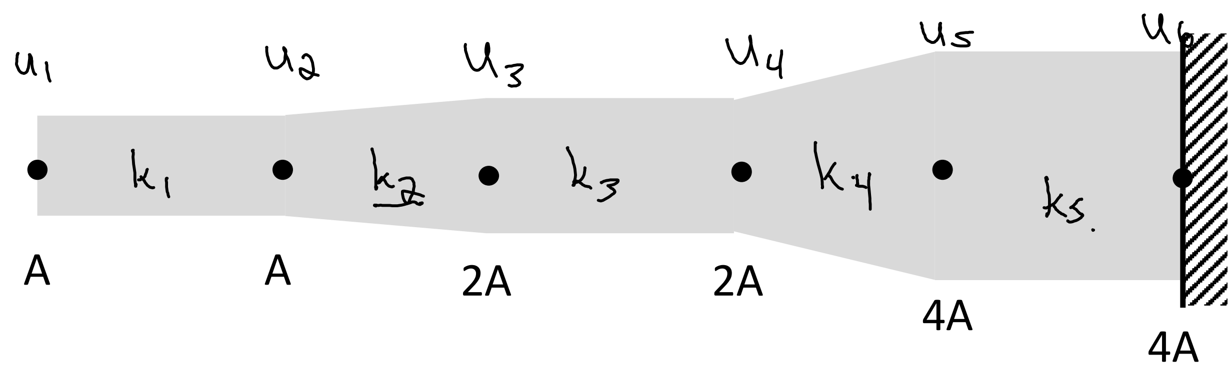
FEM Review



All elements have a modulus of E and a length of L. What is the value of the third row and third column of the stiffness matrix?

$$[K] = \frac{EA}{L}\begin{bmatrix} k_1 & - & - & \cdots \\ - & k_1+k_2 & - & \\ - & - & ?\ k_2+k_3 & \\ \vdots & & & \ddots \end{bmatrix}$$

$$k_2 = \frac{E(A+2A)}{2L} = \frac{3EA}{2L} \qquad k_3 = \frac{2EA}{L}$$

$$k_2 + k_3 = 3.5\,\frac{EA}{L}$$

12. Thin-walled pressure vessels

Objectives:
To study the combined axial and hoop stress state in the sidewalls of cylindrical vessels and in spherical pressure vessels.

Background:
Relationship between the resultant normal force F due to constant normal stress σ acting over an area A:

$$\sigma = \frac{F}{A}$$

Lecture topics:

a) Axial stress.
b) Hoop stress.
c) Combined state of stress.

Lecture Notes

We have so far understood the stresses, and deformations of thin rods/beams in (a) axial deformation, (b) torsion, and (c) in bending. In this class we will consider one more type of structure that is more “two-dimensional” compared the one-dimensional beam models.

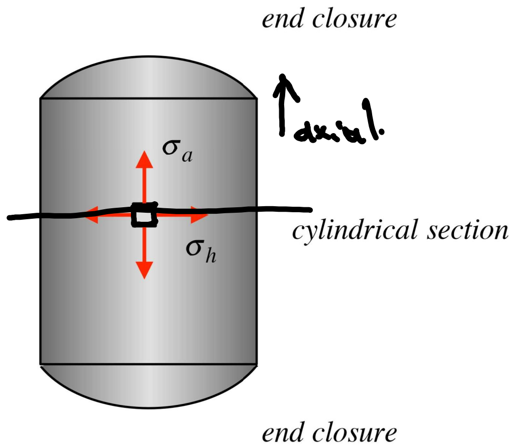
Thin-walled pressure vessels have a number of applications:

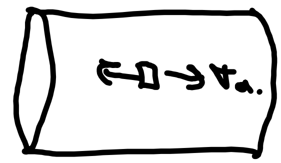
- Vacuum chambers ✓
- Pressure vessels used for storing various kinds of fluids under high pressure
- Natural gas containers, hot air balloons, coke cans, gel and aerosol cans, chemical and nuclear reactors, oil refining containers, soap bubbles
- Liquid fuel containers in space vehicles
- Submarine hulls

To prevent the explosion or breakage of these pressure vessels it is important to design these to keep stresses within an acceptable level.

Cylindrical pressure vessels

Consider a thin-walled circular-cross section pressure vessel with an internal pressure of p, inner radius r and wall thickness t.





Circumferential cut through vessel

For equilibrium of the upper section of the vessel:

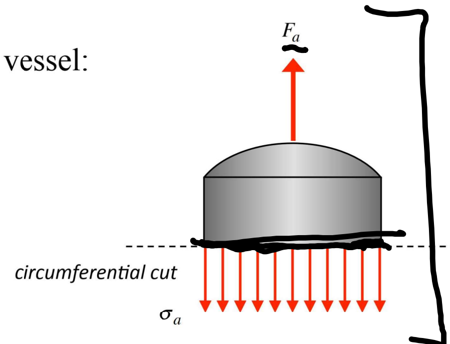
$$\sum F = F_a - \sigma_a A = 0 \;\;\Rightarrow$$

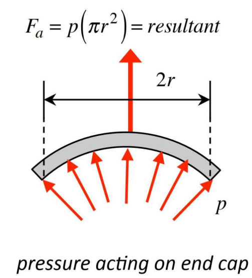
$$\sigma_a = \frac{F_a}{A} = \frac{\pi r^2 p}{\pi\left(r+t\right)^2 - \pi r^2}$$

