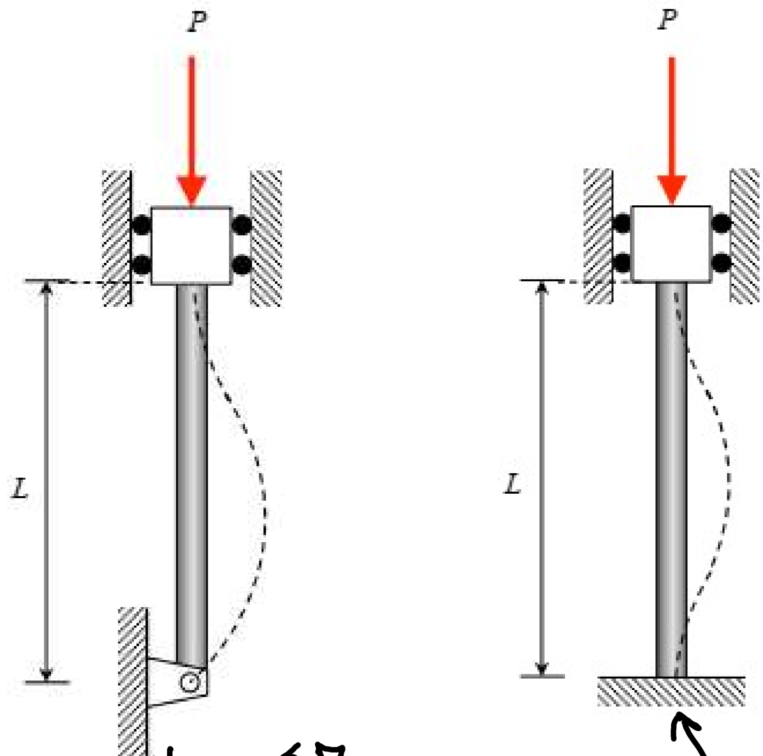
Course evaluations: currently at 54/85

40*(54/85) = 25.4

Buckling: Quick Check



Leff = 0.7   Leff = 0.5

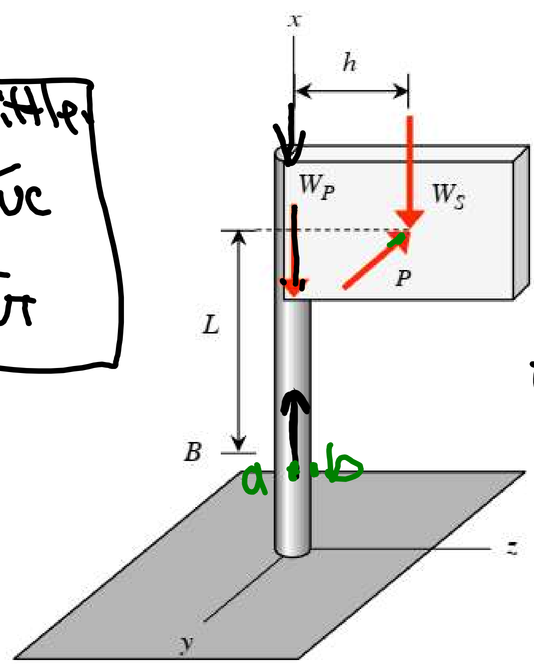
Which boundary conditions can withstand a larger force before buckling?

$$P_{cr} = \pi^2 \frac{EI}{L_{eff}^2} \qquad \sigma_{cr} = \pi^2 \frac{E}{\left(L_{eff}/r\right)^2}$$

Example 14.12

Wind blowing on a sign produces a resultant force P in the –y direction at the point indicated. The support pole for the sign weights W_P and the sign weighs W_S. The pole is a pipe with outer and inner diameters of d and d_i, respectively. Determine the principal stresses at points a and b on the outer surface of the pole at location B along the pole's length.

