

Stress Transformations

At the max normal stress, the shear stress is _____.

At the max shear stress, the normal stress is _____.

- *zero, zero*

- *zero, σ_{avg}*

- *zero, σ_{p1}*

- ~~*τ_{max} , zero*~~

- ~~*τ_{max} , σ_{avg}*~~

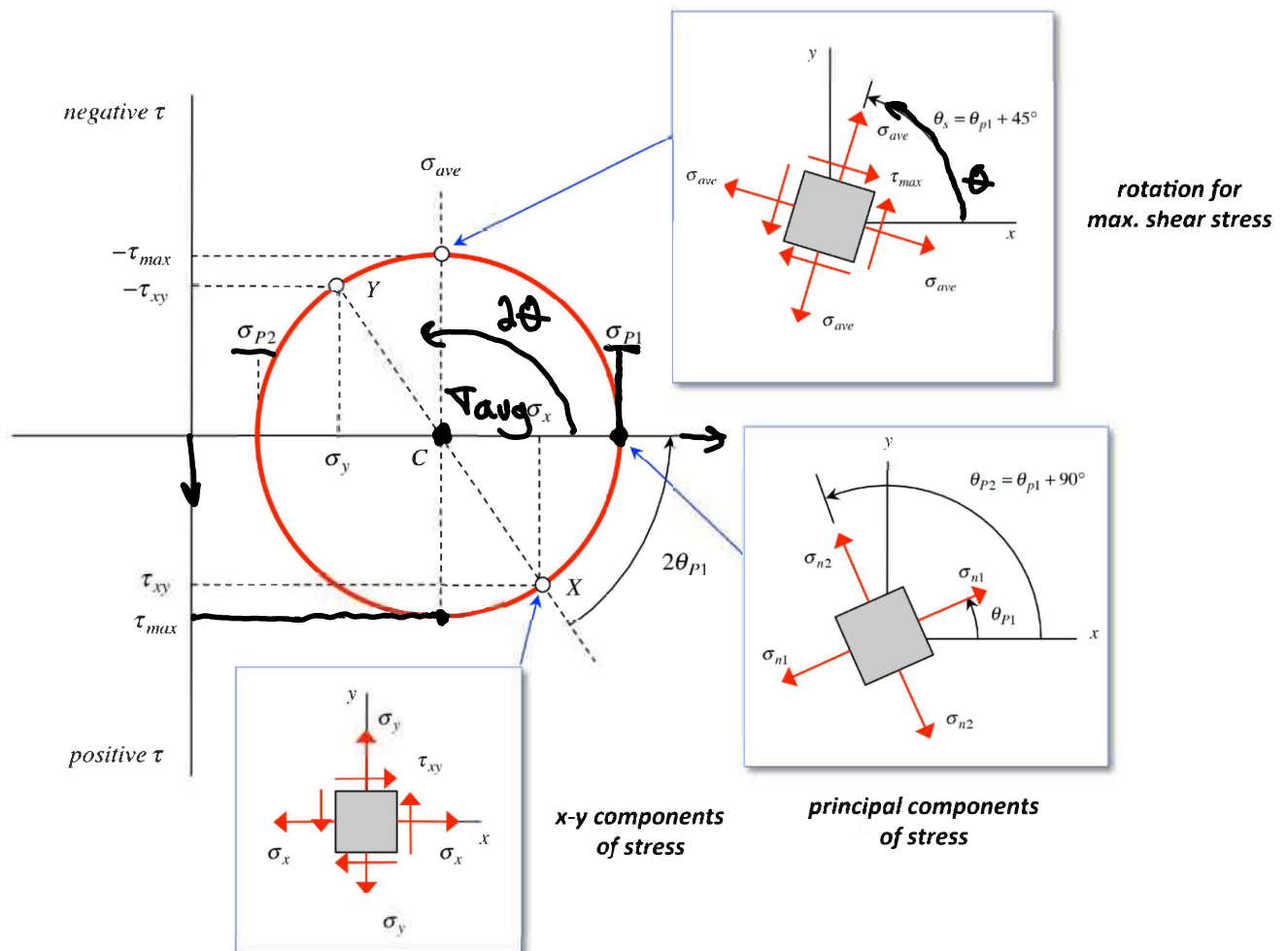
- ~~*τ_{max} , σ_{avg}*~~

- Angle of rotation vs. angle in the $\sigma - \tau$ plane. Note that the stress transformation equations are all written in terms of the angle 2θ , where θ is the physical angle of rotation in the x - y plane:

$$\left. \begin{aligned} \sigma &= \sigma_{ave} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau_{xy} \sin 2\theta \\ \tau &= - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta \end{aligned} \right\}$$