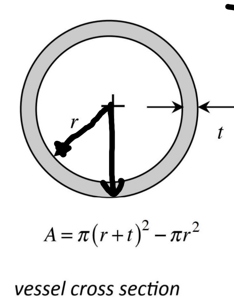
$$= \frac{\pi r^2 p}{\pi\left(r^2 + 2rt + t^2\right) - \pi r^2}$$

$$= \frac{r^2 p}{2rt + t^2}$$

$$= \frac{r^2 p}{2rt\underbrace{\left(1 + t/2r\right)}_{1}} \approx \frac{pr}{2t}; \quad \frac{t}{r} << 1$$







σ_a is the *axial* component of normal stress in the vessel due to the internal pressure.

$$\sigma_a = \frac{pr}{2t}$$

Longitudinal cut through vessel

For equilibrium of the left portion of a hoop section of the vessel (of height Δx):

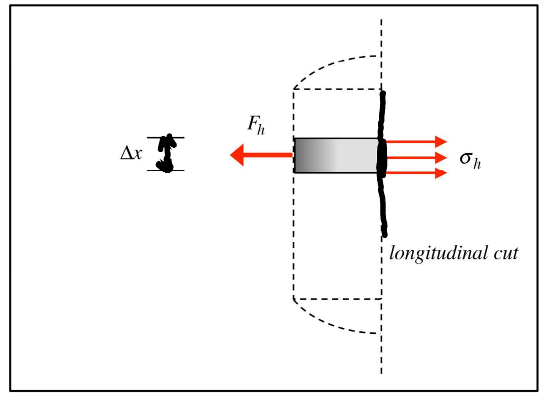
$$\sum F = F_h - \sigma_h A = 0 \quad \Rightarrow$$

$$\sigma_h = \frac{F_h}{A} = \frac{2rp\Delta x}{\underline{2t\Delta x}} = \frac{pr}{t}$$

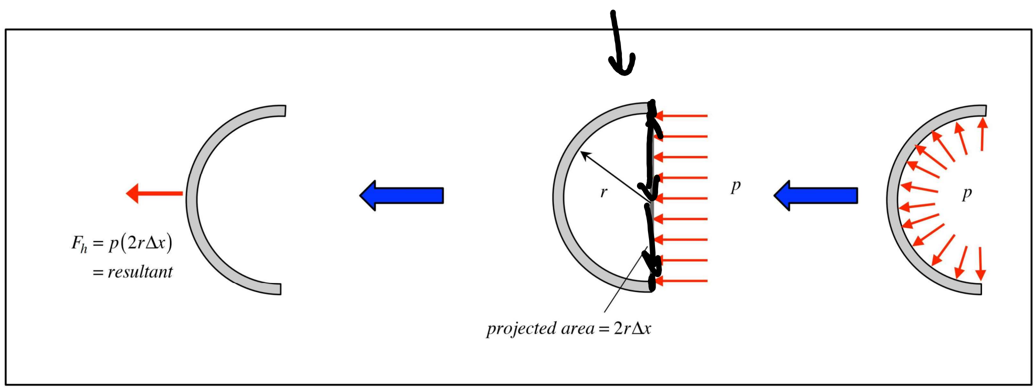
σ_h is the "*hoop*" component of normal stress in the vessel due to the internal pressure.

Note that the axial component of stress is exactly half of the hoop component of stress in a cylindrical pressure vessel.

SIDE view



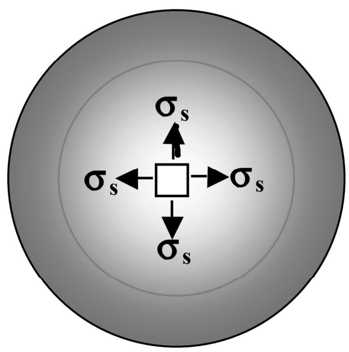
TOP view

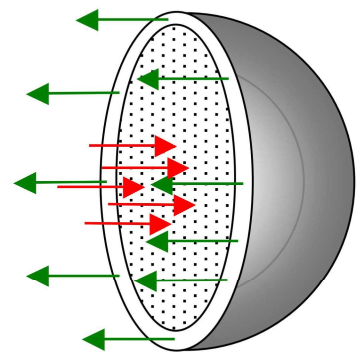


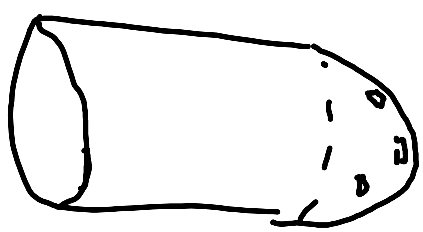
Spherical pressure vessels

Consider a thin-walled spherical pressure vessel with an internal pressure of p, inner radius r and wall thickness t. Using an equilibrium relationship on the hemispherical section of the tank gives a normal stress of:

$$\boxed{\sigma_s = \frac{pr}{2t}}$$

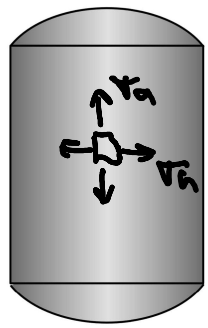






Example 12.1

A steel propane tank for a barbecue grill has a 12-in inside diameter and a wall thickness of 1/8 in. The tank is pressurized to 200 psi. Determine the axial and hoop components of stress in the wall of the tank.



