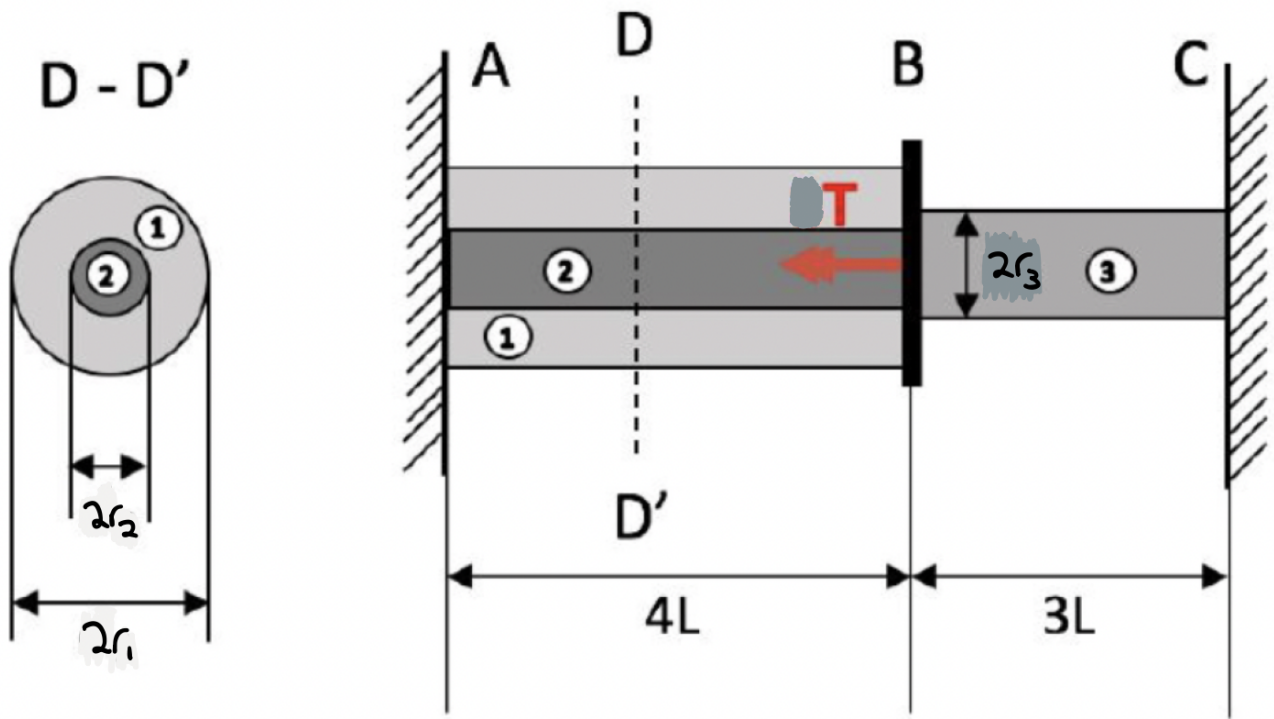


Problem 8.1 (10 points)

A stepped shaft ABC is made of members 1, 2 and 3. Member 1 is a hollow shaft and members 2 and 3 are solid shafts. Members 1 and 2 are fit tightly together. The shear modulus of members 1, 2 and 3 are G , $2G$ and G respectively.

- Determine whether the structure is statically determinate or indeterminate.
- Clearly state your choice of redundant load(s) for the problem.
- Use Castigliano's theorem to find the torque on each member.
- Use Castigliano's theorem to find the angle of rotation at B.

