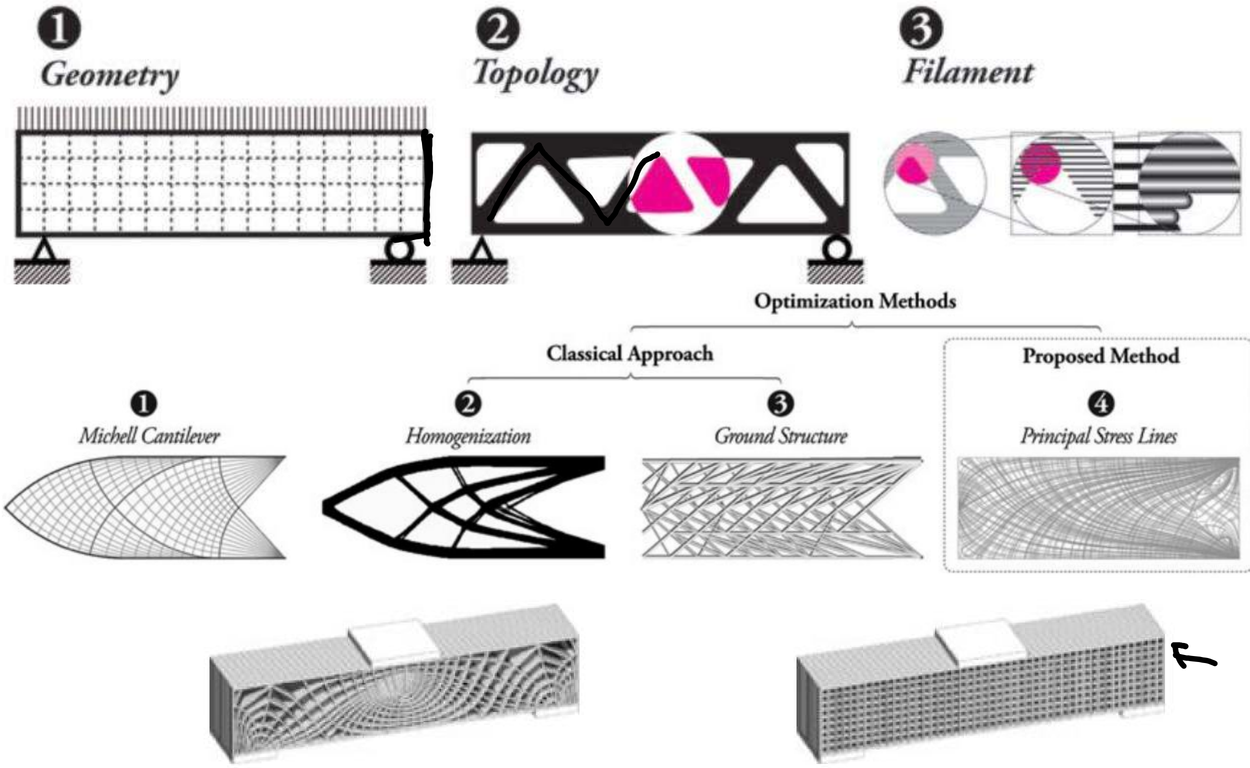
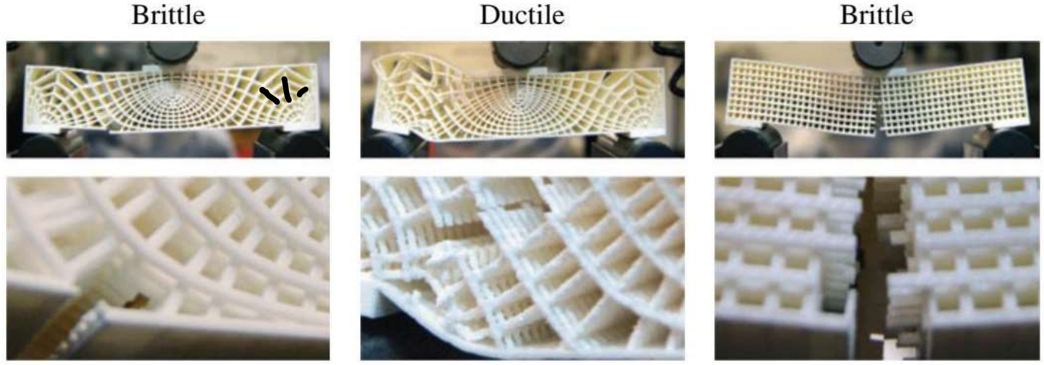