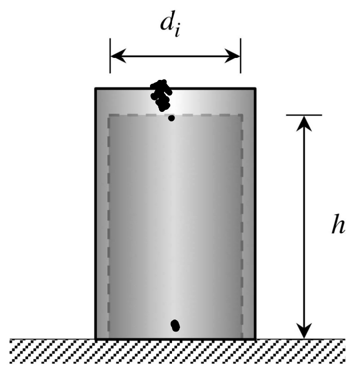
$$\sigma_a = \frac{pr}{2t} \qquad \sigma_h = \frac{pr}{t}$$

$$\sigma_a = \frac{200(6)}{2(1/8)} \qquad \sigma_h = \frac{200(6)}{(1/8)}$$

$$\sigma_a = 4800 \text{ psi} \qquad \sigma_h = 9600 \text{ psi}$$

Example 12.2

A vertical standpipe has an inside diameter of $d_i = 3m$ and is filled with water to depth of h = 5m. If the allowable hoop stress is 80MPa, what is the minimum wall thickness of the tank?



$$\sigma_h = \frac{pr}{t} \Rightarrow t_{min} = \frac{pr}{\sigma_{allow}}$$

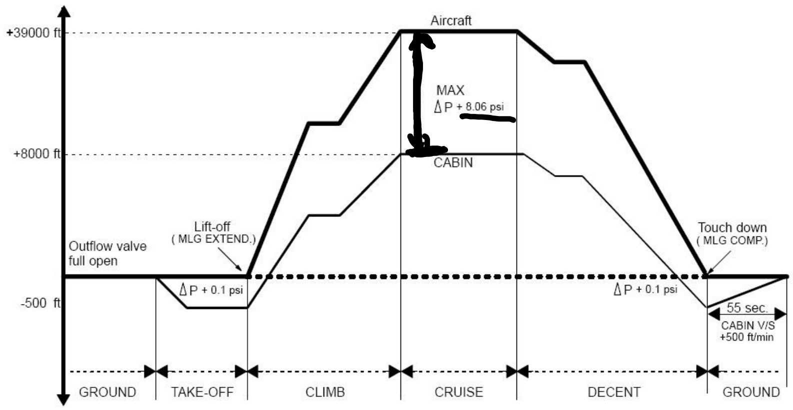
$$p = \rho g h$$

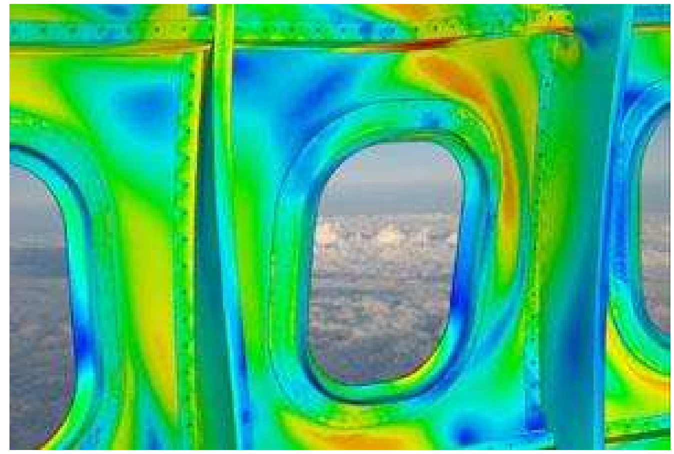
$$t_{min} = \frac{(\rho g h) r}{\sigma_{allow}}$$

$$t_{min} = \frac{(1000)(9.8)(5)(1.5)}{80 \times 10^6 \text{ Pa}}$$

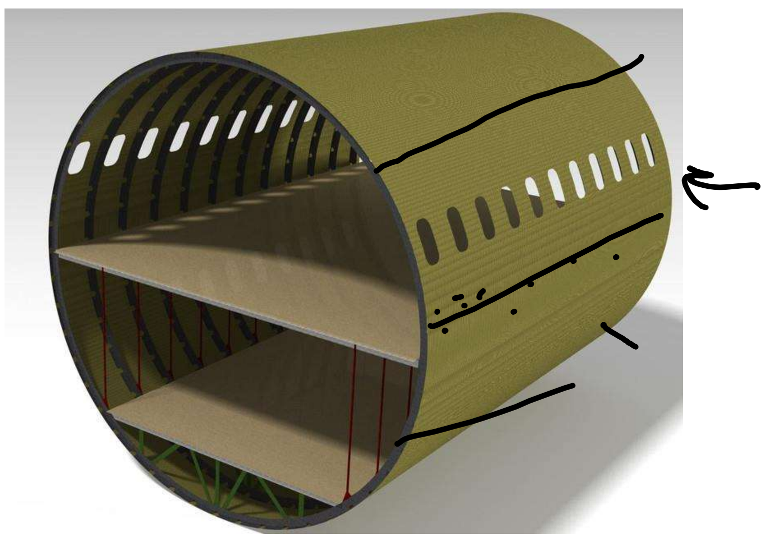
$$t_{min} = 0.9 \text{ mm}$$

Airplane as a Pressure Vessel





https://aviation.stackexchange.com/questions/19291/what-is-the-pressure-in-a-civil-aircraft-fuselage-at-flight-ceiling



An airplane exhibits a pressure difference of 56 kPa in the fuselage at 39 000 ft cruising altitude. The radius of the fuselage is 2 m. The tensile yield strength of aircraft grade aluminum is 276 MPa.

2.0.

a) What thickness is required to achieve a factor of safety of 2.5?

b) Rivets hold the fuselage together. The rivets have a diameter of 3.175 mm and a shear strength of 95 MPa. What density of rivets are required to reach a factor of safety of 2.5?