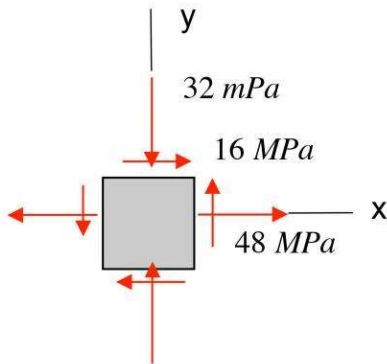
As a result, a physical angle of rotation of θ corresponds to an angle of rotation of 2θ in the $\sigma - \tau$ plane.

Both of these are demonstrated in the following figure. This figure contains much detailed information concerning the construction of Mohr's circle and the relationship of rotations in the physical $x - y$ plane to rotations in the $\sigma - \tau$ plane. In addition, we can readily see the locations of the principal stresses and maximum in-plane shear stress. Study this figure, and then move onto the next page where we have listed a series of steps that are convenient for constructing Mohr's circle from a state of stress $(\sigma_x, \sigma_y, \tau_{xy})$.



Example 13.4

Draw the in-plane Mohr's circle for the plane stress state shown below. What are the principal stresses and the maximum in-plane shear stress?



$$\sigma_{avg} = \frac{48 - 32}{2} = 8 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

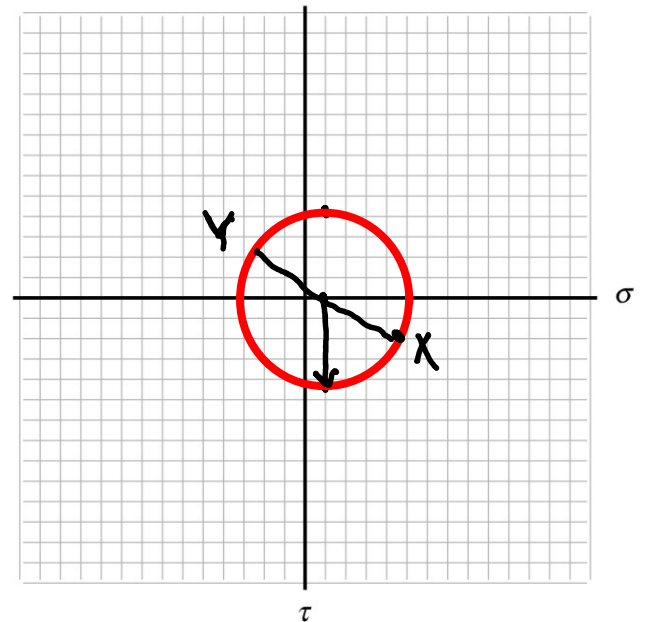
$$R = \sqrt{\left(\frac{48 + 32}{2}\right)^2 + 16^2} = 43 \text{ MPa.}$$

$$X = (\sigma_x, \tau_{xy}) = (48, 16)$$

$$Y = (\sigma_y, -\tau_{xy}) = (-32, -16).$$

$$\sigma_{p1} = \sigma_{avg} + R = 8 + 43 = 51 \text{ MPa}$$

$$\sigma_{p2} = \sigma_{avg} - R = -35 \text{ MPa.}$$

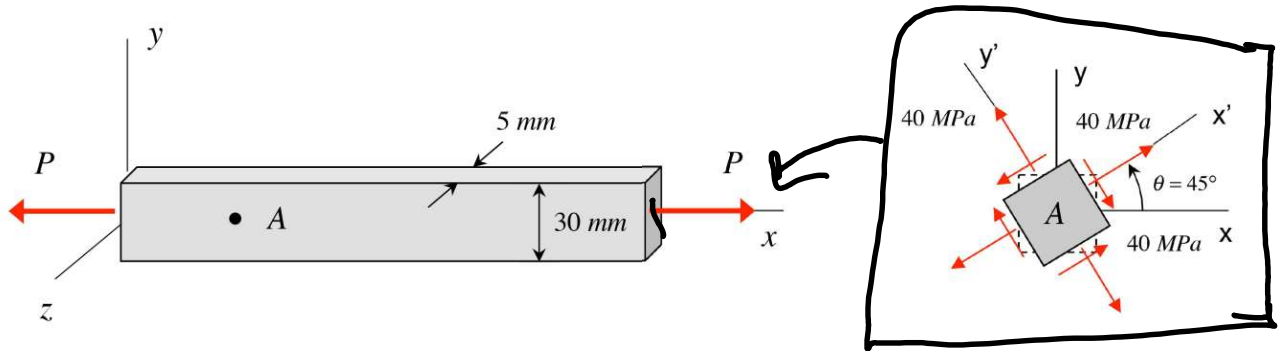


1 unit = 10 MPa.

