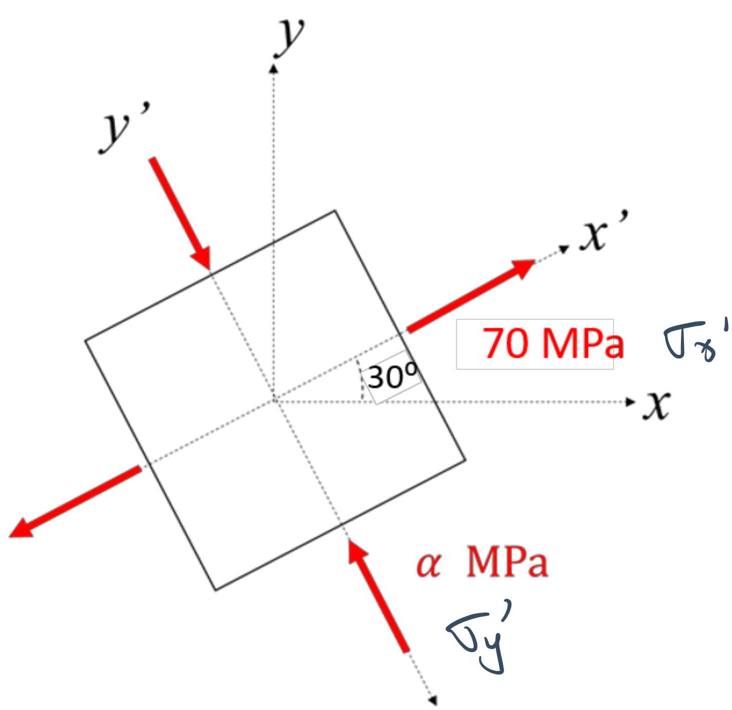
ME 323: Mechanics of Materials
Fall 2023

Homework 10
Due Friday, November 17th

Problem 10.1 (10 points)

The stress element shown below represents the state of stress measured along the x'y' axis in a component loaded under plane stress. No information is known about the stress α except that it is compressive.

a) Determine the magnitude of the maximum compressive normal stress α that can be applied, if the component is made of a material which can withstand a maximum in-plane shear stress of 90 MPa.
b) Determine the stress components when the element is oriented along the x-y axes.
c) Draw a stress element oriented along the maximum in-plane shear stress directions. (Show the angle of this rotated element with respect to the axis x')



a). $\tau_{max} = 90\,MPa \Rightarrow \dfrac{|\sigma_{x'} - \sigma_{y'}|}{2} = 90$

$\therefore \sigma_{y'} = 250\,MPa$ or $-110\,MPa$.

As we are interested in maximum compressive stress.

$\sigma_{y'} = -110\,MPa \Rightarrow \alpha = |\sigma_{y'}| = 110\,MPa$

b).

$$\sigma_x = \frac{\sigma_{x'}+\sigma_{y'}}{2} + \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta + \cancelto{0}{\tau_{x'y'}}\sin 2\theta$$

$$= 25 \text{ MPa}$$

$$\sigma_y = \frac{\sigma_{x'}+\sigma_{y'}}{2} - \frac{\sigma_{x'}-\sigma_{y'}}{2}\cos 2\theta - \cancelto{0}{\tau_{x'y'}}\sin 2\theta$$

$$= -65 \text{ MPa}$$

$$\tau_{xy} = -\left(\frac{\sigma_{x'}-\sigma_{y'}}{2}\right)\sin 2\theta + \cancelto{0}{\tau_{x'y'}}\cos 2\theta$$