brittle
σ_{UC}
σ_{UT}

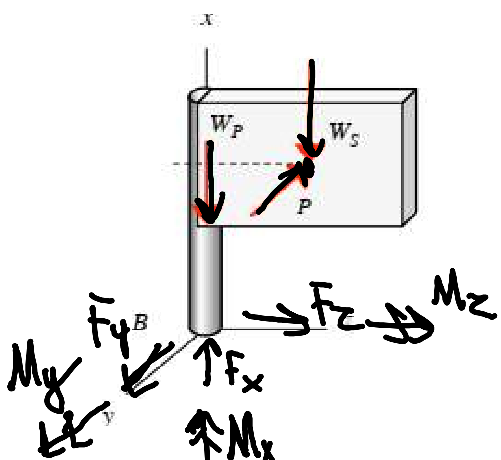


A. Determine the state of stress on the stress elements located at points a and b. (**Ch14: combined loading**)

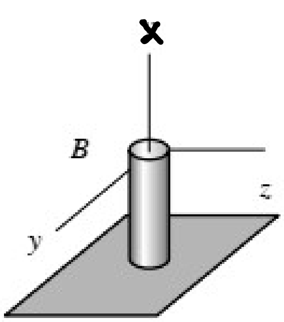
B. Draw the Mohr's circle and determine the principal stresses and angle of principal stress. Which point has a larger absolute maximum shear stress? (**Ch13: stress transformations**)

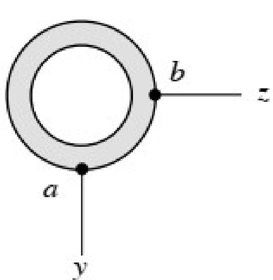
C. The pole is made of a polymer with a yield strength of 9500 psi. Have the material elements failed at either point "a" or point "b"? (**Ch15: Failure methods**)

σ_x



$$W_P = 160\ lb$$
$$W_S = 125\ lb$$
$$P = 75\ lb$$





pipe cross section at B

$$h = 40\ in$$
$$L = 220\ in$$
$$d_o = 3.5\ in$$
$$d_i = 3.068\ in$$

$$\Sigma F = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} - W_p\hat{i} - W_s\hat{i} - P\hat{j} = 0$$

$$\hat{i} \Rightarrow F_x = W_p + W_s$$

$$\hat{j} \Rightarrow F_y = P$$

$$\hat{k} \Rightarrow F_z = 0$$

$$(\Sigma M)_B = T_x\hat{i} + M_y\hat{j} + M_z\hat{k} + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = 0$$

$$\vec{r}_1 = (L, 0, h) \qquad \vec{F}_1 = (-W_s, -P, 0)$$

$$\vec{r}_2 = (?, 0, 0) \qquad \vec{F}_2 = (-W_p, 0, 0).$$

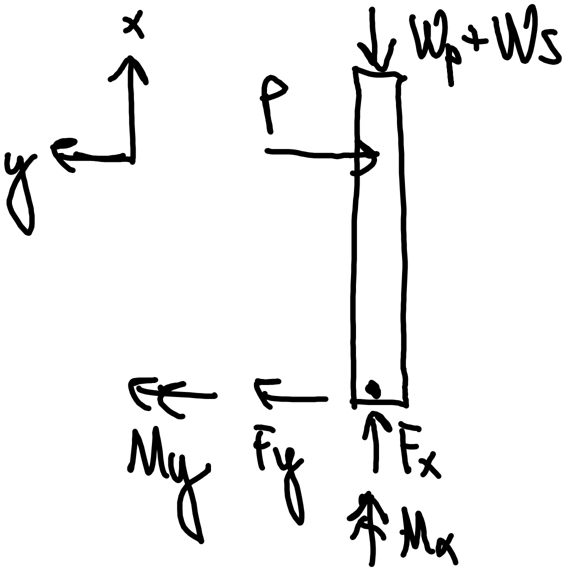
$$\vec{r}_1 \times \vec{F}_1 = (Ph, -W_s h, -PL)$$

$$0 = T_x\hat{i} + M_y\hat{j} + M_z\hat{k} + Ph\hat{i} - W_s h\hat{j} - PL\hat{k}$$

