0 & -6 & 6 \end{pmatrix}}$$

(c) $$\boxed{\{F_{total}\} = \begin{pmatrix} -F_A \\ 2P \\ 0 \\ -P \\ F_E \end{pmatrix}}$$

Alternate answer : $\pm F_A$, $\pm F_E$ depending on FBD in part (a).

(d) $$\boxed{[K] = \frac{EA}{L}\begin{pmatrix} 8 & -3 & 0 \\ -3 & 11 & -8 \\ 0 & -8 & 14 \end{pmatrix}}$$

(e) $$\boxed{\{F\} = \begin{pmatrix} 2P \\ 0 \\ -P \end{pmatrix}}$$

(f) $$\frac{EA}{L}\begin{pmatrix} 8 & -3 & 0 \\ -3 & 11 & -8 \\ 0 & -8 & 14 \end{pmatrix}\begin{pmatrix} u_2 \\ u_3 \\ u_4 \end{pmatrix} = P\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$$

Solving this system,

$$\boxed{\begin{aligned} u_2 = u_B &= 0.263 \frac{PL}{EA} \\ u_3 = u_C &= 0.034 \frac{PL}{EA} \\ u_4 = u_D &= -0.052 \frac{PL}{EA} \end{aligned}}$$

(g) Using the "deleted equations" of $[K_{total}]\, u = \{F_{total}\}$,

$$\frac{EA}{L}(\cancel{5u_1}^{0} - 5u_2) = -F_A \Rightarrow \boxed{F_A = 1.313P}$$

$$\frac{EA}{L}(-6u_4 + \cancel{6u_5}^{0}) = F_E \Rightarrow \boxed{F_E = 0.313P}$$

(Sign may vary depending on their FBD)

9.4

1. (a) 15×15

(b) 14×14

2. i. (a)

ii. (b)

iii. (b)

iv. (a)

$$K = \begin{pmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{pmatrix}$$

$\Rightarrow \quad k_1 = 1, \quad k_2 = 2, \quad k_3 = 3$

$\Rightarrow \quad a_1 = 0$

$a_2 = -2$

$a_3 = -2$

$a_4 = 0$