$$\tau_{max} = R = 43 \text{ MPa.}$$

Example 13.5

Draw the in-plane Mohr's circle for the plane stress state shown below. Determine the axial load P acting on the bar.



$$\sigma_{avg} = \frac{40 + 40}{2} = 40 \text{ MPa.}$$

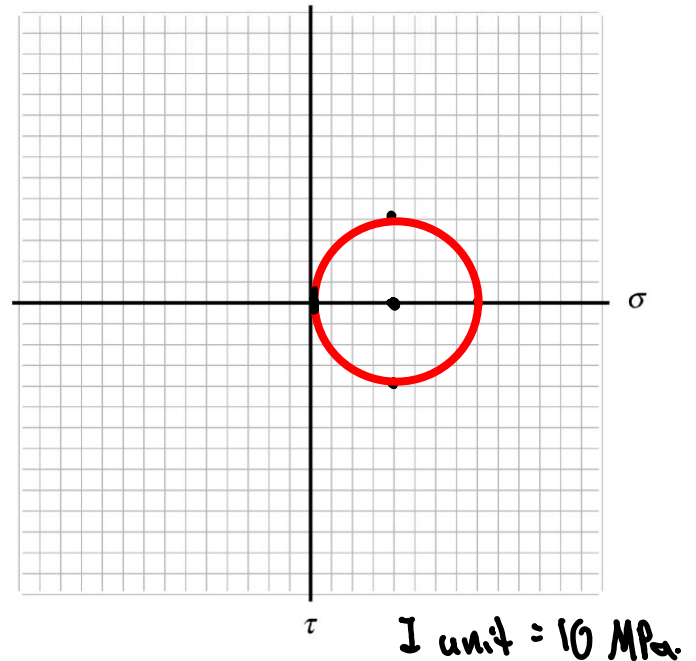
$$R = \sqrt{\left(\frac{40 - 40}{2}\right)^2 + 40^2} = 40 \text{ MPa.}$$

$$\sigma_1 = 40 + 40 = 80 \text{ MPa}$$

$$\sigma_2 = 40 - 40 = 0$$

$$\tau_x = \frac{P}{A}$$

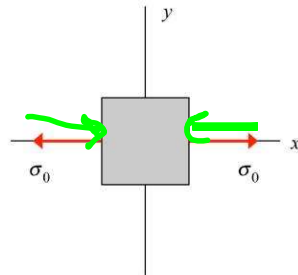
$$P = 80 \text{ MPa} (0.005 \text{ m})(0.03 \text{ m}) = 12 \text{ kN.}$$



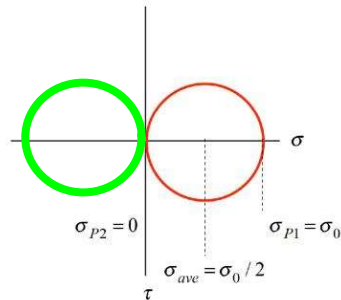
Mohr's circle examples – some special stress states

Uniaxial stress (e.g., uniaxial load on member)

$$(\sigma_x, \sigma_y, \tau_{xy}) = (\sigma_0, 0, 0)$$



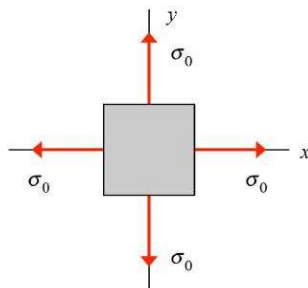
stress element



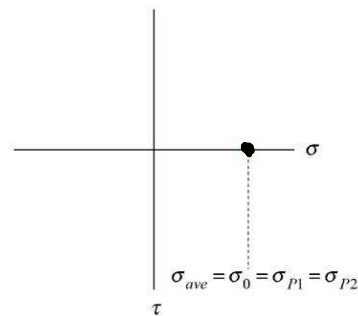
Mohr's circle

Hydrostatic stress (e.g., thin-walled spherical pressure vessel)