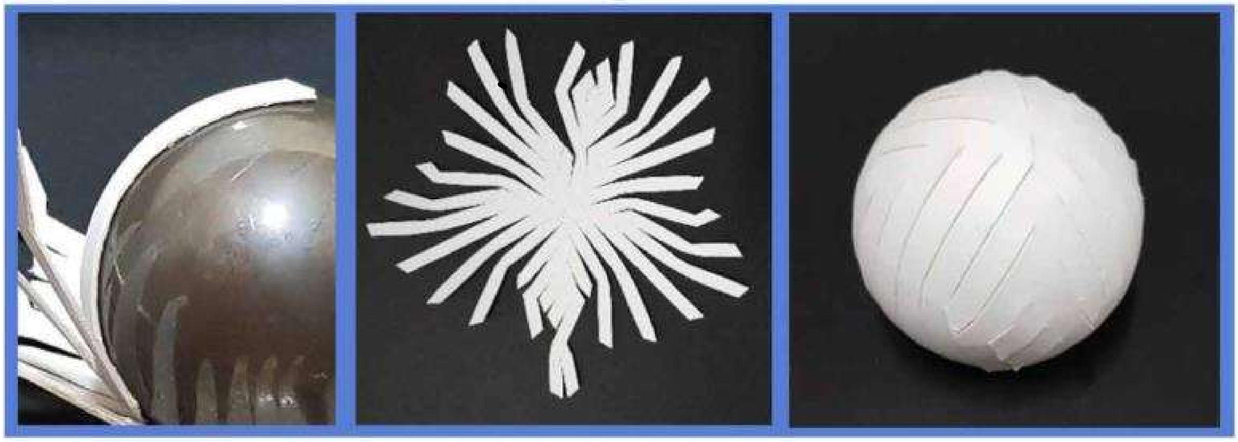
Printing Along Principal Stress Lines