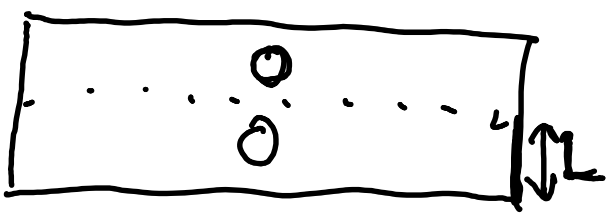
a) $\sigma_a = \frac{pr}{2t}$ $\qquad \sigma_h = \frac{pr}{t} = \frac{\sigma_Y}{FS} = \frac{\sigma_Y}{2.5}$

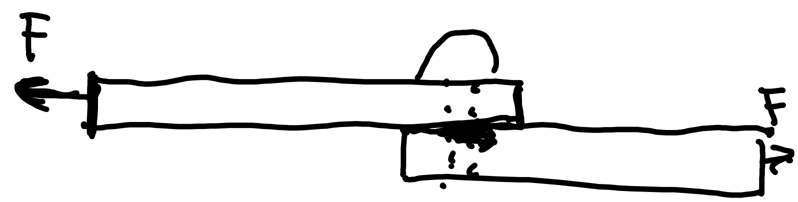
$$\frac{(56\times10^3)(2)}{t} = \frac{276\times10^6 \text{ Pa}}{2.5}$$

$$t = 0.001 \text{ m} = 1 \text{ mm}$$

b) $\sigma_h = 110.4$ MPa.

$\sigma_a = 55.2$ MPa.





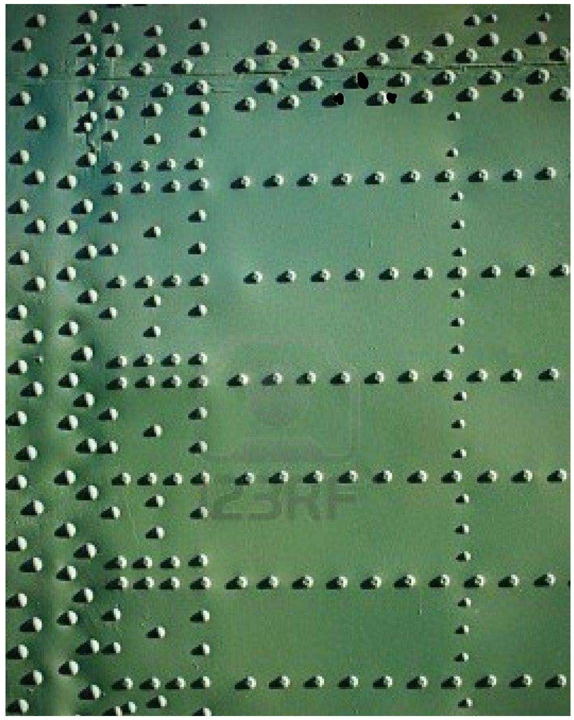
$$\tau_{rivet} A_{rivet} = \sigma_h A_n$$

$$\left(\frac{\tau_s}{2.5}\right)\pi r^2 = \sigma_h (0.001) L$$

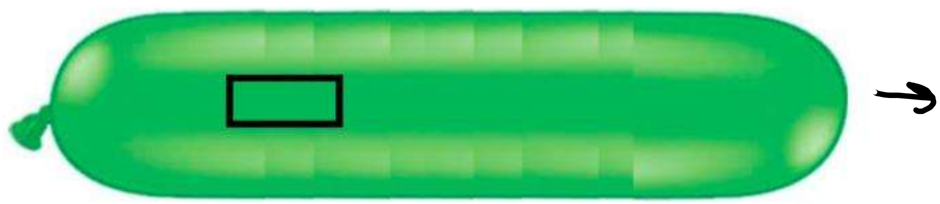
$$L = \left(\frac{\tau_s}{2.5}\right)\pi r^2 \left(\frac{1}{\sigma_h (0.001)}\right)$$

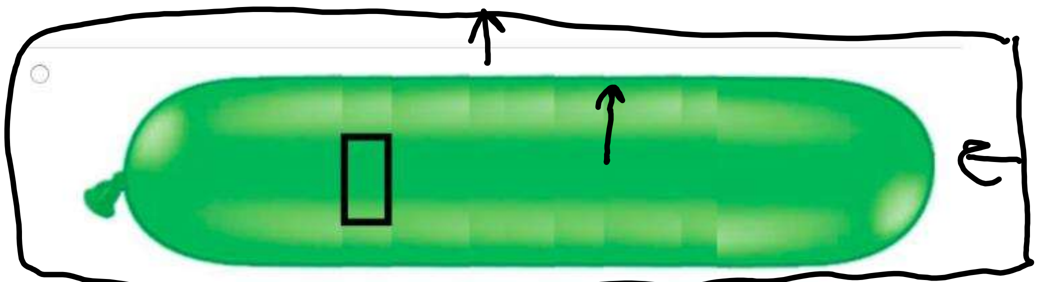
$$L = 0.0027 \text{ m} = 2.7 \text{ mm}.$$

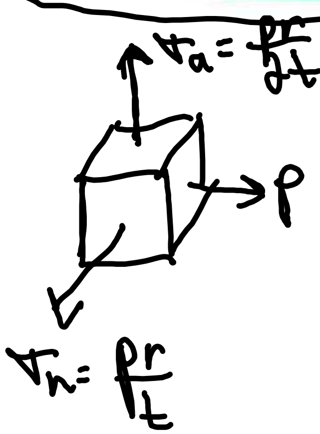
Example rivet pattern
for airplane fuselage.