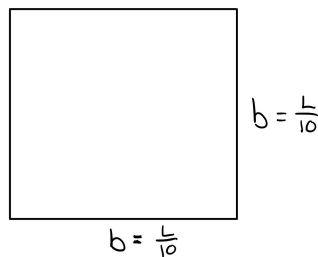
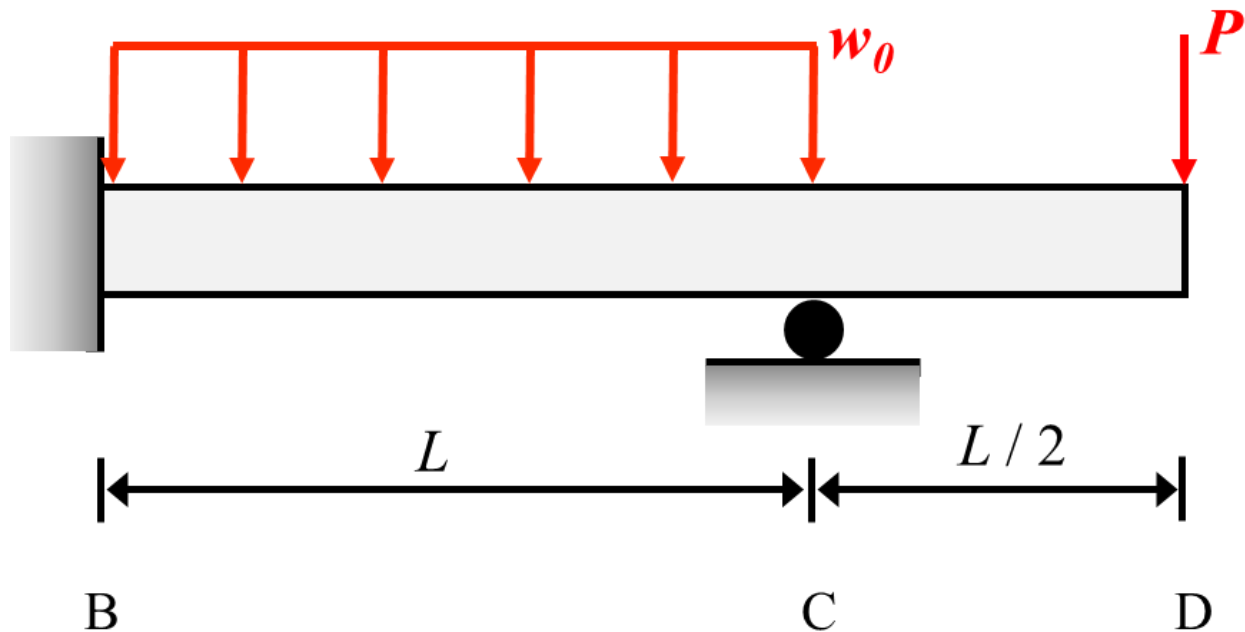


Use $r_1 = 2R$, $r_2 = R$, $r_3 = R$

Problem 8.2 (10 points)

A Beam BCD is loaded with a distributed load w_0 between B and C and a point force P at D. The beam has elastic modulus E and has a square cross section with side length $b = L / 10$. Neglect the shear strain energy due to bending.

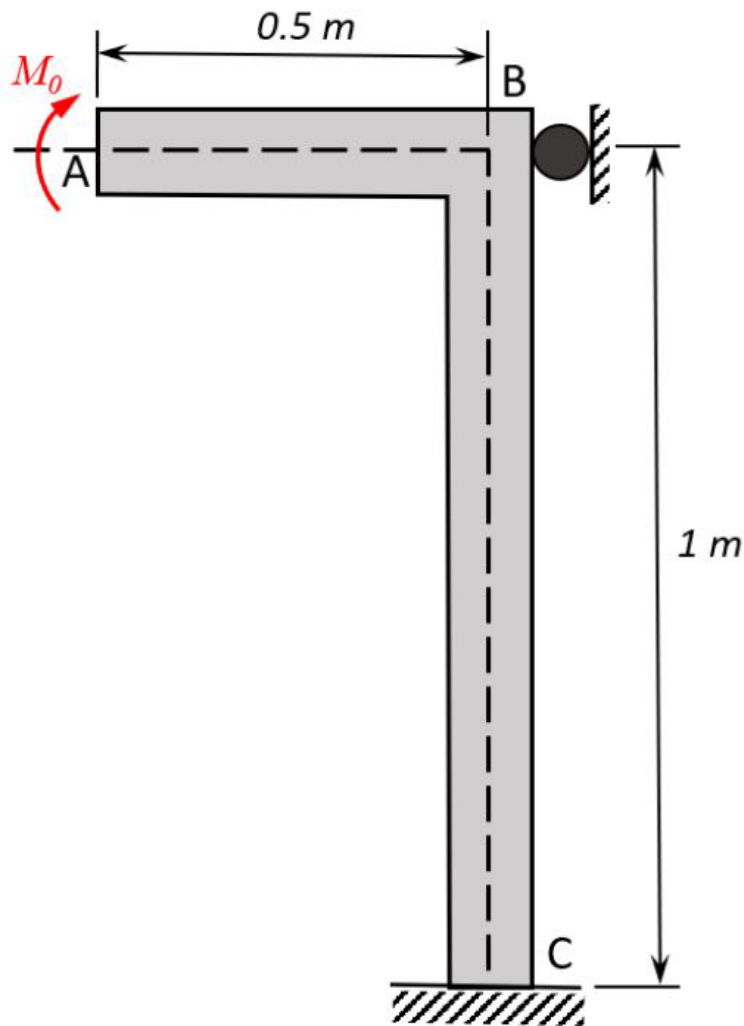
Using Castigliano's theorem, determine the reactions at the wall at B and the roller at C. Express your answer in terms of w_0 , P , L , and E . Explicitly state your choice of redundant load(s).



