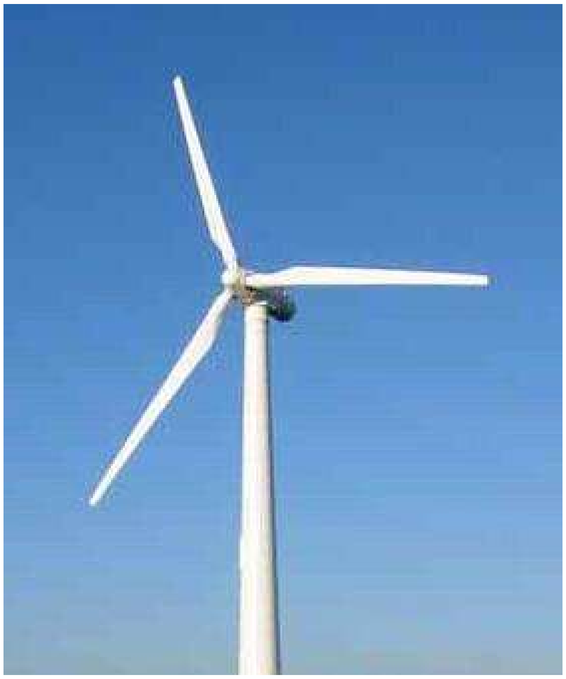
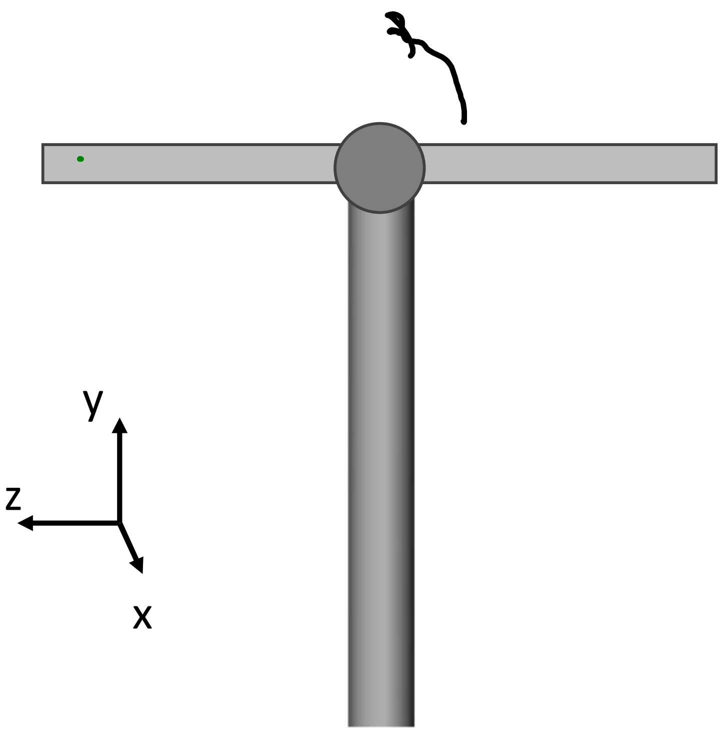
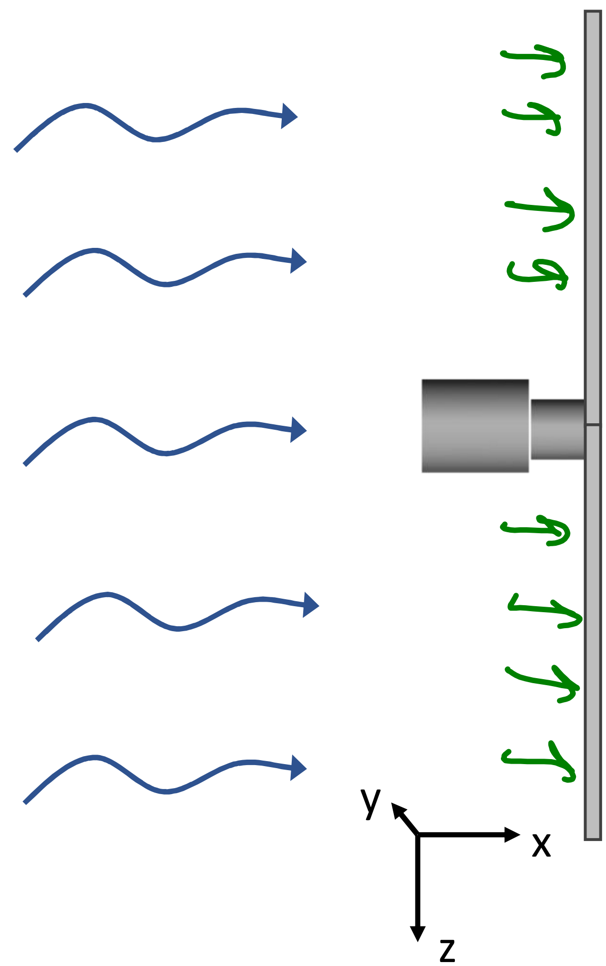
Combined Loading in Windmills



