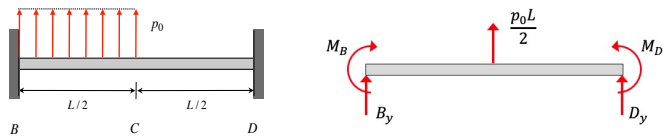
Example 11.20

Determine the reactions acting on the beam loaded as below using superposition.

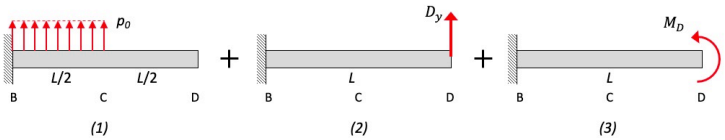


1. Equilibrium

$$(1)\quad \Sigma M_B = -M_B + \frac{p_0 L}{2}\left(\frac{L}{4}\right) + D_y L + M_D = 0 \quad \Longrightarrow \quad M_B = M_D + D_y L + \frac{p_0 L^2}{8}$$

$$(2)\quad \Sigma F_y = B_y + D_y + \frac{p_0 L}{2} = 0 \quad \Longrightarrow \quad B_y = -D_y - \frac{p_0 L}{2}$$

2. Load/deflection: *superposition*



The beam deflection can be written as:

$$v(x) = v_1(x) + v_2(x) + v_3(x)$$

where:

$$v_1(x) = \begin{cases} ---------- & ; \quad 0 < x < L/2 \\ \dfrac{p_0 L^3}{192\,EI}\left(4x - \dfrac{L}{2}\right) & ; \quad \dfrac{L}{2} < x < L \end{cases}$$

$$v_2(x) = \frac{D_y x^2}{6EI}(3L - x) \quad ; \quad 0 < x < L$$

$$v_3(x) = \frac{M_D x^2}{2EI} \quad ; \quad 0 < x < L$$

Therefore, for $L/2 < x < L$:

$$v(x) = \frac{p_0 L^3}{192\,EI}\left(4x - \frac{L}{2}\right) + \frac{D_y x^2}{6EI}(3L - x) + \frac{M_D x^2}{2EI}$$

$$\theta(x) = \frac{dv}{dx} = \frac{4p_0 L^3}{192\,EI} + \frac{D_y}{6EI}(6Lx - 3x^2) + \frac{M_D x}{EI}$$

3. Compatibility*: satisfy the BCs are the right end*

$$(3)\quad v(L) = 0 = \frac{7p_0 L^4}{384\,EI} + \frac{D_y L^3}{3EI} + \frac{M_D L^2}{2EI} \quad \Longrightarrow \quad \frac{2}{3}D_y L + M_D = -\frac{7p_0 L^2}{192}$$

$$(4)\quad \theta(L) = 0 = \frac{4p_0 L^3}{192\,EI} + \frac{D_y L^2}{2EI} + \frac{M_D L}{EI} \quad \Longrightarrow \quad \frac{1}{2}D_y L + M_D = -\frac{4p_0 L^2}{192}$$

4. Solve*: 4 equations/4 unknowns*

Subtract (4) from (3):

$$\left(\frac{2}{3} - \frac{1}{2}\right) D_y L = \left(-\frac{7}{192} + \frac{4}{192}\right) p_0 L^2 \quad \Longrightarrow \quad D_y = -\frac{18}{192} p_0 L$$

From (4):

$$M_D = -\frac{1}{2} D_y L - \frac{4p_0 L^2}{192} = \frac{5p_0 L^2}{192}$$

From (2):

$$B_y = -D_y - \frac{p_0 L}{2} = -\frac{78}{192} p_0 L$$

From (1):

$$M_B = M_D + D_y L + \frac{p_0 L^2}{8} = \frac{5p_0 L^2}{192} - \frac{18}{192} p_0 L^2 + \frac{p_0 L^2}{8} = \frac{11p_0 L^2}{192}$$