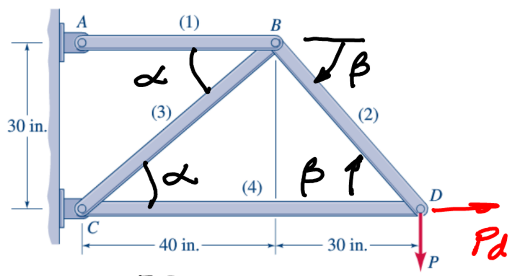
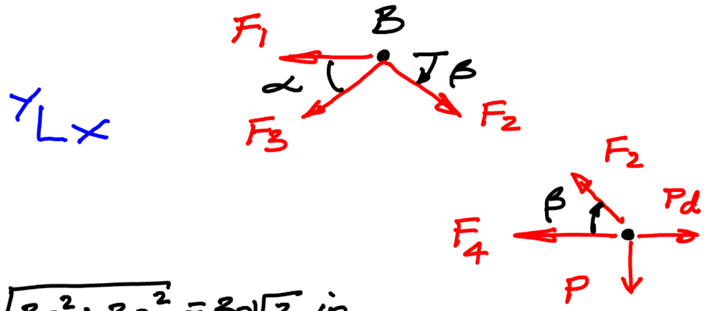
Problem D

Determine the vertical and horizontal deflection of the truss at joint D. All members of the truss have a cross-sectional area of A and are made of a material with a Young's modulus of E.





$$\alpha = \tan^{-1}\frac{30}{40} = 36.87^\circ \quad ; \; L_2 = \sqrt{30^2+30^2} = 30\sqrt{2} \text{ in}$$

$$\beta = \tan^{-1}\frac{30}{30} = 45^\circ \quad ; \; L_3 = \sqrt{30^2+40^2} = 50 \text{ in}$$

Load carried by truss members

- Joint D: $\sum F_y = F_2 \sin\beta - P = 0 \Rightarrow F_2 = P/\sin\beta$

  $\sum F_x = -F_2 \cos\beta - F_4 + P_d = 0 \Rightarrow F_4 = P_d - P\cot\beta$

- Joint B: $\sum F_y = -F_3 \sin\alpha - F_2 \sin\beta \Rightarrow F_3 = -F_2 \frac{\sin\beta}{\sin\alpha} = -P/\sin\alpha$

  $\sum F_x = -F_3 \cos\alpha + F_2 \cos\beta - F_1 = 0$

  $\hookrightarrow F_1 = F_2 \cos\beta - F_3 \cos\alpha = P(\cot\beta + \cot\alpha)$

Strain energy

$$U = \frac{F_1^2 L_1}{2EA} + \frac{F_2^2 L_2}{2EA} + \frac{F_3^2 L_3}{2EA} + \frac{F_4^2 L_4}{2EA}$$

- $V_D = \left.\frac{\partial U}{\partial P}\right|_{P_d=0}$

$$= \left[\frac{L_1}{EA} F_1 \frac{\partial F_1}{\partial P} + \frac{L_2}{EA} F_2 \frac{\partial F_2}{\partial P} + \frac{L_3}{EA} F_3 \frac{\partial F_3}{\partial P} + \frac{L_4}{EA} F_4 \frac{\partial F_4}{\partial P}\right]_{P_d=0}$$

$$= \frac{P}{EA}\left[(\cot\alpha + \cot\beta)^2 L_1 + \left(\frac{1}{\sin\beta}\right)^2 L_2 + \left(\frac{1}{\sin\alpha}\right)^2 L_3 + (\cot\beta)^2 L_4\right]$$

- $u_D = \left.\frac{\partial U}{\partial P_d}\right|_{P_d=0}$

$$= \left[\frac{L_1}{EA} F_1 \cancelto{0}{\frac{\partial F_1}{\partial P_d}} + \frac{L_2}{EA} F_2 \cancelto{0}{\frac{\partial F_2}{\partial P_d}} + \frac{L_3}{EA} F_3 \cancelto{0}{\frac{\partial F_3}{\partial P_d}} + \frac{L_4}{EA} F_4 \frac{\partial F_4}{\partial P_d}\right]_{P_d=0}$$

$$= -\frac{P L_4}{EA}\cot\beta$$