$$(\sigma_x, \sigma_y, \tau_{xy}) = (\sigma_0, \sigma_0, 0)$$



stress element



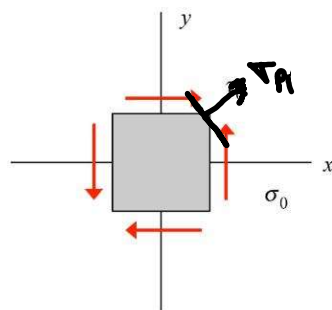
Mohr's circle

$$\tau_{avg} = \tau_0$$

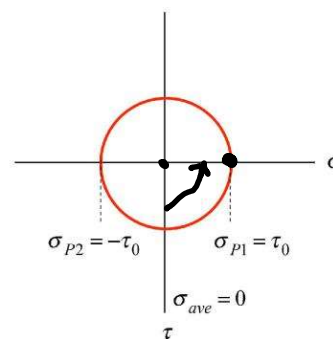
$$R = \sqrt{\left(\frac{\sigma_0 - \sigma_0}{2}\right)^2 + 0}$$

Pure shear stress (e.g., axial torque on member)

$$(\sigma_x, \sigma_y, \tau_{xy}) = (0, 0, \tau_0)$$



stress element

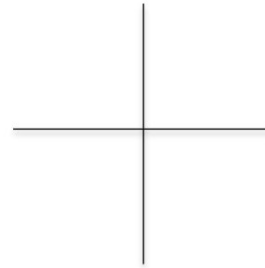
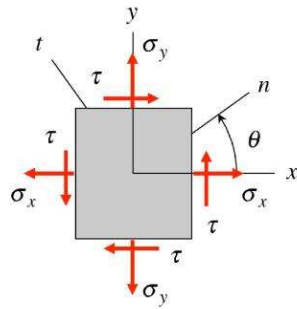


Mohr's circle

$$\tau_{avg} = 0$$

$$R = \tau$$

Reflection: Stress transformations and Mohr's circle (for a state of plane stress)



a) What is a stress element?

b) Why are we interested in stress transformations?

When things fails.

c) There are the *two important parameters* that we need to represent a state of stress – what are they?

d) What are principal stresses? How are these related to the two parameters mentioned in c) above?

most positive and negative normal stresses.

e) What is the maximum in-plane shear stress? How is this related to the two parameters mentioned in c) above?

f) Where is the center of Mohr's circle? What is the radius of Mohr's circle?

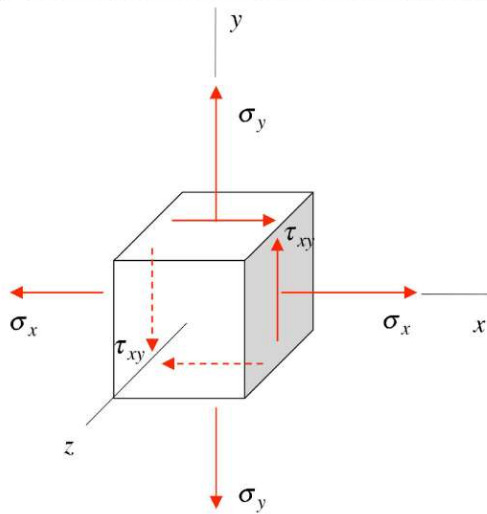
g) Why do we choose the “*positive*” direction of τ as *downward*?

Same direction real axis and Mohr's circle.

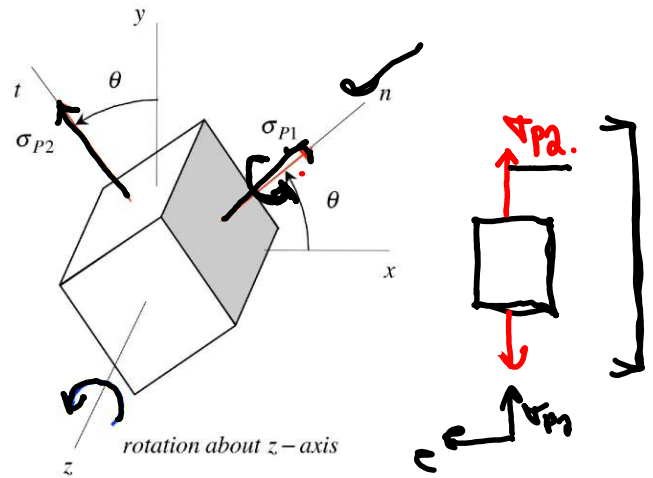
h) How do we know that a rotation of θ in the physical world correspond to a rotation of 2θ in Mohr's circle?