A square was drawn on a cylindrical balloon before inflating it. The balloon was then inflated. Which one is the correct shape resulting from the original square after inflation?







$$\varepsilon_a = \frac{1}{E}\left(\cancel{\frac{pr}{2t}} - \nu\left(\frac{pr}{t} - p\right)\right) \qquad \nu = \frac{1}{2}.$$

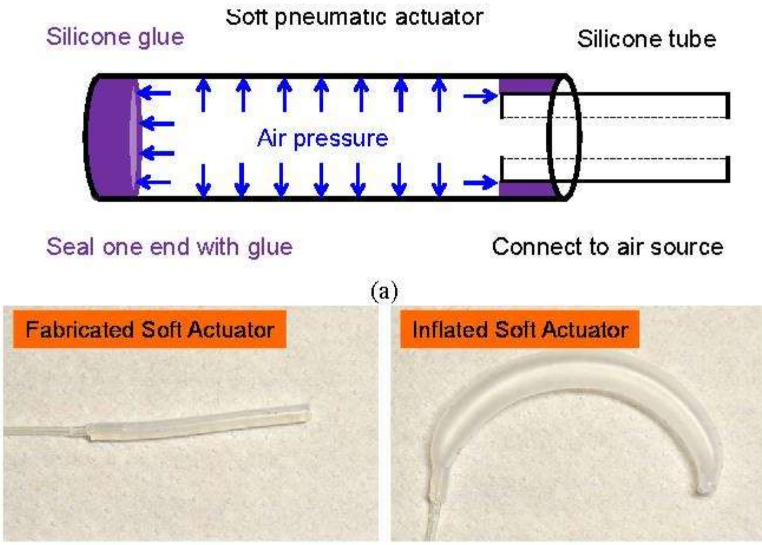
$$\varepsilon_a = \frac{1}{E}(\nu p)$$

$$\varepsilon_h = \frac{1}{E}\left(\frac{pr}{t} - \nu\left(\frac{pr}{2t} - p\right)\right)$$

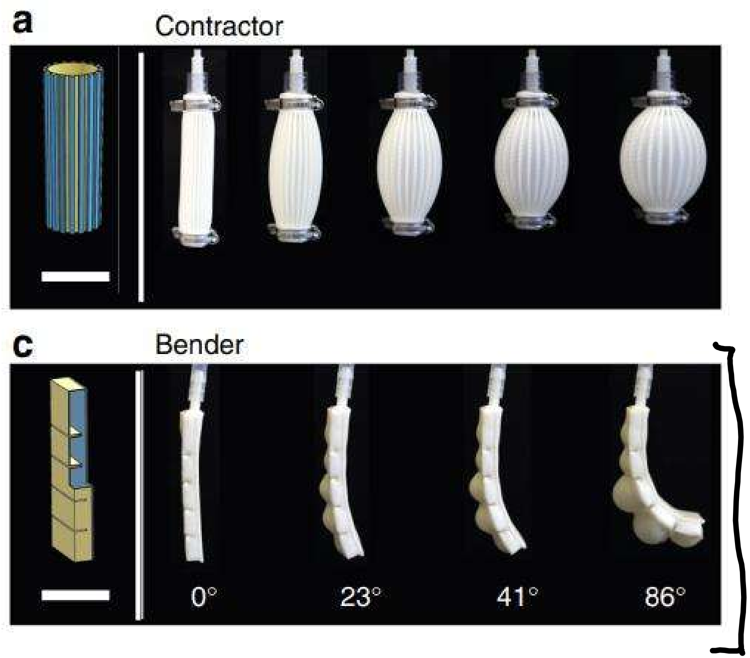
$$\varepsilon_h = \frac{1}{E}\left(\underbrace{\frac{pr}{t} - \frac{pr}{4t}} + \frac{p}{2}\right) . \; r \gg t.$$

$$\varepsilon_h \approx \frac{1}{E}\left(\frac{3pr}{4t}\right) \leftarrow$$

Pneumatic Actuators



Jin Guo et al, IEEE ICMA, 2017



Schaffner et al, Nat Comm, 9:878, 2018.

1. Starting from the equations for stresses in chambers and the generalized strain equation, derive the relationships for the strain in the axial and hoop directions in a pressure vessel.

2. For an elastomer with a Poisson’s ratio of 0.5, what is the strain in the axial and radial directions as a function of pressure?

3. How could you modify the materials properties or device structure ot improve the actuation strain and/or actuation stress in the axial direction?

