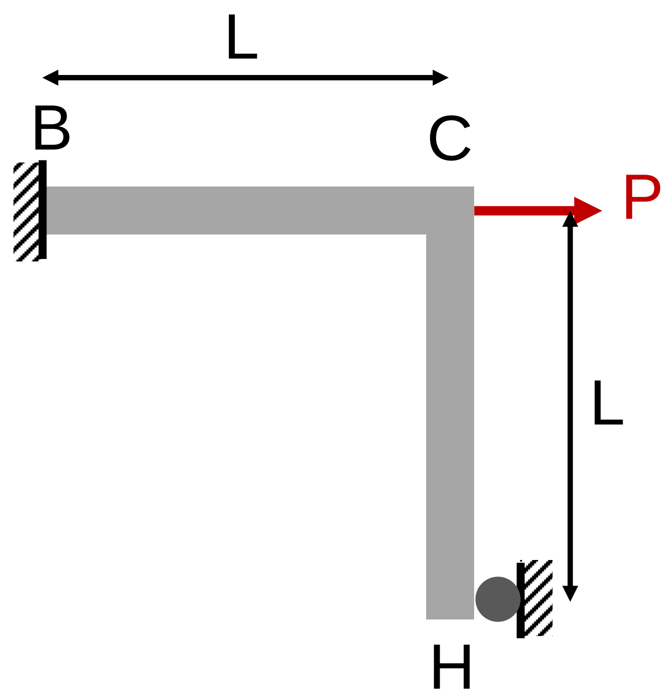
Lecture 25 Quiz

An L-shaped beam is subject to an applied force as shown in the diagram below. You are asked to use Castigliano's to solve for the angular change at H (θ_H).



(a) Draw the free body diagram.
(b) Each section can have up to 4 energy terms (axial, torsion, flexural shear), giving a total number of potential terms that is 4x(number of sections). How many energy terms are non-zero in this structure?
(c) Which equations would need to be solved to solve for the angular change at H? (don't need to solve the equations) e.g. $\frac{\delta U}{\delta K} = 0$

Deflection analysis – Castigliano's method

The procedure for deflection analysis using Castigliano's method:

i) First determine if you need to include any "dummy" loads (recall that the Castigliano's method can produce deflections/rotations at points on the structures at which applied forces/moments act and in directions in which these forces/moments act). *Add in ALL of the needed dummy loads from the start*; this can save you a lot of time down the road.

ii) Draw a free body diagram (FBD) of the entire structure, and from this FBD write down the equilibrium equations; these equilibrium equations will be written in terms of the external reactions.

- If *DETERMINATE*, solve these equations for the external reactions.
- If *INDETERMINATE*, establish the "order" N_R of the indeterminancy (i.e., equal to the number of additional equations needed to solve for external reactions). From your external reactions, choose a set of *n* redundant reactions (R_i ; $i = 1,2,...,N_R$). *Write the remaining reactions in terms of these* N_R *redundant reactions*.

iii) Divide beam into sections: $x_i < x < x_{i+1}$. This section division is dictated by: support reactions, beam geometry changes, and/or load changes (concentrated forces/moments, line load definition changes, etc.).

iv) For each section, draw an FBD of either the left or right side of the body from a cut through that section of the beam. From this FBD, determine the distribution of bending moment $M_i(x)$, shear force $V_i(x)$ and axial force $F_{Ni}(x)$ through that section of the structure. Using these, write down the strain energy in that section of the structure using:

$$U_i = \frac{1}{2EI}\int_{x_i}^{x_{i+1}} M_i^2\,dx + \frac{f_s}{2GA}\int_{x_i}^{x_{i+1}} V_i^2\,dx + \frac{1}{2EA}\int_{x_i}^{x_{i+1}} F_{Ni}^2\,dx$$

From these strain energy terms, write down the total strain energy for the structure: $U = U_1 + U_2 + U_3 + \dots$. It is recommended that you do NOT expand out the "squared" terms in these integrals at this point.