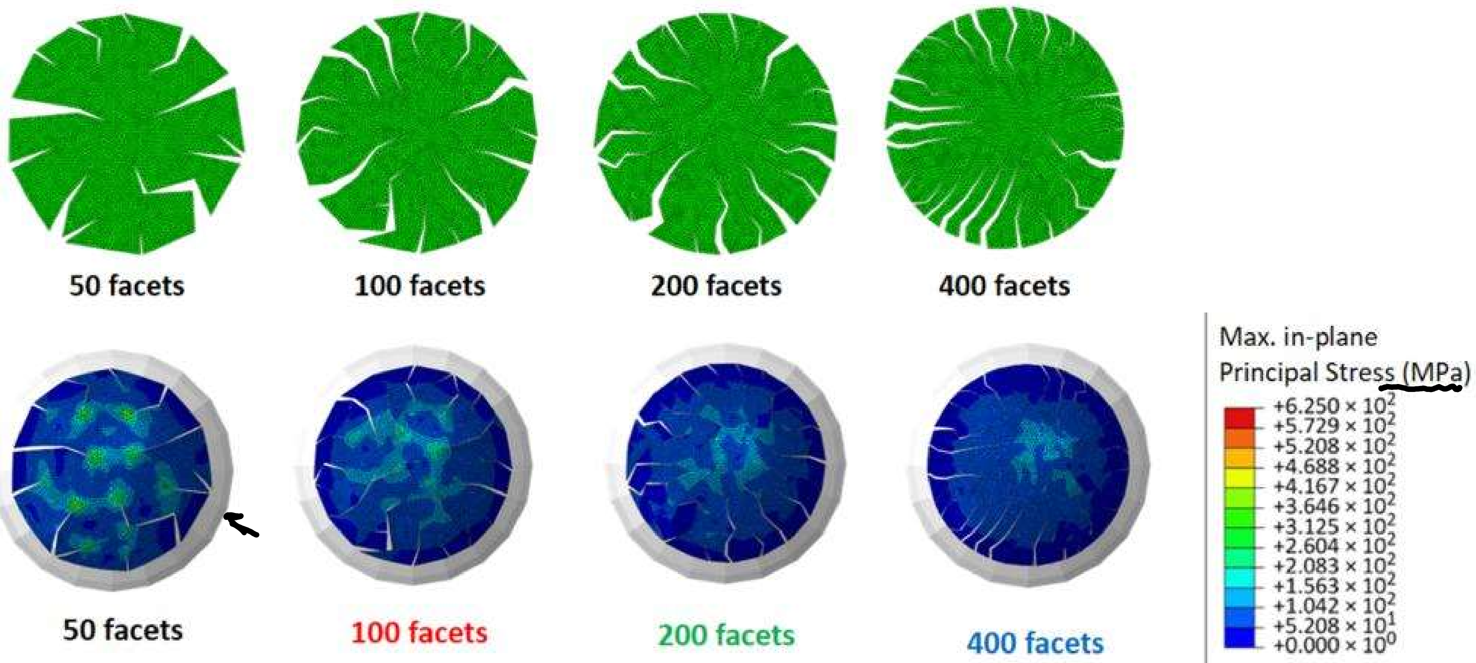


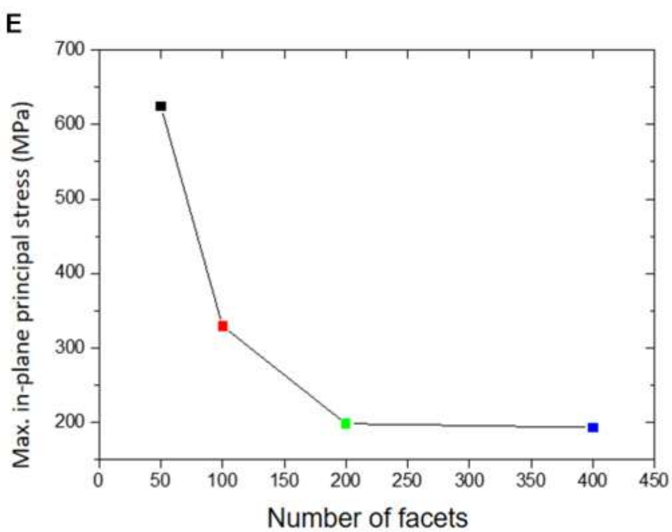
Case	*SLAM-XY*	*SLAM-XZ*	*Grid-XY*
Predicted normalized yield load and displacement	2495.1 N/N at 0.79 mm		2250.1 N/N at 5.12 mm
Average normalized yield load and displacement	2537.7 N/N at 2.26 mm	541.6 N/N at 0.64 mm	1783.0 N/N at 2.93 mm
Average normalized ultimate load and displacement	2847.8 N/N at 2.92 mm	810.9 N/N at 2.46 mm	2799.9 N/N at 7.54 mm
Average elastic stiffness normalized by specimen mass	1111.8 (N/N)/mm	836.4 (N/N)/mm	601.6 (N/N)/mm
Failure mode description	Simultaneous gross section failures (tension) at concentrated location	Delamination between layers (tension) along Y axis, progressive failure at multiple locations	Gross section (tension), multiple locations
Failure type	Brittle	Ductile	Brittle
Failure profiles Global geometry Member view			

Tam and Mueller, 3D Printing and Additive Manufacturing, "Additive Manufacturing Along Principal Stress Lines", 2017.

Computational Wrapping







Lee et al, Sci. Adv., "Computational wrapping: A universal method to wrap 3D-curved surfaces with nonstretchable materials for conformal devices", 6:eaax6212, 2020.

Course Roadmap

Ch 13: Mohr's Circles ✓

- Given the loading conditions at a point, what are the stress states at different angles?
- At what angle does the max normal stress and max shear stress occur?

