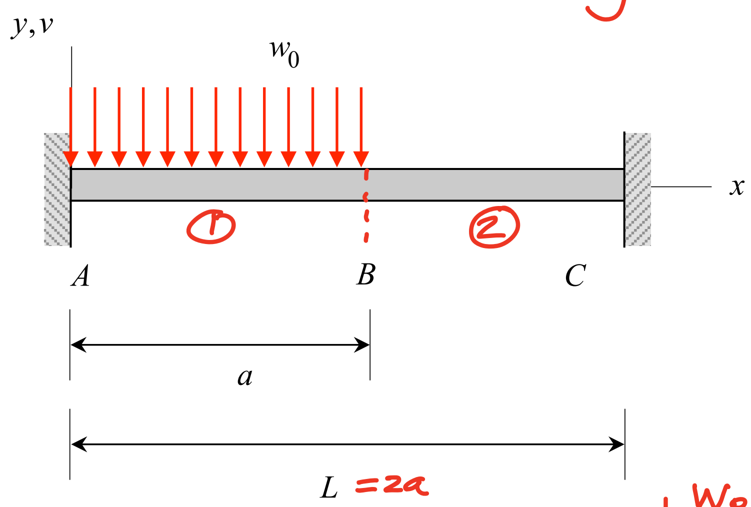
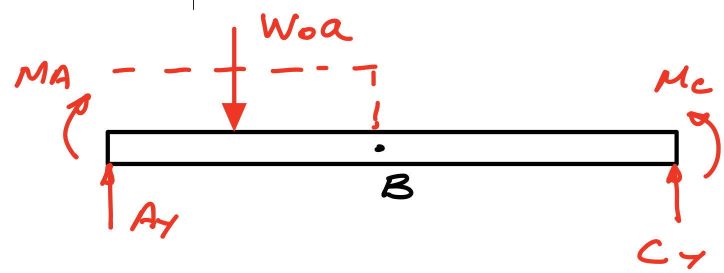
Problem B

Find the vertical deflection of the beam at point B and the angle of rotation of the beam at B. Let E and I be the Young's modulus and second area moment of the beam cross section, respectively, of the beam. Let $L = 2a$. Ignore shear effects.



<u>Equilibrium</u>



(1) $\sum M_A = M_C - M_A + C_y(2a) - (W_0 a)\frac{a}{2} = 0$

(2) $\sum F_y = A_y + C_y - W_0 a = 0$

- INDETERMINATE
- Will <u>choose</u> M_A and A_y as redundant loads

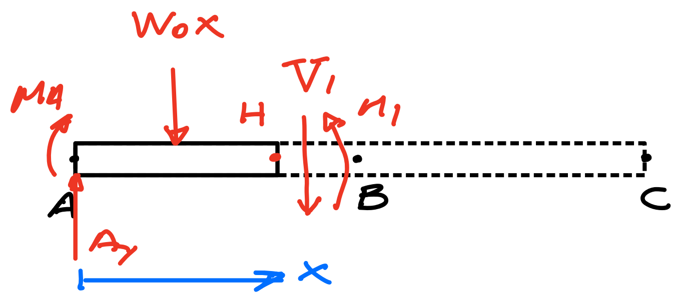
$\therefore$

(1) $C_y = -A_y + W_0 a$

(2) $M_C = M_A - (-A_y + W_0 a)(2a) + \frac{1}{2} W_0 a^2 = M_A + 2a A_y - \frac{3}{2} W_0 a^2$

↳ in terms of redundant loads ✓

<u>Strain energy</u>



- <u>Section AB</u> − 1

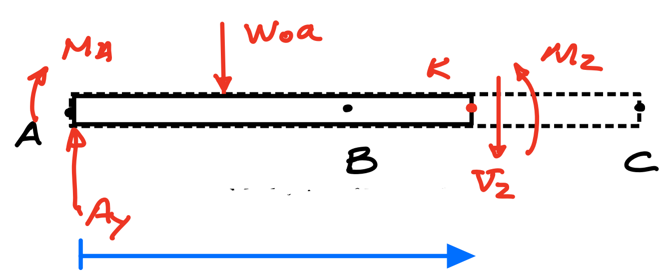
$\sum M_H = M_1 - A_y x + (W_0 x)\frac{x}{2} - M_A$