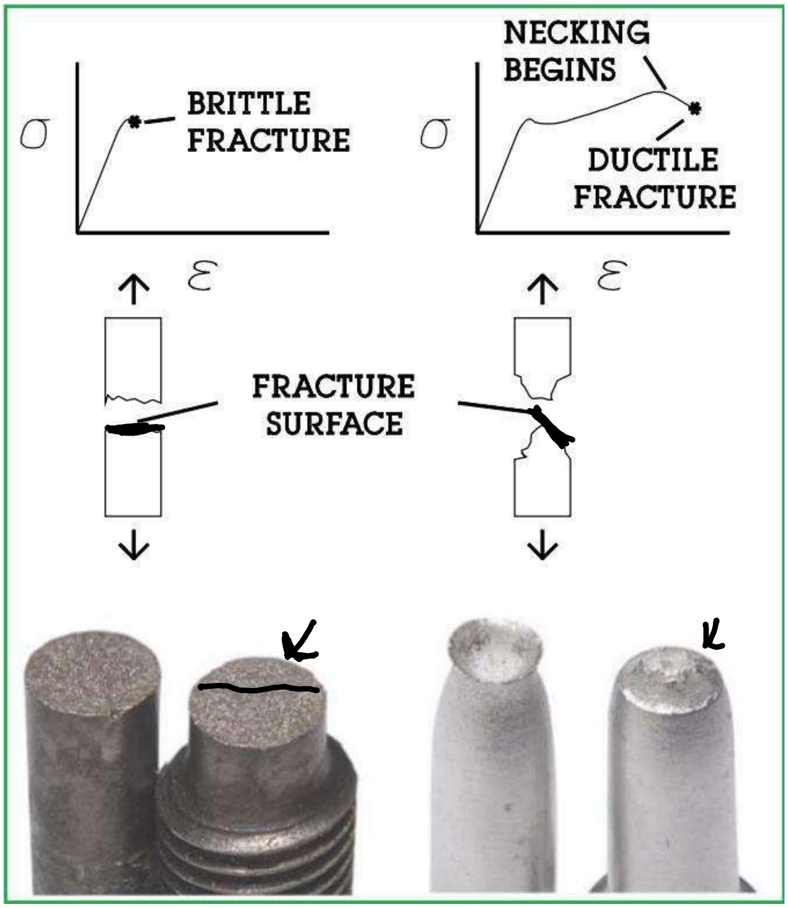
ME 323 - MECHANICS OF MATERIALS

Schedule for Fall 2023

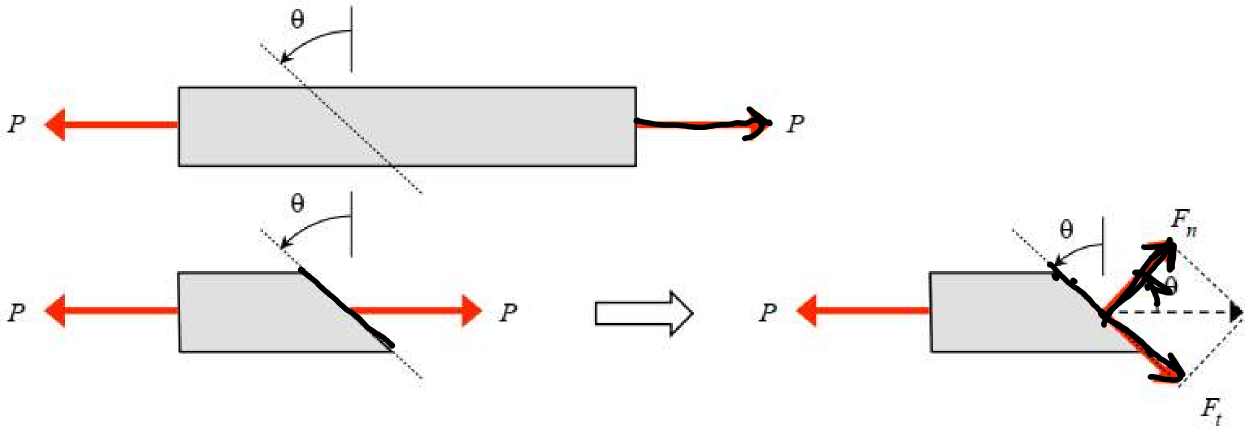
PER	*DATE*	*TOPIC*	*READING**	*HWK DUE*
1 M	21-Aug	Introduction; Static equilibrium	Chap. 1	
2 W	23-Aug	Normal stress and strain; Mechanical properties	Chap. 2	
3 F	25-Aug	Shear stress and strain – direct shear	Chap. 3	
4 M	28-Aug	Stress – introduction to design of deformable bodies	Chap. 4	
5 W	30-Aug	Stress and strain – general definitions	Chap. 5	
6 F	1-Sep	Axial members – determinate structures	Chap. 6	HW 1
M	**4-Sep**	***Labor Day – no class***		
7 W	6-Sep	Axial members – indeterminate structures	Chap. 6	
8 F	8-Sep	Axial members – planar trusses	Chap. 6	HW. 2
9 M	11-Sep	Axial members – thermal effects	Chap. 7	
10 W	13-Sep	Torsion members – stresses in circular bars	Chap. 8	
11 F	15-Sep	Torsion members – statically determinate structures	Chap. 8	HW 3
12 M	18-Sep	Torsion members – statically indeterminate structures	Chap. 8	
13 W	20-Sep	Beam stresses – equilibrium and flexural stresses	Chap. 10	
14 F	22-Sep	Beam stresses – flexural and shear stresses	Chap. 10	HW 4
15 M	25-Sept	Review		
W	***27 Sept***	***Examination 1, 8-10pm (no lecture on Wednesday)***		
16 F	29-Sep	Beam stresses – shear stresses	Chap. 10	
17 M	2-Oct	Shear force/bending moment diagrams - determinate structures	Chap. 9	
18 W	4-Oct	Beam deflections – statically determinate structures	Chap. 11	
19 F	6 -Oct	Beam deflections – statically indeterminate structures	Chap. 11	HW 5
M	**9-Oct**	***October Break - no class***		
20 W	11-Oct	Beam deflections – superposition methods	Chap. 11	
21 F	13-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW. 6
22 M	16-Oct	Energy methods – Castigliano's theorems	Chap. 16	
23 W	18-Oct	Energy methods – Castigliano's theorems	Chap. 16	
24 F	20-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW 7
25 M	23-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
26 W	25-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
27 F	27-Oct	Energy methods – introduction to finite element methods	Chap. 17	HW 8
28 M	30-Oct	Review		
W	**1-Nov**	***Examination 2, 8-10p.m. (no lecture on Wednesday)***		
29 F	3-Nov	Energy methods – introduction to finite element methods	Chap. 17	
30 M	6-Nov	Thin-walled pressure vessels – axial and hoop stresses	Chap. 12 ✓	
31 W	8-Nov	Stress transformation – principal /maximum shear stresses	Chap. 13	
32 F	10-Nov	Stress transformation – Mohr's circle	Chap. 13	HW 9
33 M	13-Nov	Stress transformation – absolute maximum shear stress	Chap. 13	
34 W	15-Nov	Stresses – combined loading	Chap. 14	
35 F	17-Nov	Stresses – combined loading	Chap. 14	HW 10
36 M	20-Nov	Stresses – combined loading	Chap. 14	
W	**22-Nov**	***Thanksgiving Vacation – no class***		
F	**24-Nov**	***Thanksgiving Vacation – no class***		
37 M	27-Nov	Failure analysis-stress theories	Chap. 15	
38 W	29-Nov	Failure analysis – stress theories	Chap. 15	
39 F	1-Dec	Failure analysis – buckling	Chap. 18	HW. 11
40 M	4-Dec	Practice with combined loadings and failure analysis		
41 W	6-Dec	Practice with combined loadings and failure analysis		
42 F	8-Dec	Review		
	TBA	***Final Examination***		