Ch 14: Combined Loading

- What are the normal and shear stresses at points on a cross section due to combined axial, torsion, and bending loading?
- Determine the principal stresses and max shear stress at these points – use Mohr's circles.

Ch 15: Failure Analysis

- Given the stress states at a point, under what condition will a 3D structure fail?

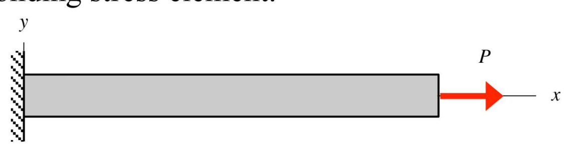
14. Stresses due to combined loadings

Objectives:

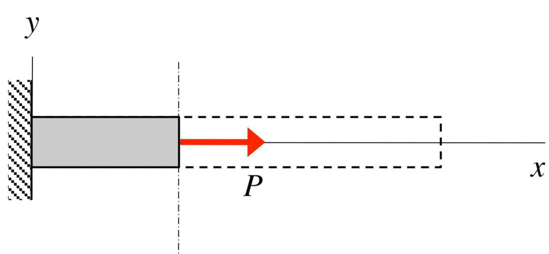
To study the combined effects of axial, torsion and bending loads on the principal and maximum shear components of stress at a point.

Background:
For each of the following three loading situations, consider the i) internal loading; ii) stress distribution; and , iii) the corresponding stress element.

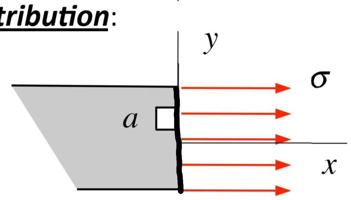
AXIAL LOADING



Internal loading:



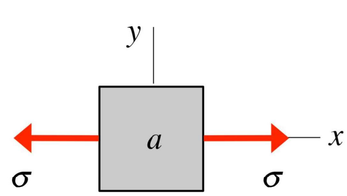
Stress distribution:



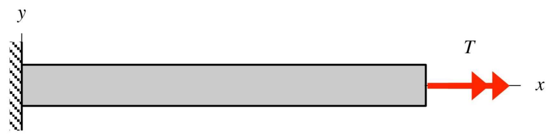
$$sign\ convention : \sigma\ \underline{positive}\ OUTWARD\ on\ face\ (tension)$$

$$\sigma = \frac{P}{A} = constant\ in\ y \quad ; \quad A = cross\text{-}sectional\ area$$

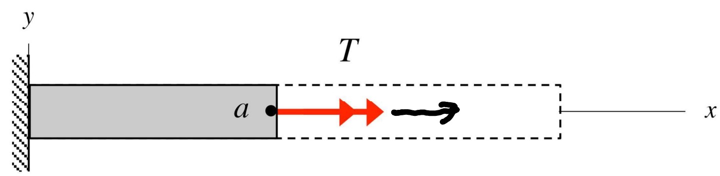
Stress element:



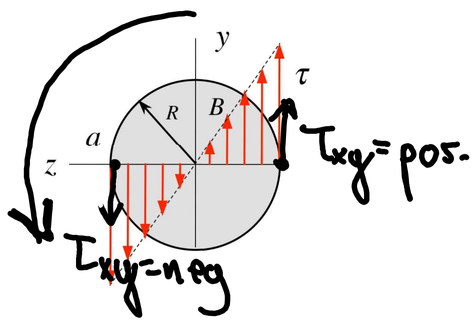
TORSIONAL LOADING



Internal loading:



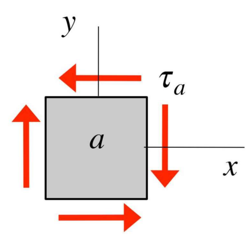
Stress distribution:



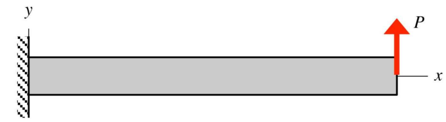
sign convention : *T* <u>*positive*</u> *OUTWARD on face* (*by right – hand rule*)

$$\tau_a = \frac{TR}{I_P} = \textit{linear in radial position} \quad ; \quad I_P = \textit{polar area moment}$$