Why does a windmill have 3 blades?





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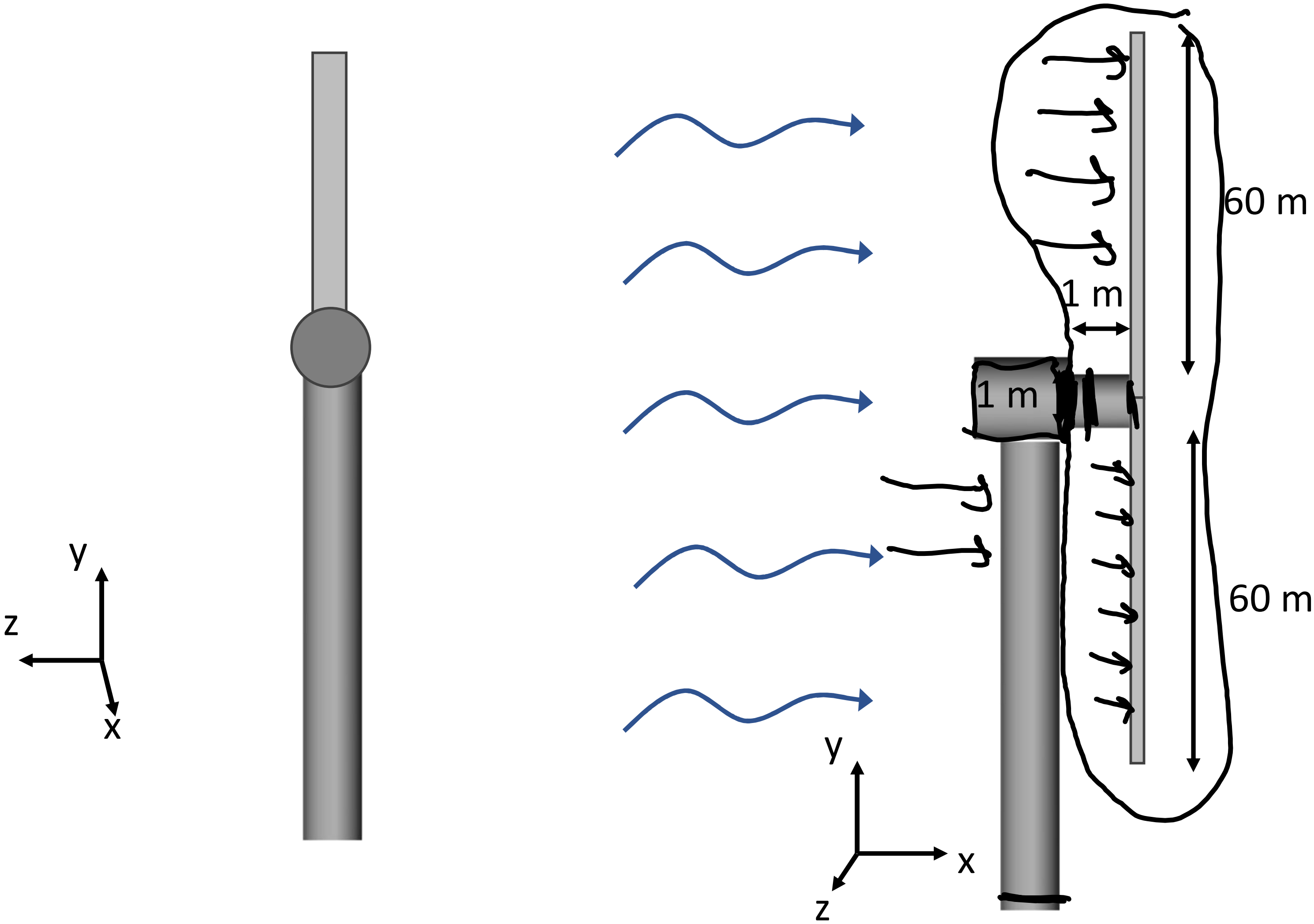
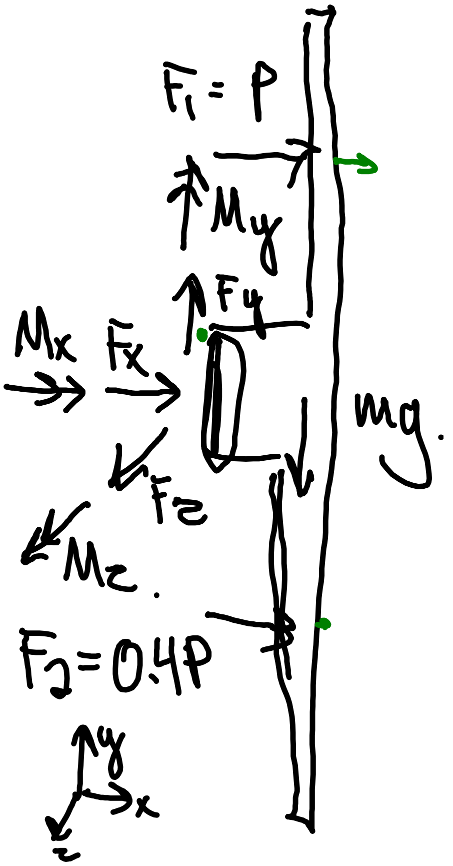