* Reading assignments from lecture book

Lecture 4.


NECKING
BEGINS
BRITTLE
FRACTURE
DUCTILE
FRACTURE
FRACTURE
SURFACE

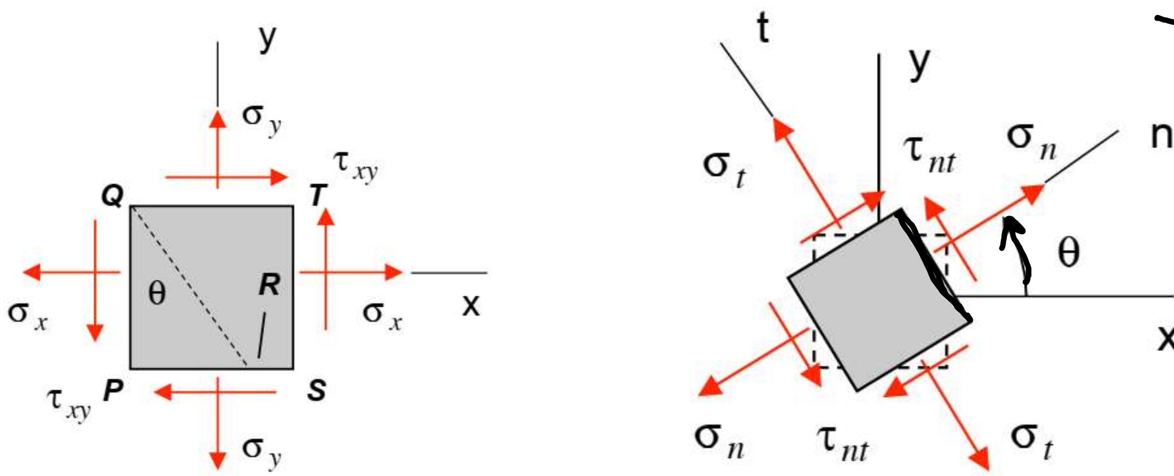
Preview of Chapter 13



1D

$$\sigma = \frac{F_n}{A_c} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta = \frac{P}{2A}\left(1+\cos 2\theta\right)$$

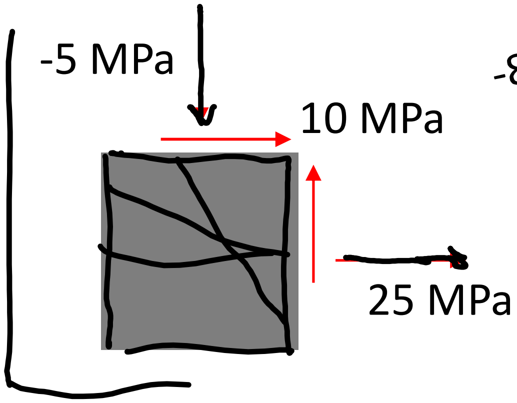
$$\tau = \frac{F_t}{A_c} = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\cos\theta\sin\theta = \frac{P}{2A}\sin 2\theta$$

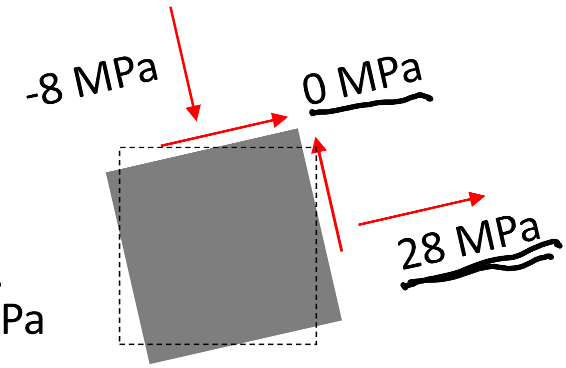


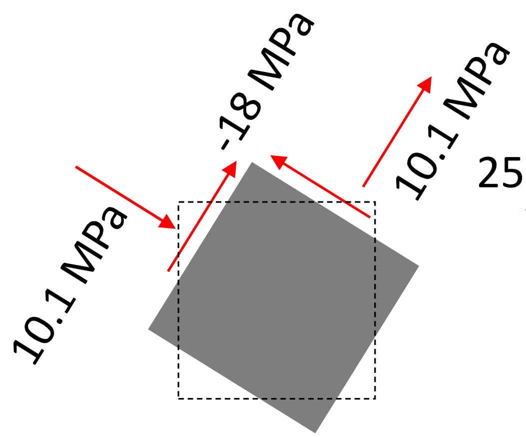
2D.