i) How can we use Mohr's circle to find the rotations of the stress element that correspond to the principal components of stress and the maximum in-plane shear stress?

e) Absolute maximum shear stress for plane stress



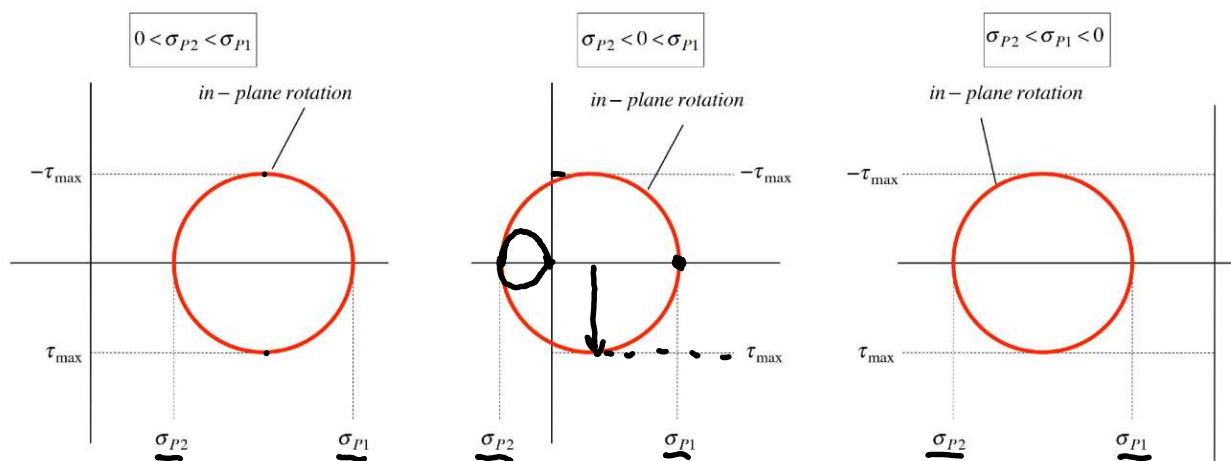
stress element in
plane stress



stress element in rotated to
orientation for principal
stresses

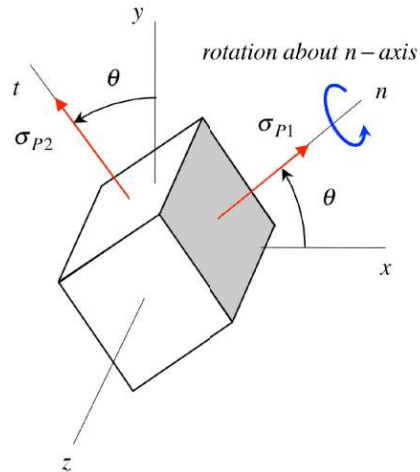
Recall for a state of plane stress, the stress element shows zero normal and shear stresses on the z -face. As we rotate the stress element about the z -axis, we observe the normal and shear stresses on the other four faces change according to our stress transformation equations. When rotated to the orientation showing the principal stresses, only the normal stresses appear on these four faces, as shown in the figure above right.

The Mohr's circle for the stress element rotation described above is shown in the following figures, considering the three possibilities on the signs of the principal stresses σ_{P1} and σ_{P2} .

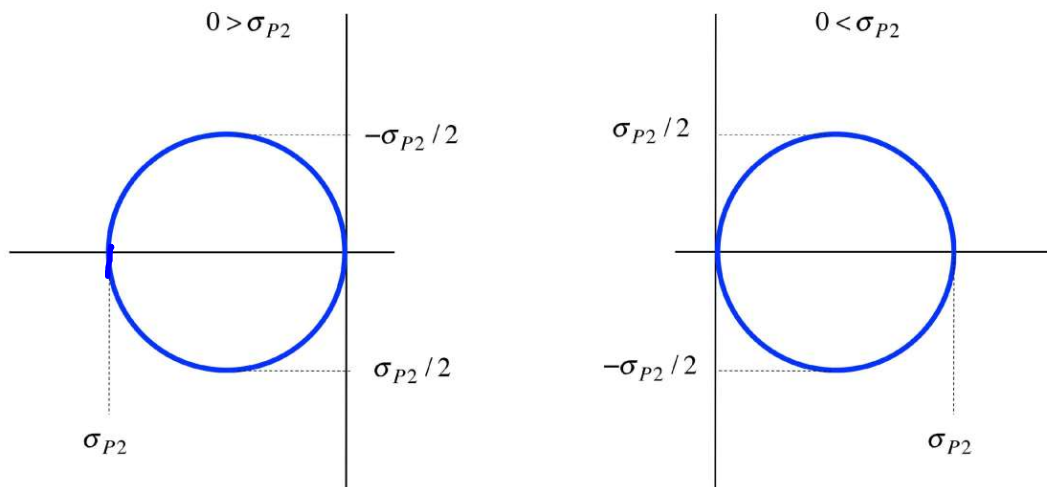


in-plane-Mohr's circles.