The rotor assembly weighs ~8 000 kg.
A distributed force of 300 N/m acts over the blades, which are each 60 m long.
Find the stresses at the interface between the hub and the nacelle.



$$\Sigma F = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} + 0.4P \hat{i} + P \hat{j} - mg \hat{j} = 0$$

$$F_x = -1.4P$$

$$F_y = mg$$

$$F_z = 0$$

$$\Sigma M = M_x \hat{i} + M_y \hat{j} + M_z \hat{k} + \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = 0$$

$r_1 = (1, 30, 0) \quad F_1 = (P, 0, 0)$

$r_2 = (1, -30, 0) \quad F_2 = (0.4P, 0, 0)$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 30 & 0 \\ P & 0 & 0 \end{vmatrix} = (0, 0, -30P)$$

$$\vec{r}_2 \times \vec{F}_2 = (0, 0, 12P)$$

$$0 = M_x \hat{i} + M_y \hat{j} + M_z \hat{k} + 12P\hat{k} - 30P\hat{k} = 0$$

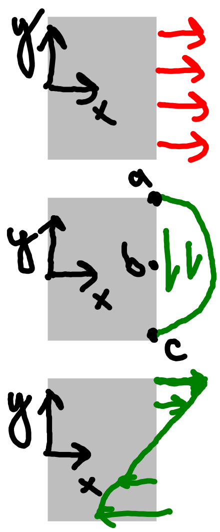
$M_x = 0 \quad M_y = 0 \quad M_z = 18P \leftarrow$

$P = (300\ N/m)(60\ m) = 18\ 000\ N \quad |M_z| = 324\ 000\ N \cdot m.$

Windmills

$F = (1.4P, -mg, 0)$

$M = (0, 0, -324\,000)$



Force	a	b	c
F_x	$\sigma_x = \frac{F_x}{A}$	$\sigma_x = \frac{F_x}{A}$	$\sigma_x = \frac{F_x}{A}$
F_y	0	$\tau_{xy} = \frac{2V}{A}$	0
M_z	$\sigma_x = \frac{-MR}{I}$ $\sigma_x = \frac{\lvert M\rvert R}{I}$	0	$\sigma_x = -\frac{\lvert M\rvert R}{I}$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = 0.0169\ m^4$