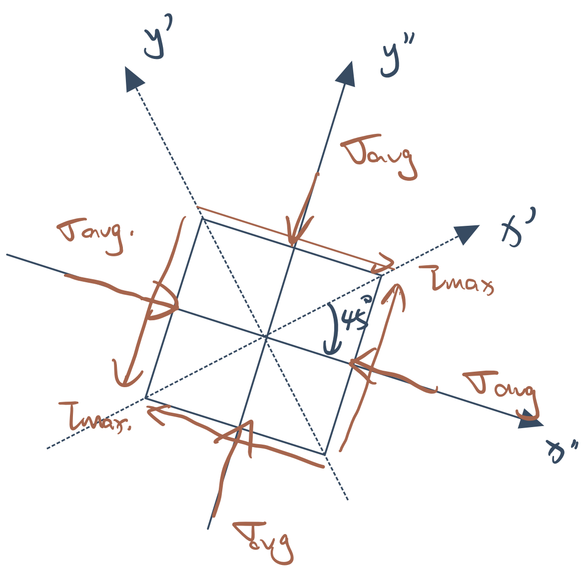
$$= 45\sqrt{3} \text{ MPa.}$$

c) $\beta = \pm 45°$ with respect to x', for a positive τ_{max}.

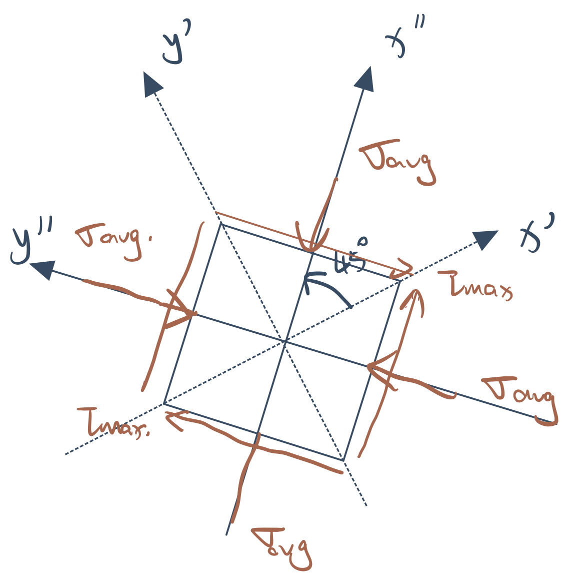
$$\tau_{x''y''} = \tau_{max} = -\frac{\sigma_{x'}-\sigma_{y'}}{2}\sin 2\beta + \cancelto{0}{\tau_{x'y'}}\cos 2\beta$$

$$= 90 \text{ MPa} \Rightarrow \beta = -45°$$

$$\sigma_{x''} = \sigma_{y''} = \sigma_{ave} = \frac{\sigma_{x'}+\sigma_{y'}}{2} = -20 \text{ MPa.}$$