Now, starting with the orientation of the stress element corresponding to the principal stresses, say we rotate the stress element about the “ n ” axis, as shown below.

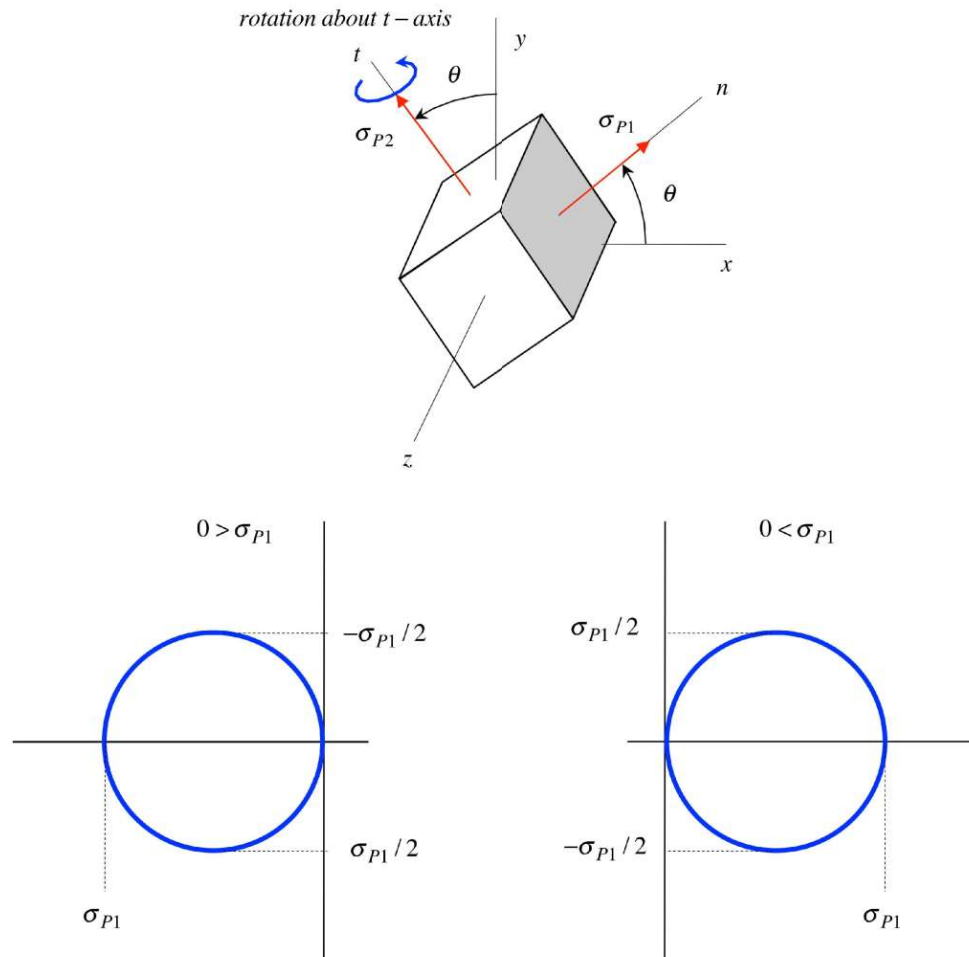


Since the z -face of the stress element is stress free, a 90° rotation produces zero normal and zero shear stress on the t -face. The Mohr's circle for this rotation is shown below, considering the two possibilities on the sign for σ_{P2} . Note that $\sigma = 0$ is an out-of-plane principal stress.