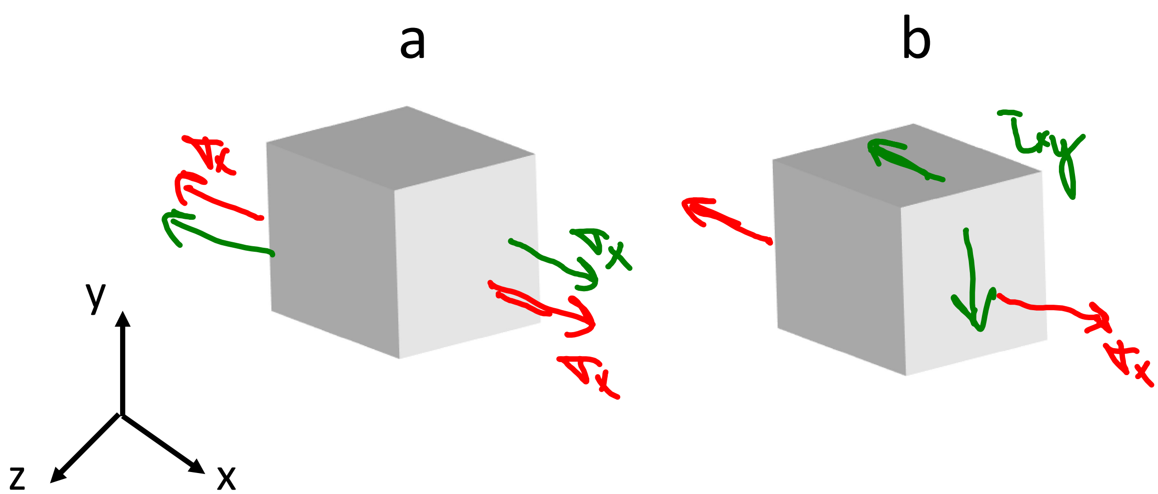
$A = \pi(r_o^2 - r_i^2) = 0.15\ m^2$

$\frac{F_x}{A} = 169\ kPa$

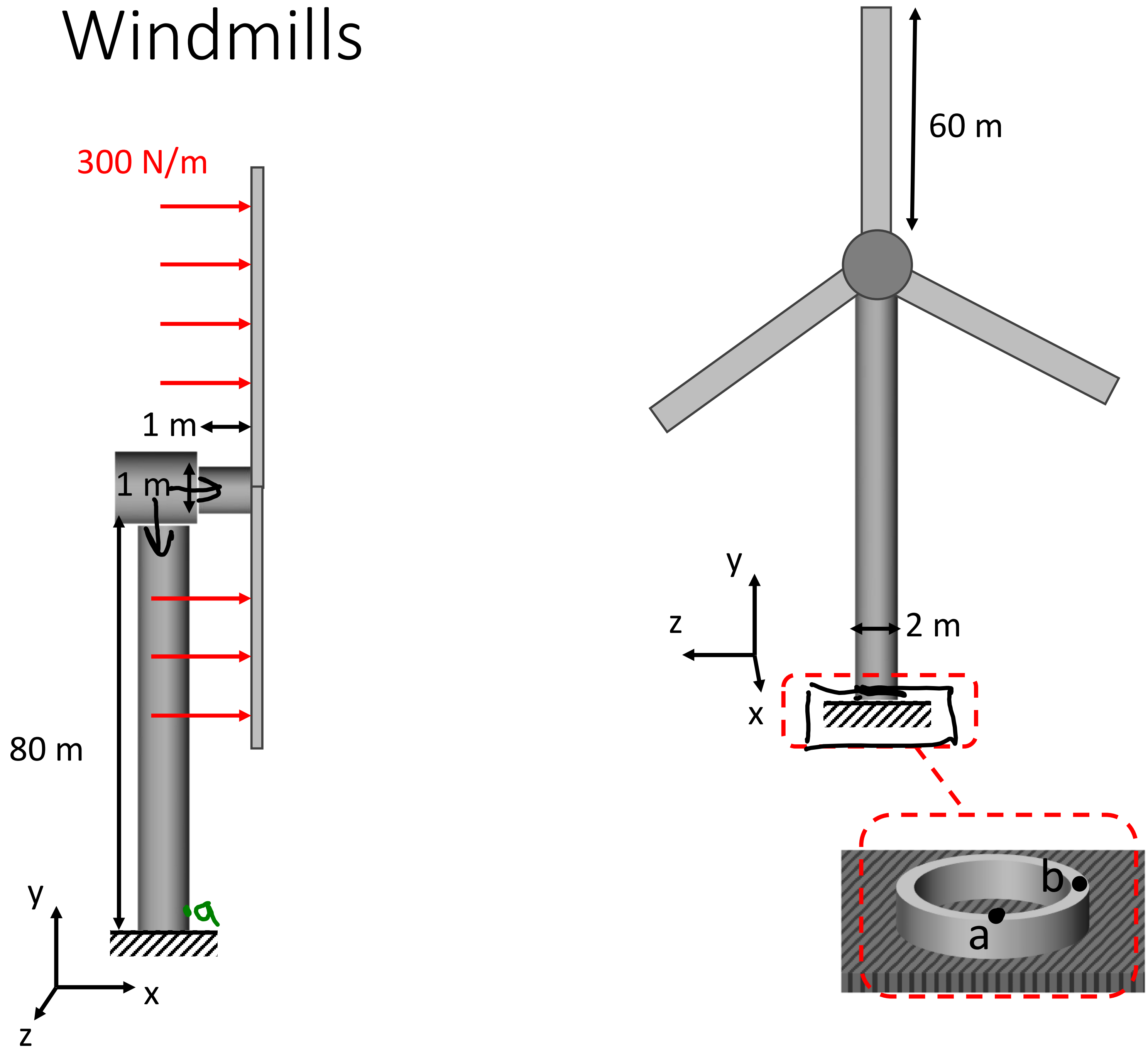
$\frac{|M|R}{I} = 9.586\ MPa.$

$\sigma_{x,a} = 9.75\ MPa$

$\sigma_{x,c} = -9.4\ MPa.$

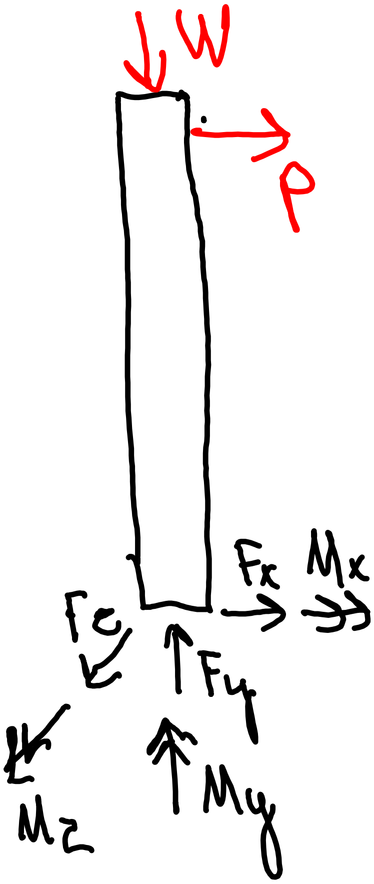


Windmills



The wind applies a distributed load of 300 N/m on each of three blades that are 60 m long. The rotor assembly and nacelle together weigh ~74 000 kg. The shaft of the windmill is 2 m in diameter at the base and is made of steel that has a thickness of 0.1 m. The weight of the shaft can be neglected compared to the weight of the rotor assembly and nacelle.