↳ $M_1(x) = A_y x - \frac{1}{2} W_0 x^2 + M_A$

- <u>Section BC</u> − 2

$\sum M_K = M_2 - A_y x + (W_0 a)\left(x - \frac{a}{2}\right) - M_A$

↳ $M_2(x) = (A_y - W_0 a)x + \frac{1}{2} W_0 a^2 + M_A$



$\therefore \; U = U_1 + U_2$

$$= \frac{1}{2EI}\int_0^a M_1^2(x)\,dx + \frac{1}{2EI}\int_a^{2a} M_2^2(x)\,dx$$

<u>Apply Castigliano</u>

Since A_y and M_A are our redundant loads, we can write the following:

- $$0 = \frac{\partial U}{\partial A_y} = \frac{1}{\cancel{EI}}\int_0^a M_1 \frac{\partial M_1}{\partial A_y}dx + \frac{1}{\cancel{EI}}\int_a^{2a} M_2 \frac{\partial M_2}{\partial A_y}dx$$

w/ $\frac{\partial M_1}{\partial A_y} = x$ & $\frac{\partial M_2}{\partial A_y} = x$

$$\therefore \; 0 = \int_0^a \left(A_y x - \tfrac{1}{2}w_0 x^2 + M_A\right)x\,dx + \int_a^{2a}\left[(A_y - w_0 a)x + \tfrac{1}{2}w_0 a^2 + M_A\right]x\,dx$$

$$= \tfrac{1}{3}A_y a^3 - \tfrac{1}{8}w_0 a^4 + \tfrac{1}{2}M_A a^2 + \tfrac{1}{3}(A_y - w_0 a)\underbrace{\left[(2a)^3 - a^3\right]}_{7a^3} + \tfrac{1}{4}w_0 a^2\underbrace{\left[(2a)^2 - a^2\right]}_{3a^2} + \tfrac{1}{2}M_A\underbrace{\left[(2a)^2 - a^2\right]}_{3a^2}$$

(3) $$0 = \tfrac{8}{3}a^3 A_y + 2a^2 M_A - \tfrac{41}{24}a^4 w_0$$

- $$0 = \frac{\partial U}{\partial M_A} = \frac{1}{\cancel{EI}}\int_0^a M_1 \frac{\partial M_1}{\partial M_A}dx + \frac{1}{\cancel{EI}}\int_a^{2a} M_2 \frac{\partial M_2}{\partial M_A}dx$$

w/ $\frac{\partial M_1}{\partial M_A} = \frac{\partial M_2}{\partial M_A} = 1$

$$\therefore \; 0 = \int_0^a \left[A_y x - \tfrac{1}{2}w_0 x^2 + M_A\right]dx + \int_a^{2a}\left[(A_y - w_0 a)x + \tfrac{1}{2}w_0 a^2 + M_A\right]dx$$

$$= \tfrac{1}{2}A_y a^2 - \tfrac{1}{6}w_0 a^3 + M_A a + \tfrac{1}{2}(A_y - w_0 a)\underbrace{\left[(2a)^2 - a^2\right]}_{3a^2} + \tfrac{1}{2}(w_0 a^2 + M_A)\underbrace{\left[2a - a\right]}_{a}$$

(4) $$0 = 2a^2 A_y + \tfrac{3}{2}a M_A - \tfrac{7}{6}a^3 w_0$$

$\therefore$ we need to solve (3) and (4) for A_y and M_A:

(5) $\tfrac{8}{3}a A_y + 2M_A = \tfrac{41}{24}a^2 w_0$

(6) $2a A_y + \tfrac{3}{2}M_A = \tfrac{7}{6}a^2 w_0$

} Solve for A_y and M_A

Once A_y and M_A are known from above, solve (1) and (2) for C_y and M_C.

Now that the reactions are known, we can then find the displacement at B. To do this, add dummy load, P_d, at B. Repeat process to determine the strain energy function for beam, U. To find displacement at B, use:

$$\delta_B = \left.\frac{\partial U}{\partial P_d}\right|_{P_d=0}$$