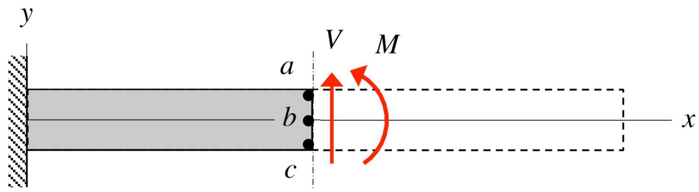
Stress element:



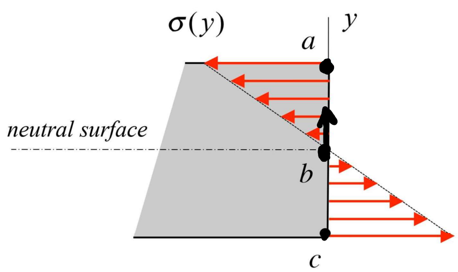
TRANSVERSE LOADING (e.g., rectangular cross section)



Internal loading:

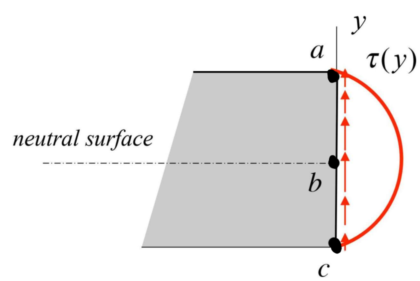


Normal stress distribution:



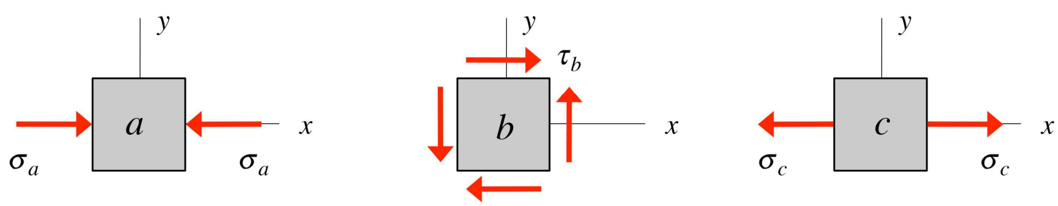
$$|\sigma_a| = \frac{M|y_a|}{I} \qquad \sigma_b = 0 \qquad |\sigma_c| = \frac{M|y_c|}{I}$$

Shear stress distribution:

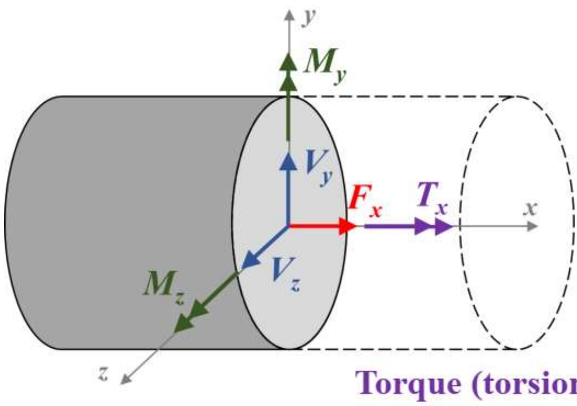


$$\tau_a = 0 \qquad |\tau_b| = \frac{3}{2}\frac{|V|}{A} \qquad \tau_c = 0$$

Stress element:



Combined Loading

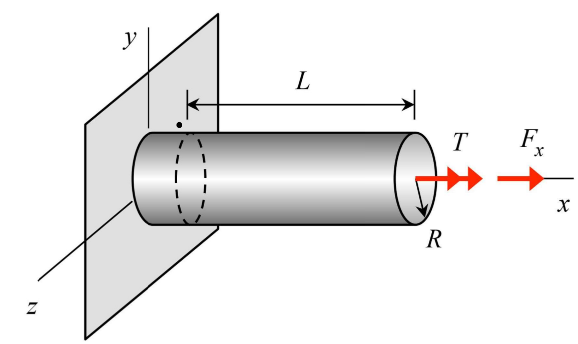


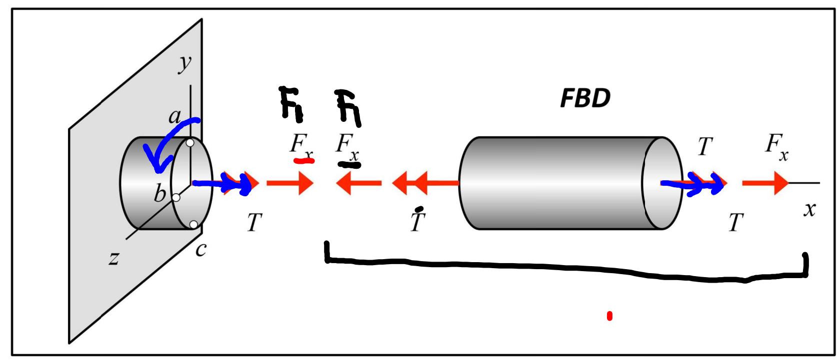
Load	Type of stress	Stress distribution	Lecture book ch.
Axial force F_x	Normal	$\sigma_x = F_x / A$	Ch. 6
Shear force V_y	Shear	$\tau_{xy} = \frac{V_y Q}{I_{zz} t}$	Ch. 10
Shear force V_z	Shear	$\tau_{xz} = \frac{V_z Q}{I_{yy} t}$	Ch. 10
Torque (torsional moment) T_x	Shear	$\tau = T\rho / I_p$	Ch. 8
Bending moment M_y	Normal	$\sigma_x = M_y z / I_{yy}$	Ch. 10
Bending moment M_z	Normal	$\sigma_x = -M_z y / I_{zz}$	Ch. 10

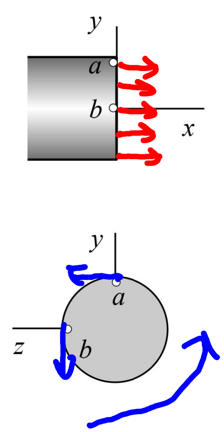
For combined loads, use superposition (possible because of the assumption of linearity).

$$\Sigma F_x = F_x - F_1 = 0$$
$$F_1 = F_x$$

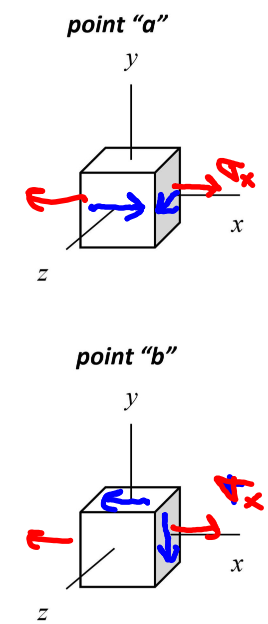
Example I

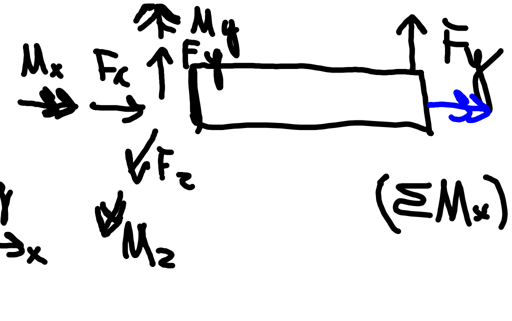






loading	stress comp. @ "a"	stress comp. @ "b"
F_x	$\sigma_x = \dfrac{F_x}{A}$	$\sigma_x = \dfrac{F_x}{A}$
T_x	$\tau_{xz} = \dfrac{TR}{I_p}$	$\tau_{xy} = \dfrac{-TR}{I_p}$