$$T_x = -Ph \qquad M_y = W_s h \qquad M_z = PL$$

On negative face: $F = (W_p + W_s, P, 0)$

$M = (-Ph, W_s h, PL).$



$$\Sigma F_x = F_x - W_p - W_s = 0$$

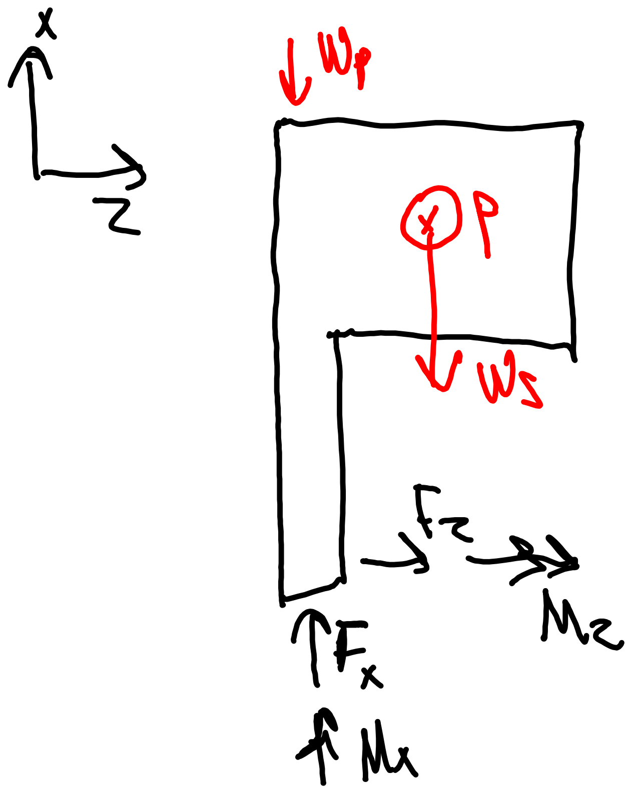
$$F_x = W_p + W_s$$

$$\Sigma F_y = F_y - P = 0$$

$$F_y = P$$

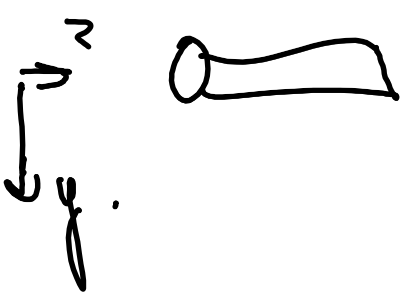
$$(\Sigma M)_z = M_z - PL = 0$$

$$M_z = PL.$$



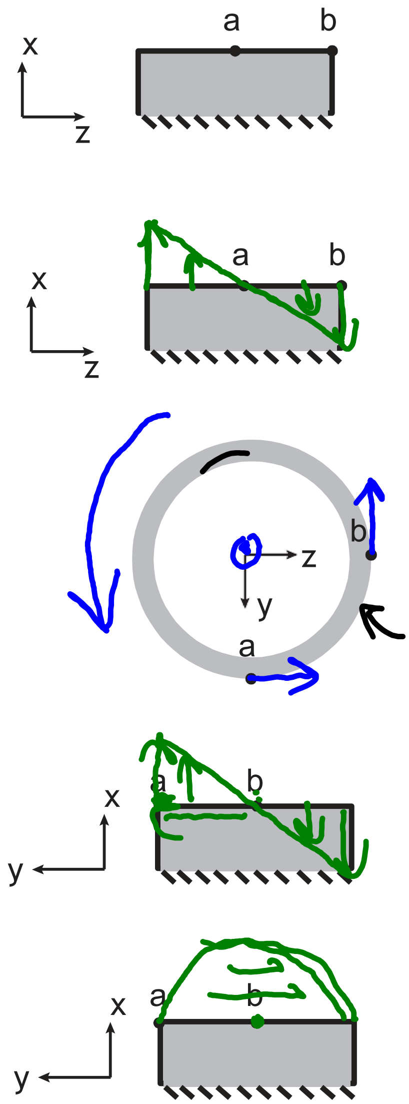
$$(\Sigma M)_y = M_y - W_s h = 0$$

$$M_y = W_s h.$$



On positive face: $F = (-W_p - W_s, -P, 0)$

$M = (Ph, -W_s h, -PL)$.