Problem 8.3 (10 points)

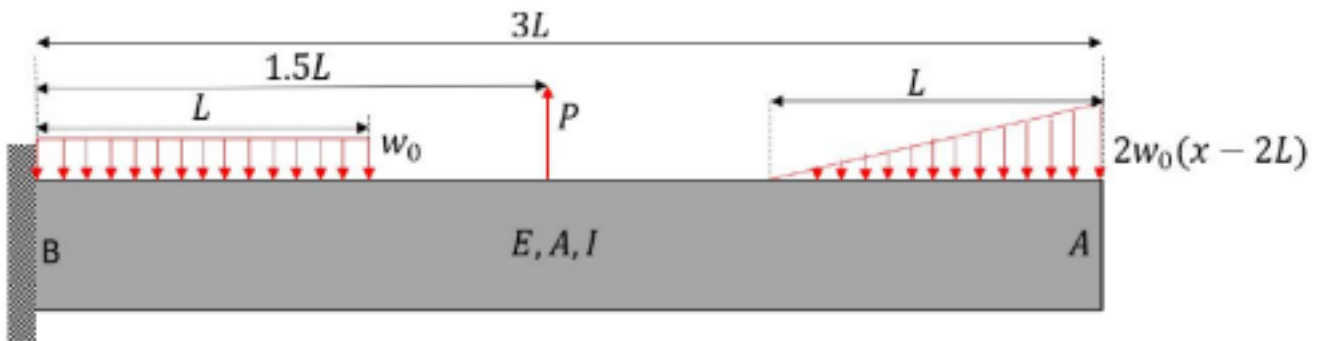
Two segments, AB and BC, with a thin walled hollow circular cross section of outer diameter a and inner diameter $0.8a$, are welded together at B to form the L-shaped ABC shown in the figure below. Use the following data in your analysis: $E = 280 \text{ GPa}$, $G = 120 \text{ GPa}$, $a = 20 \text{ mm}$, $M_0 = 1000 \text{ Nm}$

- (a) Determine the reactions.
- (b) Use Castigliano's Second Theorem to determine the slope θ at point A.



Problem 8.4 (5 points)

Consider the cantilevered beam with distributed loads as shown below. Ignoring shear effects:



Select the general form that the equation for flexural energy due to bending will take (note that $f_i(x)$ represents some *continuous* function $M^2(x)/(2EI)$ for the length of integration):

(a) $U = \int_0^L f_1(x)dx + \int_L^{2L} f_2(x)dx + \int_{2L}^{3L} f_3(x)dx$

(b) $U = \int_0^L f_1(x)dx + \int_L^{1.5L} f_2(x)dx + \int_{1.5L}^{3L} f_3(x)dx$

(c) $U = \int_0^L f_1(x)dx + \int_L^{1.5L} f_2(x)dx + \int_{1.5L}^{2L} f_3(x)dx + \int_{2L}^{3L} f_4(x)dx$

(d) $U = \int_0^L f_1(x)dx + \int_0^{1.5L} f_2(x)dx + \int_0^{2L} f_3(x)dx + \int_0^{3L} f_4(x)dx$