ME 323: Mechanics of Materials — *Homework Set 9*

Fall 2023 — *Due: Friday, November 10*

Problem 9.1 (10 points)

A beam ABC of length $2L$ is loaded with a distributed load over half its length as shown in the figure below. The beam is fixed at A and supported by a roller at C. The beam has a modulus of elasticity E and second area moment of the cross section I.

Compute the following:

a) The free body diagrams of the entire beam.
b) The equilibrium equations for the entire beam.
c) The reaction forces at A and C. (Hint: make use of the principle of superposition to avoid integration.)
d) The bending moment $M(x)$ in terms of p_0, E, I, x, L.
e) The shear force $V(x)$ in terms of p_0, E, I, x, L.

Plot the following:

f) The shear force diagram.
g) The bending moment diagram.

Box in your answers at each step.

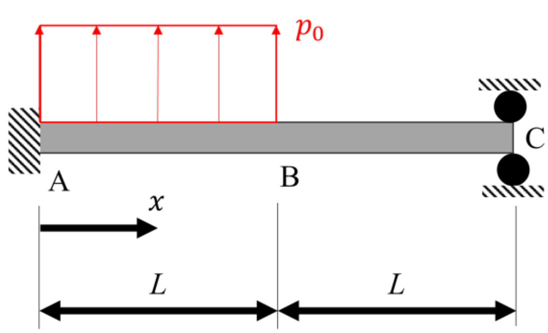

Fig. 9.1

Problem 9.2 (10 points)

A rod AC is made up of two segments AB and BC. Both segments have a Young's modulus of elasticity E and cross-sectional area A. The rod is subjected to axial loads at connectors A and B, as shown below. Compute the displacements u_A and u_B at A and B using the finite element method. One finite element is used to model each segment. Follow the given outline:

a) Draw a diagram labelling your nodes and elements.
b) Draw the free body diagram of the entire rod AC.
c) Calculate the total stiffness matrix $[K_{total}]$.
d) Calculate the total forcing vector $\{F_{total}\}$.
e) Calculate the stiffness matrix [K] after imposing the appropriate boundary conditions.
f) Calculate the forcing vector {F} after imposing the appropriate boundary conditions.
g) Write down the system of equations [K]{u}={F} and solve for u_A and u_B in terms of FL/EA.

Box in your answers at each step.

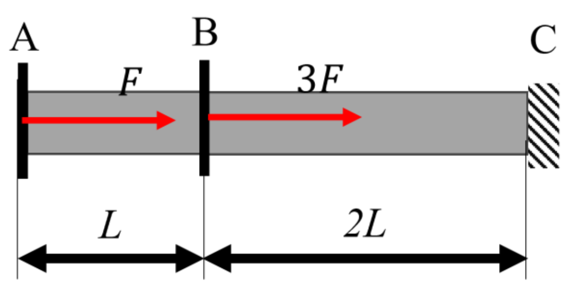

Fig. 9.2

Problem 9.3 (10 points)

A rod is made up of four segments: AB, BC, CD, and DE. Their geometric and material properties are given below:

Segment	Cross section area	Elastic modulus
AB	Varies linearly from $6A$ to $4A$	E
BC	Varies linearly from $4A$ to $2A$	E
CD	$4A$ (constant)	2E
DE	Varies linearly from $2A$ to $4A$	2E

The finite element method will be used to compute several quantities of interest. Consider four elements (1-4) and 5 nodes (A-E) as labelled in the figure. Follow the given outline:

a) Draw a free body diagram of the entire rod AE.
b) Write down the total stiffness matrix $[K_{total}]$.
c) Write down the total force vector $\{F_{total}\}$.
d) Calculate the stiffness matrix [K] after imposing the appropriate boundary conditions.
e) Calculate the forcing vector {F} after imposing the appropriate boundary conditions.
f) Write down the system of equations [K]{u}={F} and solve for the displacements. Express your answer in terms of PL/EA.
g) Compute the reaction forces at A and E. Express your answers in terms of P.

Box in your answers at each step.

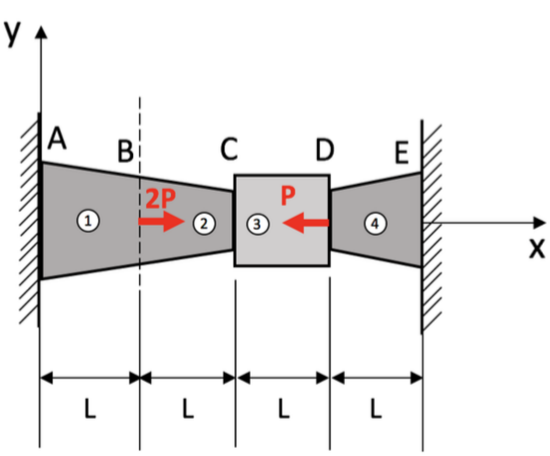

Fig. 9.3

Problem 9.4 - Conceptual (5 points)

1. Consider a rod fixed to the wall at A shown below. Let the rod be discretized into 15 nodes and 14 elements.
   i. What is the size of the total stiffness matrix? (0.5 points)
      a) 14 x 14 b) 15 x 15 c) 16 x 16 d) 17 x 17
   ii. What is the size of the stiffness matrix after imposing the boundary conditions? (0.5 points)
      a) 14 x 14 b) 15 x 15 c) 16 x 16 d) 17 x 17

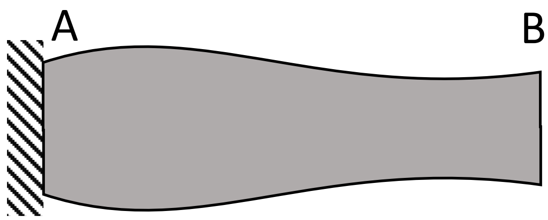

Fig. 9.4.1

2. Consider a different rod discretized into 4 nodes (A,B,C,D) and 3 elements as shown below. The total stiffness matrix has been partially computed as

$$[K] = \begin{pmatrix} 1 & -1 & 0 & a_4 \\ -1 & 3 & a_3 & 0 \\ 0 & a_2 & 5 & -3 \\ a_1 & 0 & -3 & 3 \end{pmatrix} \times 10^9 \text{ N/m}$$

   What are the values of a_1, a_2, a_3, and a_4 in N/m? (1 point per question)
   i. $a_1 =$
      a) 0 b) -10^9 c) 10^9 d) -3×10^9 e) 3×10^9
   ii. $a_2 =$
      a) 0 b) -2×10^9 c) 2×10^9 d) -5×10^9 e)-5×10^9
   iii. $a_3 =$
      a) 0 b) -2×10^9 c) 2×10^9 d) -5×10^9 e)-5×10^9
   iv. $a_4 =$
      a) 0 b) -10^9 c) 10^9 d) -3×10^9 e) 3×10^9

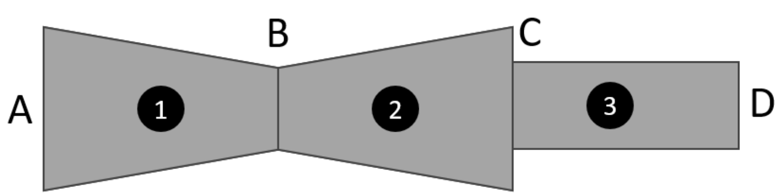

Fig. 9.4.2