Force	a	b
F_x	$\sigma_x = -\frac{\lvert F_x \rvert}{A}$	$\sigma_x = -\frac{\lvert F_x \rvert}{A}$
M_y	0	$\sigma_x = \frac{\lvert M_y \rvert (d/2)}{I}$
T_x	$\tau_{xz} = \frac{T_x(d/2)}{I_p}$	$\tau_{xy} = -\frac{T_x(d/2)}{I_p}$.
M_z	$\sigma_x = \frac{M_z(d/2)}{I}$.	0
F_y	0	$\tau_{xy} = -\frac{2F_y}{A}$

$\tau_{max} = \frac{3}{2}\frac{V}{A}$ (rectangular)

$\tau_{max} = \frac{4}{3}\frac{V}{A}$ (circular)

$\tau_{max} = \frac{2V}{A}$ (hollow circular).

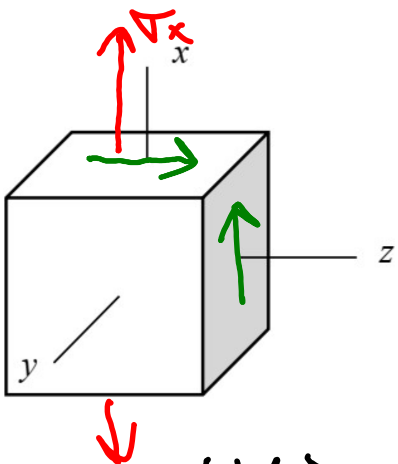
$I_p = \frac{\pi}{2} R^4$

$I = \frac{\pi}{4} R^4$

$I_p = \frac{\pi}{2}(r_o^4 - r_i^4) = 6.034 \text{ in}^4$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = 3.017 \text{ in}^4$

stress element at "a"



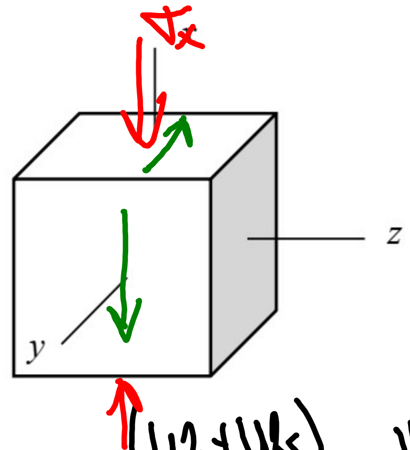
$$\sigma_x = \frac{-(W_p+W_s)}{A} + \frac{PL(d/2)}{I}$$

$$\sigma_x = -128\text{psi} + 9561\text{ psi}$$

$$\sigma_x = 9433\text{psi}$$

$$\tau_{xz} = \frac{Phd_o}{2I_p} = 871\text{psi}$$

stress element at "b"



$$\sigma_x = -\frac{(W_p+W_s)}{A} - \frac{W_s h d_o}{2I}$$

$$\sigma_x = -128 - 2897\text{psi}$$

$$\sigma_x = -3025\text{psi}$$

$$\tau_{xy} = \frac{-Phd_o}{2I} - \frac{2P}{A}$$

$$\tau_{xy} = -871\text{psi} - 67\text{psi}$$

$$= -938\text{psi}$$

a

$$\sigma_{avg} = \frac{\sigma_x}{2} = 4716\ psi$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = 4796\ psi$$