v) If the problem is *INDETERMINATE*, first set up the additional algebraic equations for the reactions of the problems using Castigliano:

$$0 = \frac{\partial U}{\partial R_i} \quad ; \quad i = 1,2,...,N_R$$

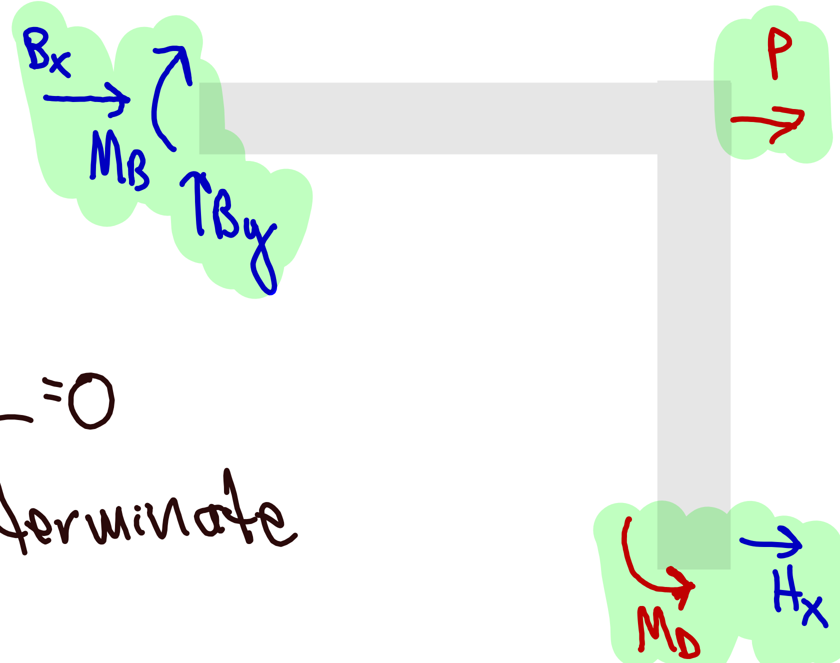
Be sure to set any dummy loads to zero in the end. Solve these equations with the equilibrium equations from i) above.

vi) Determine the desired deflections/rotations using Castigliano's method: $\delta_i = \partial U / \partial P_i$. Be sure to set any dummy loads to zero in the end.

(i) Determine if dummy loads are required

--> Need dummy moment at H

(a) (ii)



$\Sigma F_y = B_y = 0$

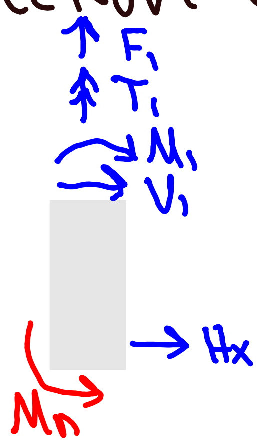
$\Sigma F_x = B_x + H_x + P = 0$

$(\Sigma M)_B = -M_B + M_D + H_x L = 0$

3 unknowns, 2 equations } indeterminate

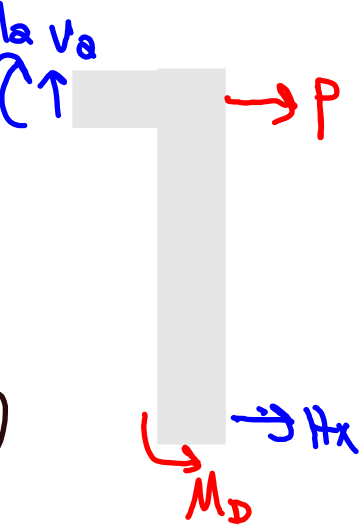
(b) (iii) + (iv)

Section CH



$F_1 = 0$ $T_1 = 0$ $M_1 \neq 0$ $V_1 \neq 0$

Section BC



$F_2 \neq 0$

$T_2 = 0$

$M_2 \neq 0$

$V_2 = 0$

$\Rightarrow$ 4 non-zero terms (M_1, V_1, F_2, M_2)

(c) (v) If indeterminate, solve for reactions:

$$\left[\frac{\partial U}{\partial H_x}\right]_{M_D=0} = 0$$

(vi) Solve for displacements:

$$\theta_H = \left[\frac{\partial U}{\partial M_D}\right]_{M_D=0}$$