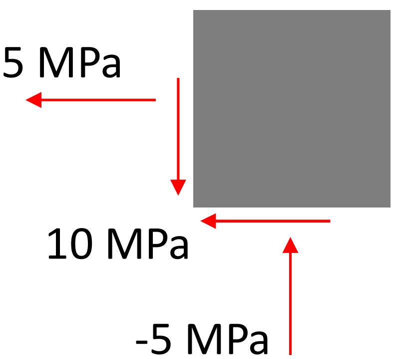
$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

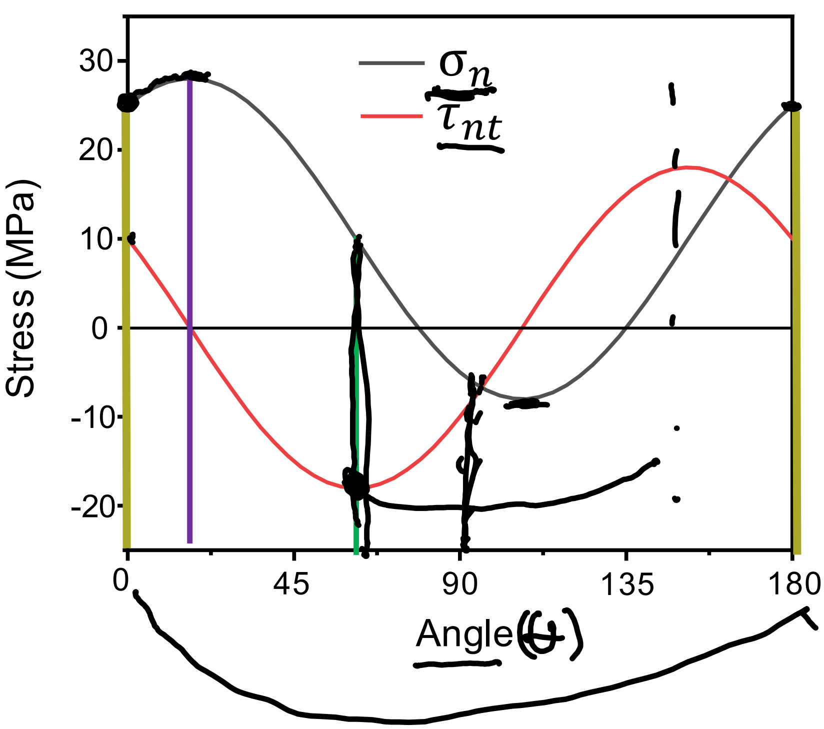
$$\tau_{nt} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

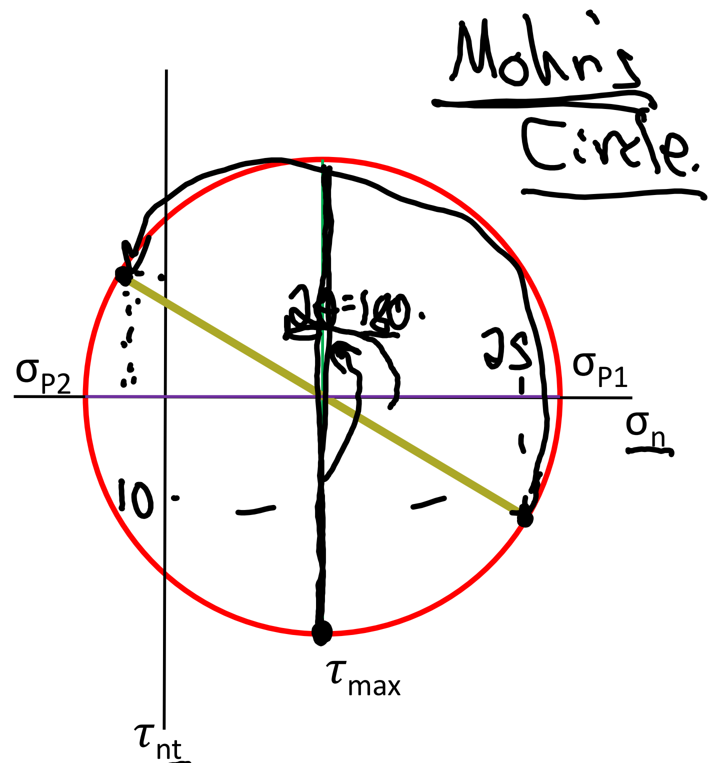

-5 MPa
10 MPa
25 MPa


-8 MPa
0 MPa
28 MPa


-18 MPa
10.1 MPa
10.1 MPa


25 MPa
10 MPa
-5 MPa


Stress (MPa)
30
20
10
0
-10
-20
σn
τnt
0
45
90
135
180
Angle (θ)


Mohr's Circle
σP2
σP1
σn
τmax
τnt
2θ=180.
25
10