$$\sigma_{p1} = 9512\ psi$$

$$\sigma_{p2} = -80\ psi$$

b

$$\sigma_{avg} = \frac{\sigma_y}{2} = -1514\ psi$$

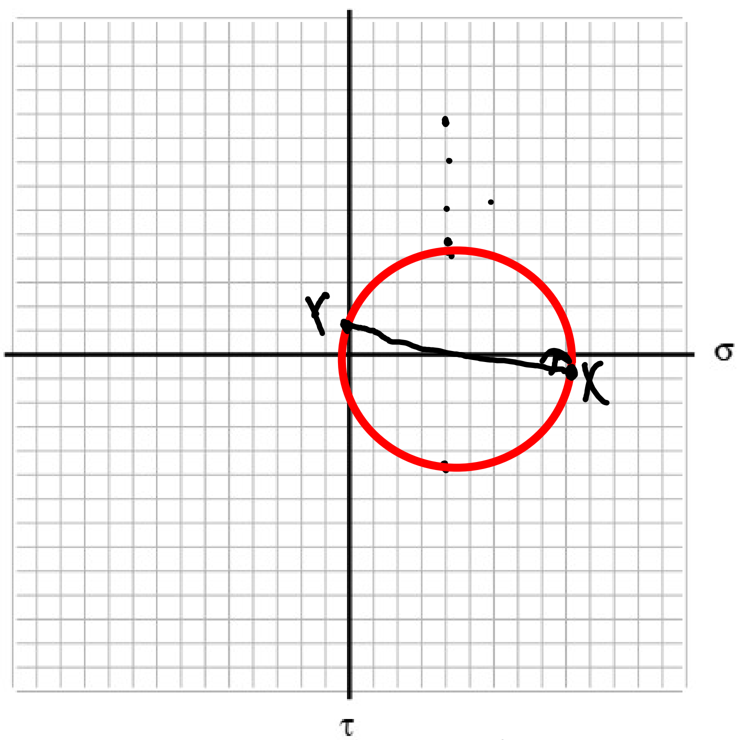
$$R = 1781\ psi$$

$$\sigma_{p1} = 267\ psi$$

$$\sigma_{p2} = -3295\ psi$$

Mohr's circle at "a"

1 unit = 1000 psi



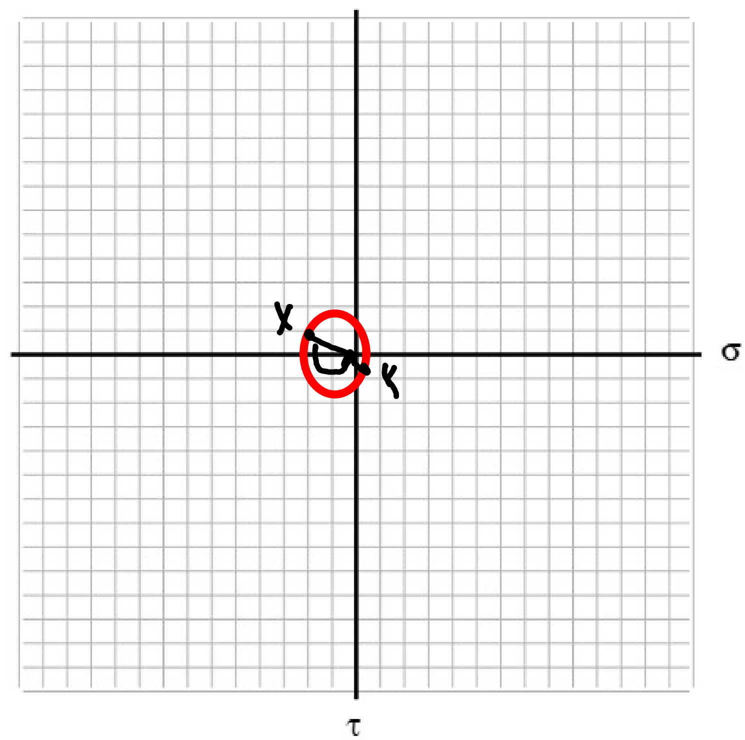
$X = (9433, 871)$

$$2\theta_p = \sin^{-1}\left(\frac{\tau_{xy}}{R}\right)$$

$$\theta_p = 9.2^\circ\ CCW.$$

$$\tau_{max,abs} = R$$

Mohr's circle at "b"



$X = (-3028, -938$

$$2\theta_{p1} = \sin^{-1}\left(\frac{\tau_{xy}}{R}\right) + 180^\circ$$

$$\theta_{p1} = 105.9^\circ\ CCW$$

a

b

$$\boxed{\text{MSS}: \tau_{max,abs} = \frac{\sigma_Y}{2}.}$$

4796psi > 4750psi

⇒ failure.

MDE: $\sigma_M = \sqrt{\sigma_{P1}^2 - \sigma_{P1}\sigma_{P2} + \sigma_{P2}^2}$

$\sigma_M = 9556.5$ psi > 9500psi

⇒ failure.