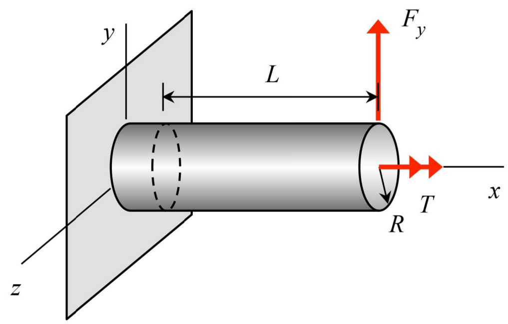


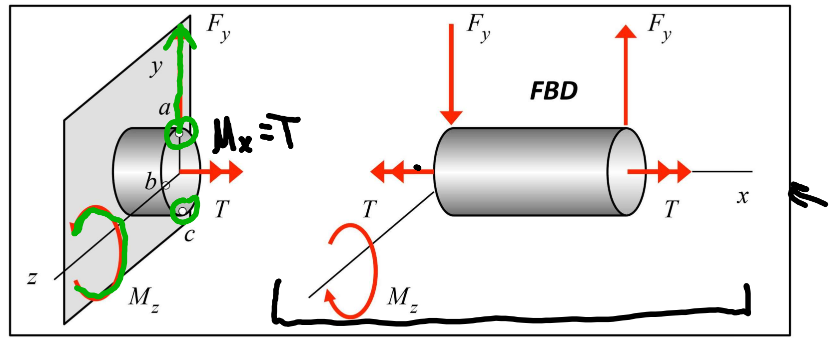


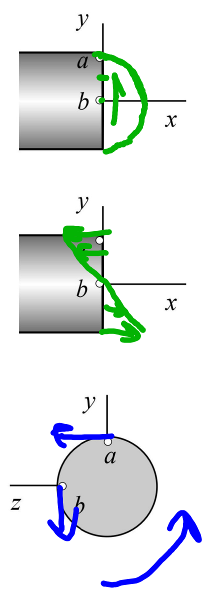
$$(\Sigma M_x) = T + M_x = 0$$

$$M_x = -T$$

Example II



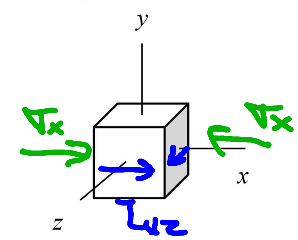




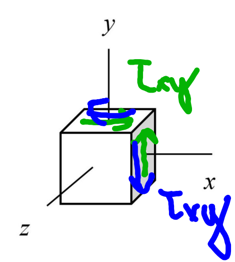
$$\sigma = -\frac{My}{I}$$

loading	stress comp. @ "a"	stress comp. @ "b"
V_y	0	$\tau_{xy} = \frac{4V}{3A}$
M_z	$\sigma_x = -\frac{\lvert M_z\rvert R}{I}$	0
T	$\tau_{xz} = \frac{TR}{I_p}$	$\tau_{xy} = -\frac{TR}{I_p}$

point "a"



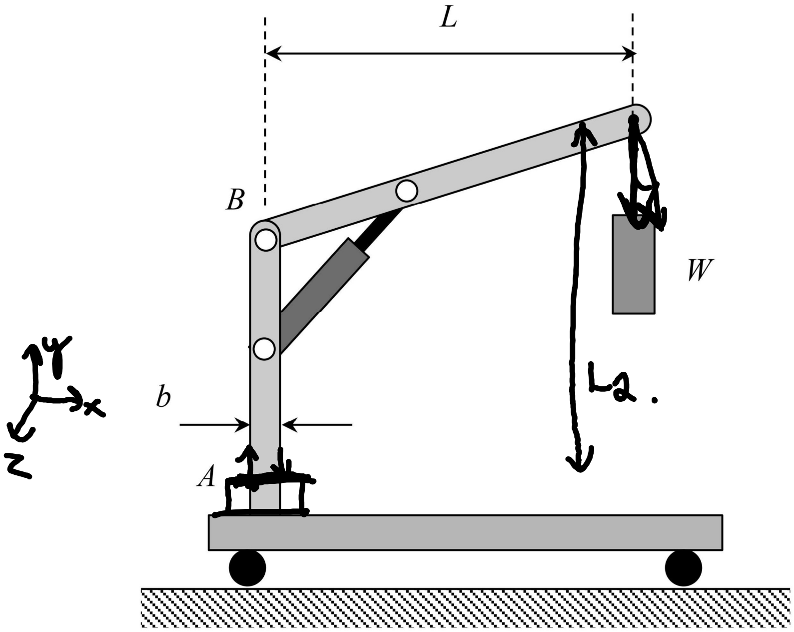
point "b"