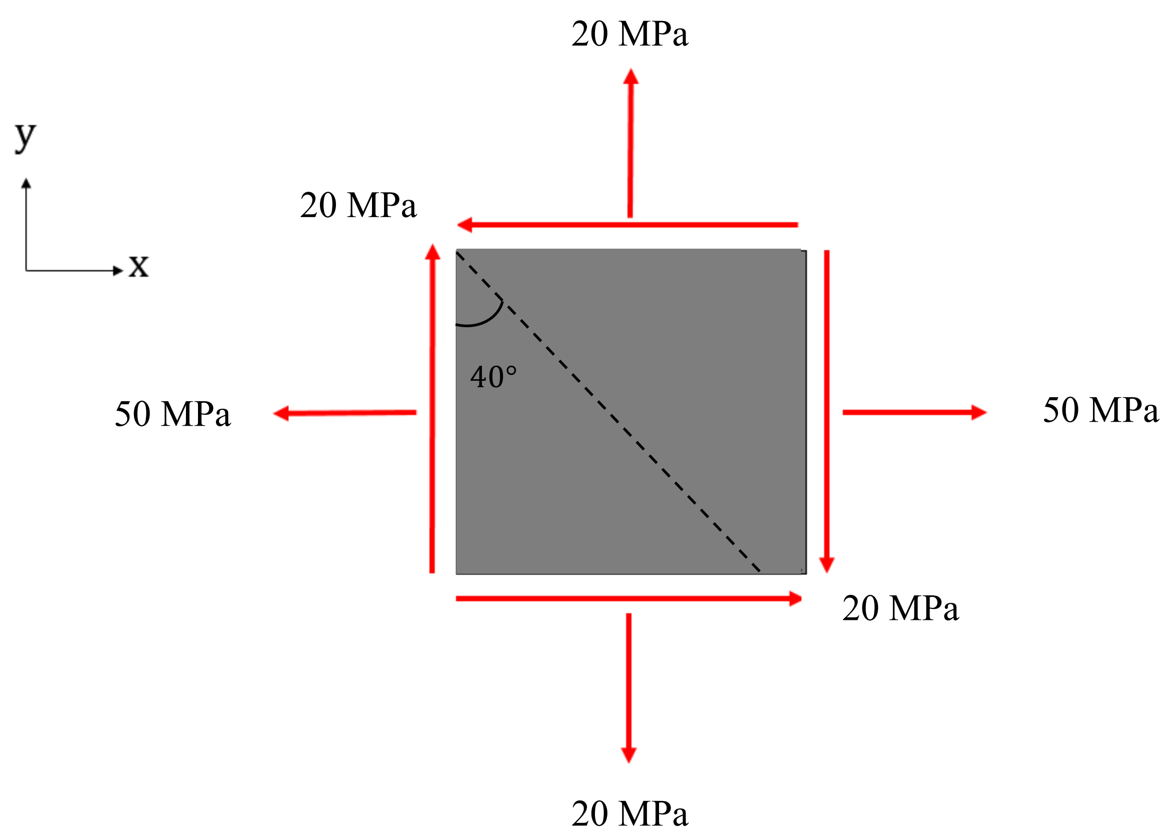
or



Problem 10.2 (10 points)

For the state of plane stress shown in the figure:

a) Draw the Mohr's circle and indicate the points that represent stresses on face x and on face y.
b) Using the Mohr's circle, determine the normal and shear stress on the inclined plane shown in the figure and label this point as N on the Mohr's circle.



$\sigma_x = 50\,\text{MPa}$

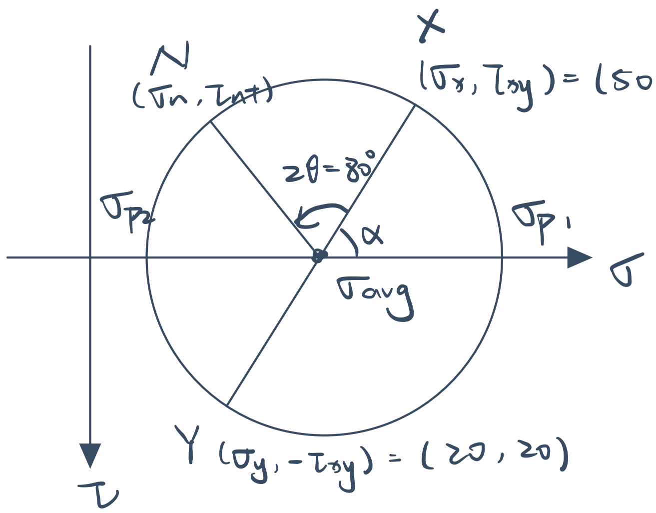
$\sigma_y = 20\,\text{MPa}$

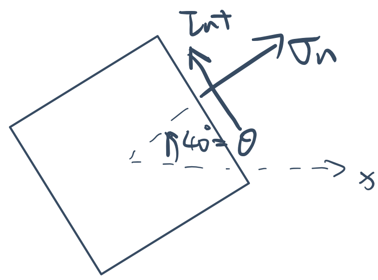
$\tau_{xy} = -20\,\text{MPa}$

$$\Rightarrow \quad \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 35\,\text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 25\,\text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$





$$\tan\alpha = \frac{20}{50-35} \Rightarrow \alpha = 0.927\,\text{rad} = 53.13°$$