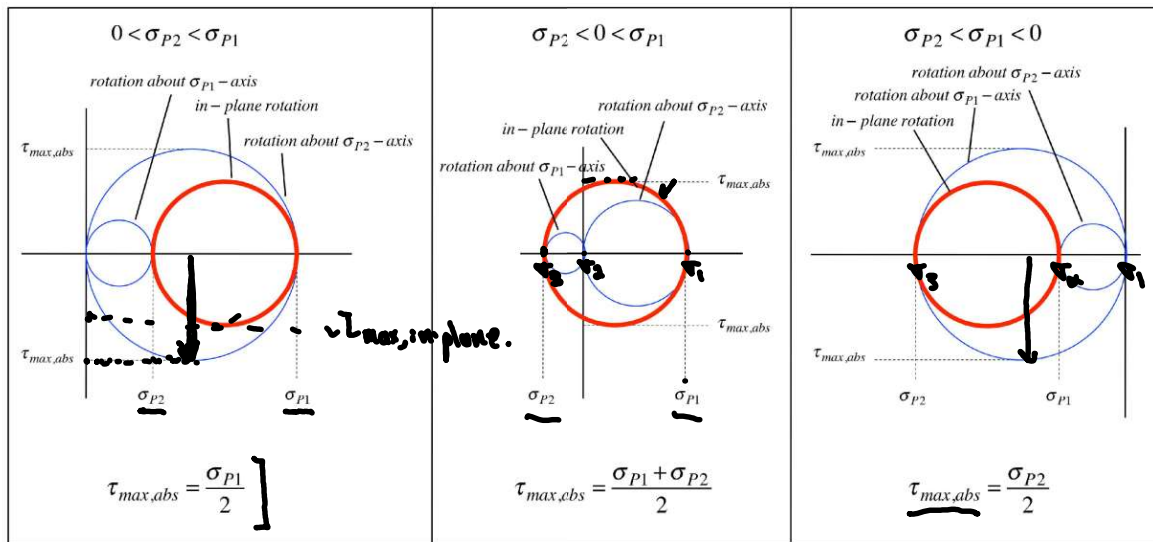


Out-of-plane Mohr's circles.

Similarly, a rotation of the stress element about the “ t ” axis, starting with the principal stresses orientation, produces the Mohr’s circle shown below, depending on the sign of σ_1 . Note that $\sigma = 0$ on the out-of-plane principal stress.



Suppose that we superimpose the Mohr’s circle for these three stress element rotations. In the end, we will have three sets of Mohr’s circles, depending on the signs of the principal stresses σ_{P1} and σ_{P2} , as shown in the following.



In conclusion, we see that the “absolute maximum shear stress”, $\tau_{max,abs}$, (the largest shear stress observed for both in-plane and out-of-plane rotations) depends on the signs of the in-plane principal stresses, σ_{P1} and σ_{P2} : if they are of the same sign, then $\tau_{max,abs}$ is half of the larger of the two, and if they are of opposite signs, then $\tau_{max,abs}$ is the average value of the two in-plane principal stresses.

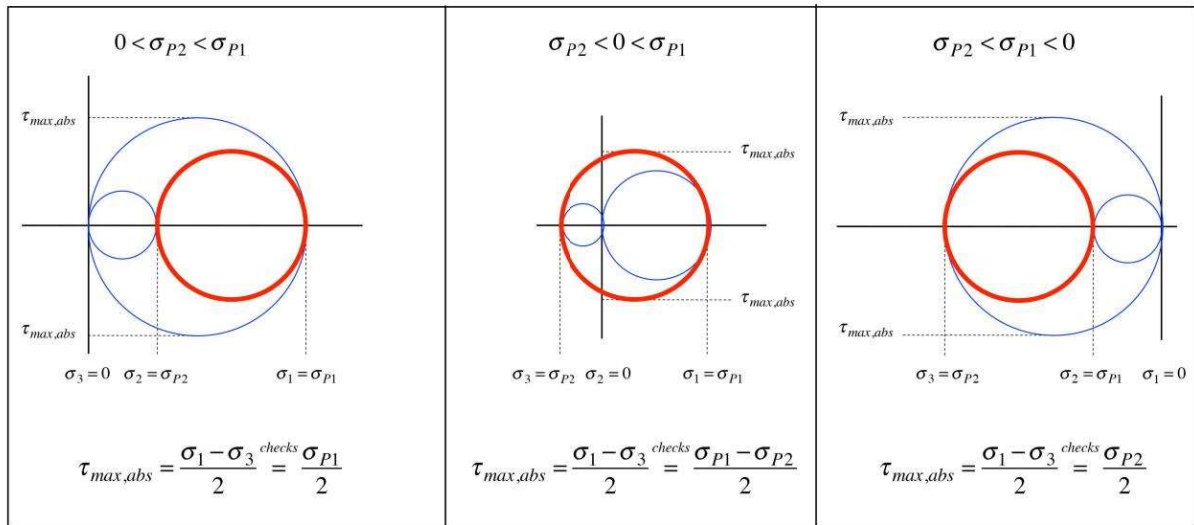
An alternate way (and easier to remember way) to express this result is to first introduce the following notation for principal stresses: consider the two in-plane principal stresses σ_{P1} and σ_{P2} , and the out-of-plane principal stress (which is zero) and rename these in the following way:

- σ_1 is the *largest* of the three
- σ_3 is the *smallest* of the three
- σ_2 is the *intermediate* of the three

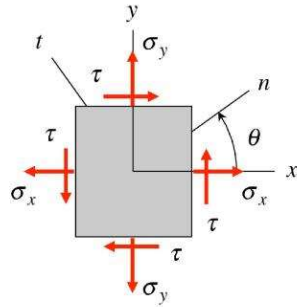
That is, $\sigma_3 \leq \sigma_2 \leq \sigma_1$. With this notation, we can simply write:

$$\tau_{max,abs} = \frac{\sigma_1 - \sigma_3}{2}$$

as summarized in the following.



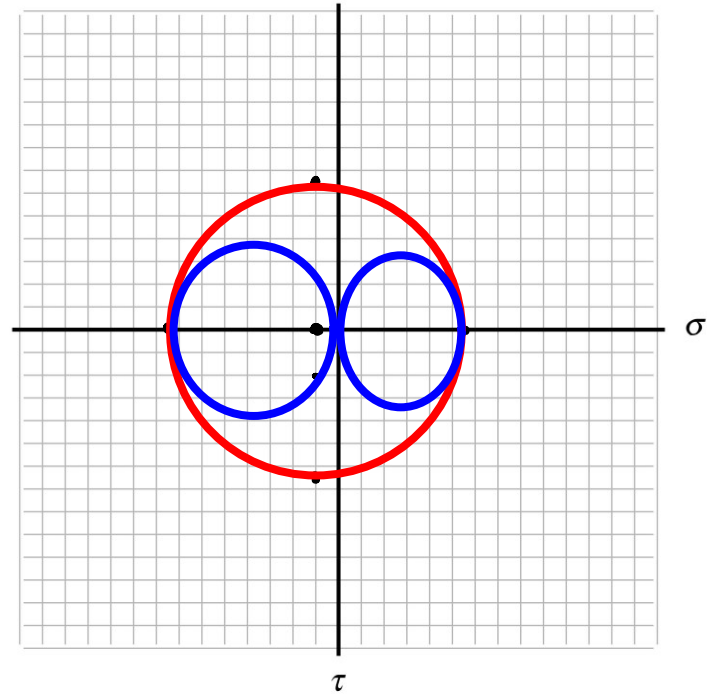
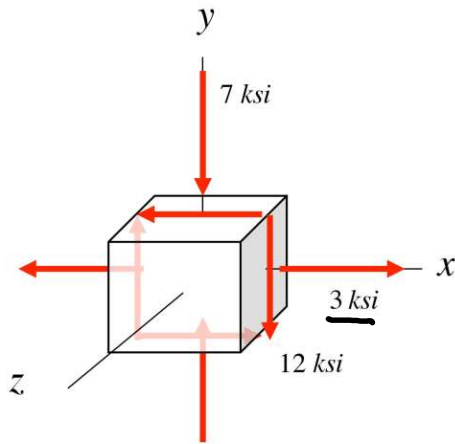
Reflection: Stress transformations and Mohr's circle (for a state of plane stress)



- What is a stress element?
- Why are we interested in stress transformations?
- There are the *two important parameters* that we need to represent a state of stress – what are they?
- What are principal stresses? How are these related to the two parameters mentioned in c) above?
- What is the maximum in-plane shear stress? How is this related to the two parameters mentioned in c) above?
- Where is the center of Mohr's circle? What is the radius of Mohr's circle?
- Why do we choose the “*positive*” direction of τ as *downward*?
- How do we know that a rotation of θ in the physical world correspond to a rotation of 2θ in Mohr's circle?
- How can we use Mohr's circle to find the rotations of the stress element that correspond to the principal components of stress and the maximum in-plane shear stress?

Example 13.6

Determine the principal stresses and the absolute maximum shear stress for the plane stress element shown.



1 unit = 2 ksi

$$\tau_{avg} = \frac{3-7}{2} = -2 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$R = \sqrt{\left(\frac{10}{2}\right)^2 + 12^2} = 13 \text{ ksi}$$

$$\tau_{max, in-plane} = \tau_{max, abs} = R = 13 \text{ ksi}$$