Find the state of stress at point **a** and point **b** at the bottom of the shaft. Draw the 3D stress element for the loading conditions.



$$\Sigma F_x = F_x + P = 0 \Rightarrow F_x = -P$$

$$\Sigma F_y = F_y - W = 0 \Rightarrow F_y = W$$

$$\Sigma F_z = F_z = 0$$

$$(\Sigma M)_x = M_x = 0$$

$$(\Sigma M)_y = M_y = 0$$

$$(\Sigma M)_z = M_z - P(80) = 0$$

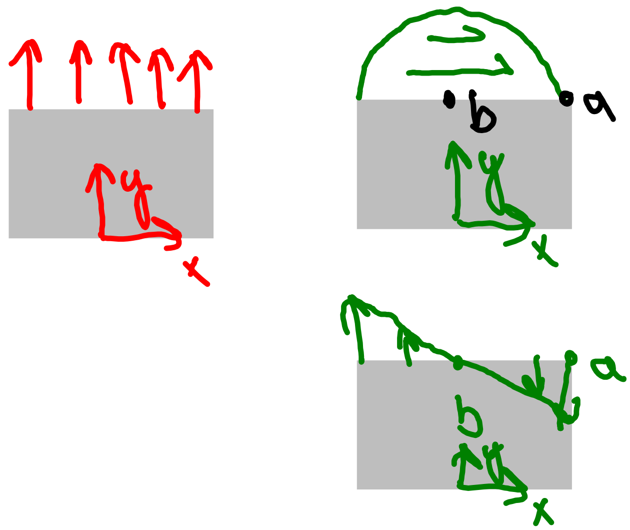
$$M_z = 80P.$$

On positive face:

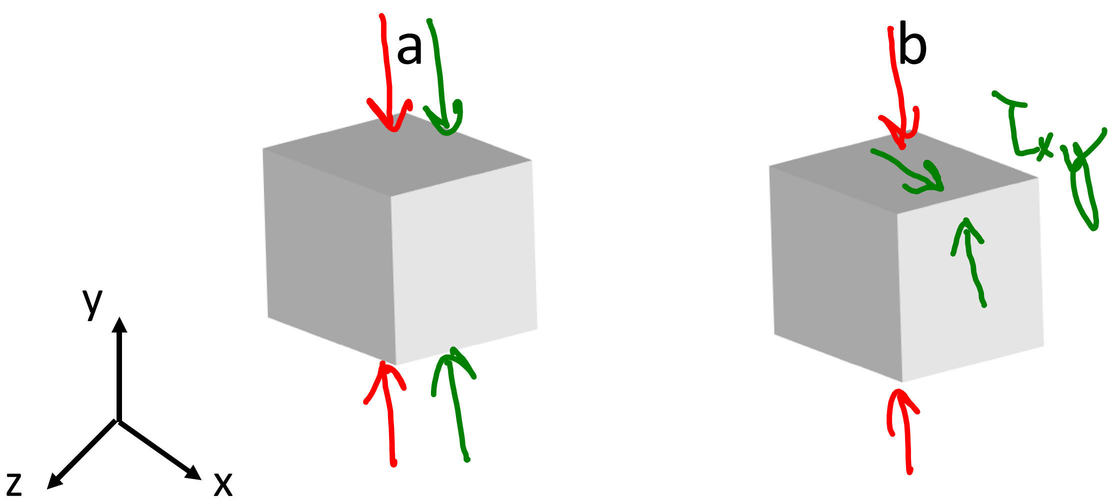
$$F = (P, -W, 0)$$

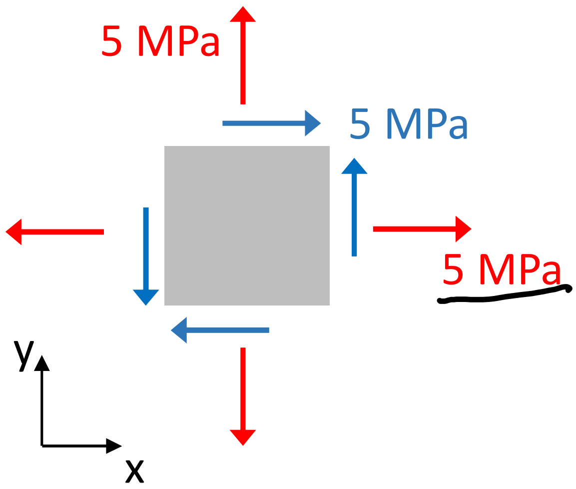
$$M = (0, 0, -80P)$$

Windmills



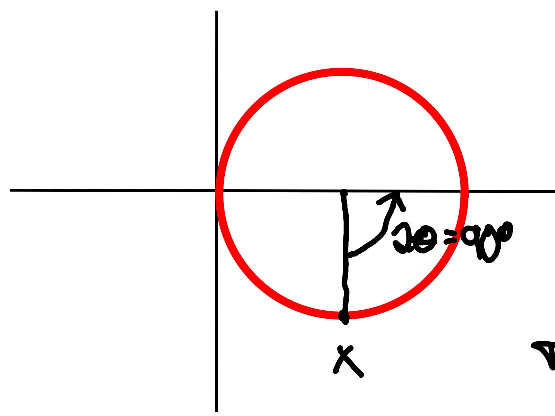
Force	a	b
F_y	$\sigma_y = -\frac{\lvert F_y \rvert}{A}$	$\sigma_y = -\frac{\lvert F_y \rvert}{A}$
F_x	0	$\tau_{xy} = \frac{2V}{A}$
M_z	$\sigma_y = -\frac{\lvert M_z \rvert R}{I}$	0





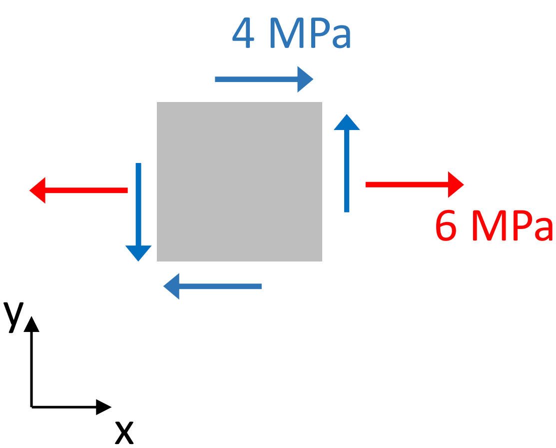
θ_{P1}

At what angle is X compared to σ_{P1}?



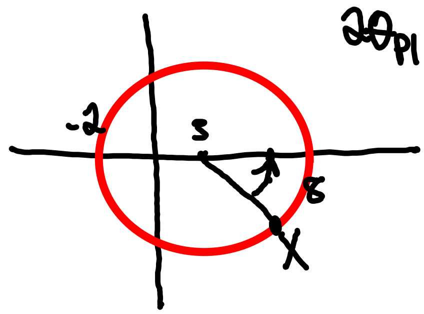
$\tau_{avg} = 5$ MPa
$R = 5$ MPa

$\theta_{P1} = 45°$.

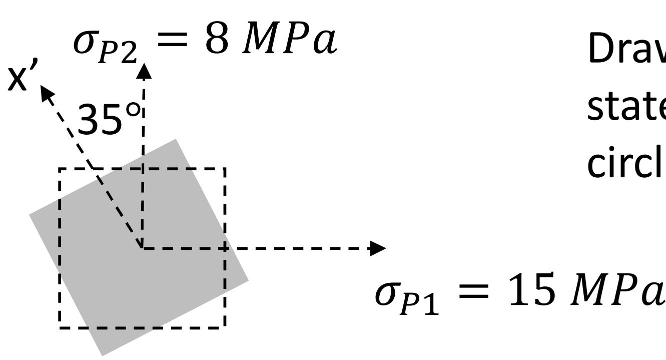


At what angle is X compared to σ_{P1}?

$2\theta_{P1} = \sin^{-1}\left(\frac{4}{5}\right) = \tan^{-1}\left(\frac{4}{3}\right)$



$\tau_{avg} = 3$ MPa $\quad R = \sqrt{\left(\frac{6}{2}\right)^2 + 4^2} = 5$ MPa



Draw the Mohr's circle for this stress state. Indicate the position of x' on the circle.