$$N = \left(R\cos(\alpha + 2\theta) + \sigma_{avg},\ -R\sin(\alpha + 2\theta)\right)$$

$$= (17.9\,\text{MPa},\ -18.25\,\text{MPa})$$

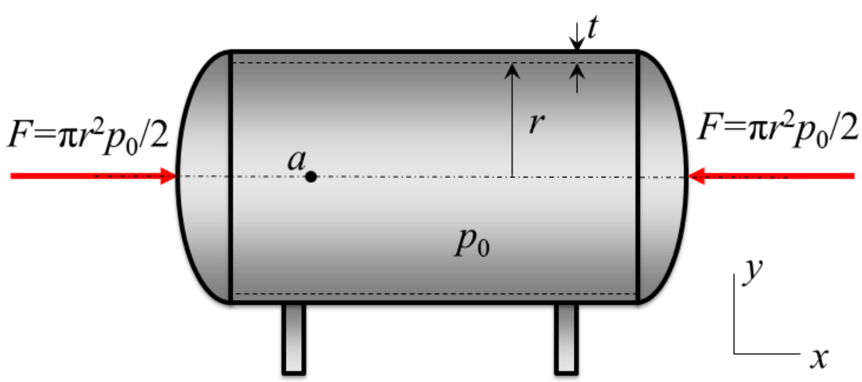
$\sigma_n = 17.9\,\text{MPa}$

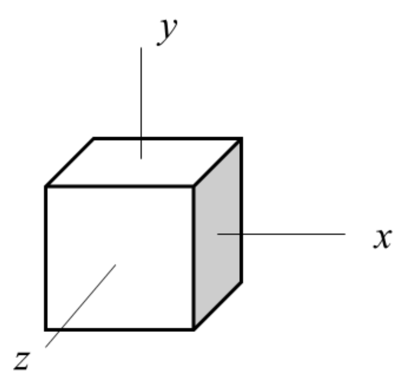
$\tau_{nt} = -18.25\,\text{MPa}$

Problem 10.3 (10 points)

The cylindrical pressure vessel shown below has an inner radius of r and a wall thickness of t, with semi-spherical end caps ($t/r \ll 1$). The pressure vessel contains a gas that is under a pressure of p_0. A pair of compressive loads (force) $F=\pi r^2 p_0/2$ is applied on the end caps of the vessel.

a) Determine the state of stress at point "a" on the front of the vessel.
b) Show the components of stress on the stress element provided.
c) Make an accurate drawing of the three Mohr's circles for this state of stress.
d) The allowable tensile stress is 100 MPa and the allowable shear stress is 40 MPa. What is the minimum wall thickness of the vessel? At this step, use $p_0 = 1$ MPa, $r = 0.5$ m in your analysis.





stress state @ "a"

a). $\sigma_a = \dfrac{P_o r}{2t}$.