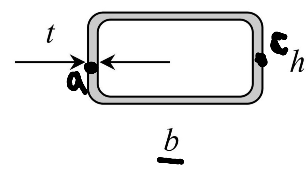


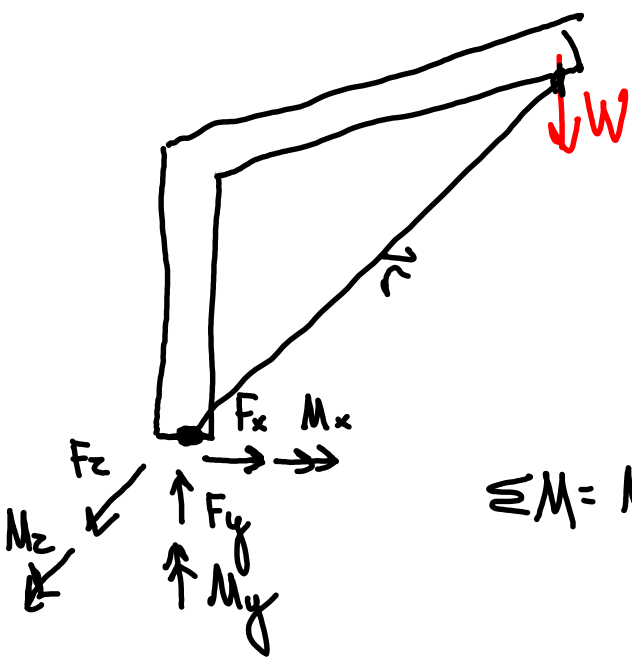
$$\tau_{xy} = \frac{4V}{3A} - \frac{TR}{I_p}.$$

Example 14.2

A crane is made up of a vertical column AB with a boom pinned to the column at B. The columnn has a tubular cross section of thickness t, as shown below. The boom supports a block with a weight of W. Determine the maximum tensile stress and maximum compressive stress near the base cross section at A when the boom is in the position shown.







$$\Sigma F = F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k} - W\hat{\jmath} = 0$$

$$F_x = 0 \qquad F_y - W = 0$$

$$F_z = 0$$

$$F_y = W$$

$$\Sigma M = M_x \hat{\imath} + M_y \hat{\jmath} + M_z \hat{k} + \vec{r} \times \vec{F} = 0$$

$$\vec{r} = (L, L_2, 0)$$

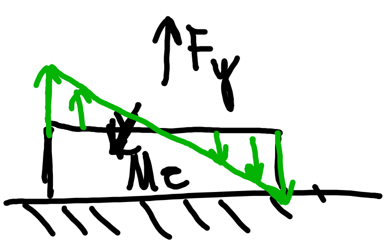
$$\vec{F} = (0, -W, 0)$$

$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ L & L_2 & 0 \\ 0 & -W & 0 \end{vmatrix} = (0, 0, -WL)$$

$$M_x\hat{i} + M_y\hat{j} + M_z\hat{k} - WL\hat{k} = 0$$

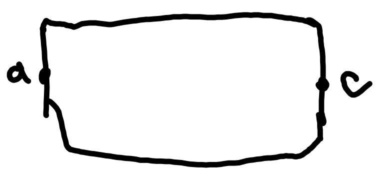
$M_x = 0$
$M_y = 0$
$M_z = WL$

$F = (0, W, 0)$ $\quad$ $M = (0, 0, WL)$ on neg face



On positive face:

$F = (0, -W, 0)$ $\quad$ $M = (0, 0, -WL)$



	a	c
F_y	$\sigma_y = -\frac{W}{A}$	$\sigma_y = -\frac{W}{A}$
M_z	$\sigma_y = \frac{-WL(-t/2)}{I}$	$\sigma_y = \frac{-WL(t/2)}{I}$.