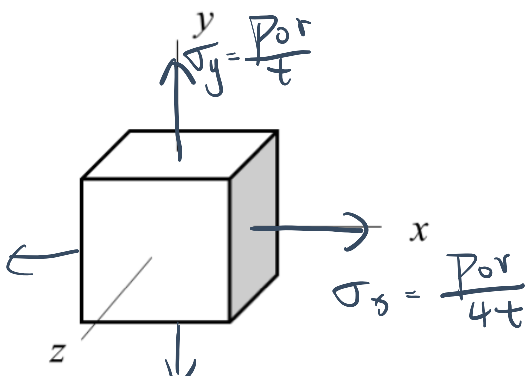
$$\sigma_x = \sigma_a - \frac{F}{A}$$

$$= \frac{P_o r}{2t} - \frac{\pi r^2 P_o / 2}{2\pi r t}$$

$$= \frac{P_o r}{4t}.$$

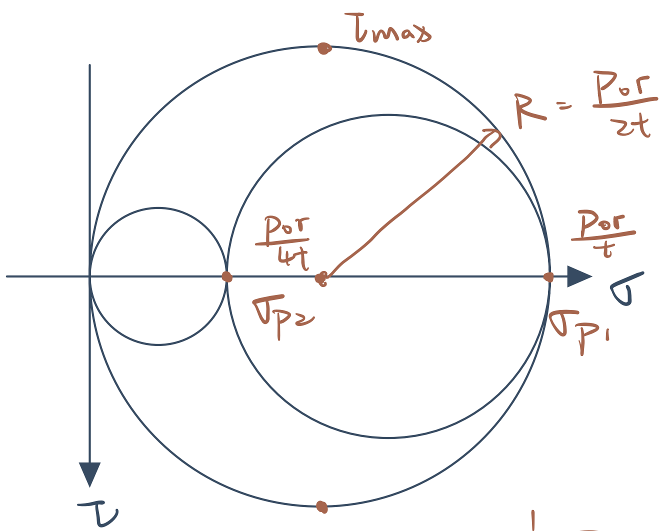
$$\sigma_h = \frac{P_o r}{t} = \sigma_y$$

b)



stress state @ "a"

c).



$$\tau_{max} = \frac{1}{2}\sigma_{p1} = \frac{P_o r}{2t} = R$$

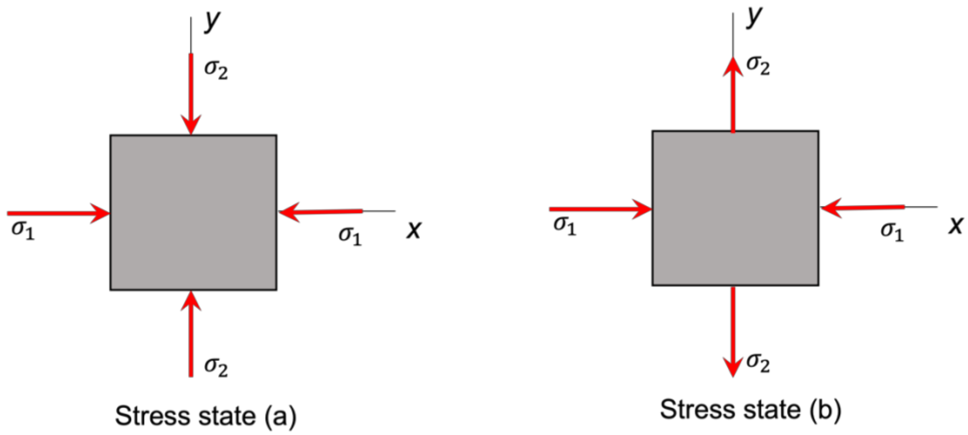
d) ① $\sigma_{p1} = \sigma_{max} = 100\,\text{MPa} = \dfrac{P_o r}{t} \Rightarrow t \geq \dfrac{P_o r}{100} = 5\text{ mm}$

② $\tau_{max} = 40\,\text{MPa} = \dfrac{P_o r}{2t} \Rightarrow t \geq \dfrac{P_o r}{80} = 6.25\text{ mm}$

$\therefore\ t_{min} = 6.25\text{ mm}$

Problem 10.4 (5 points)

Consider stress states (a) and (b) shown above, with $|\sigma_1| > |\sigma_2|$. Let $\left(|\tau|_{max,abs}\right)_a$ and $\left(|\tau|_{max,abs}\right)_b$ represent the absolute maximum shear stress corresponding to stress states (a) and (b), respectively. Choose the response below that describes the relative sizes of these stresses.



i. $\left(|\tau|_{max,abs}\right)_a > \left(|\tau|_{max,abs}\right)_b$
ii. $\left(|\tau|_{max,abs}\right)_a = \left(|\tau|_{max,abs}\right)_b$
iii. $\left(|\tau|_{max,abs}\right)_a < \left(|\tau|_{max,abs}\right)_b$

a): $\sigma_2 < \sigma_1 < 0$

b): $\sigma_1 < 0 < \sigma_2$.

$$|\tau_{max}| = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$