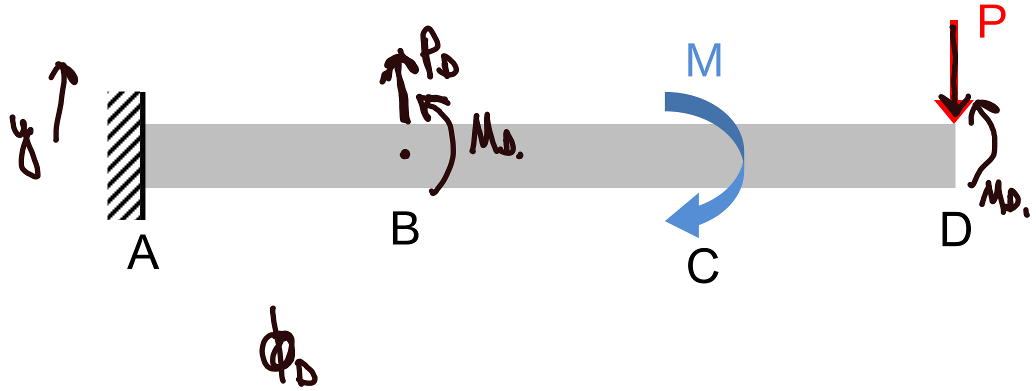
M	**9-Oct**	***October Break - no class***		
20 W	11-Oct	Beam deflections – superposition methods	Chap. 11	
21 F	13-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW. 6
22 M	16-Oct	Energy methods – Castigliano's theorems	Chap. 16	
23 W	18-Oct	Energy methods – Castigliano's theorems	Chap. 16	
24 F	20-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW 7
25 M	23-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	Quiz.
26 W	25-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
27 F	27-Oct	Energy methods – introduction to finite element methods	Chap. 17	HW 8
28 M	30-Oct	Review		
W	**1-Nov**	***Examination 2, 8-10p.m. (no lecture on Wednesday)***		
29 F	3-Nov	Energy methods – introduction to finite element methods	Chap. 17	

Castigliano's Review

Question 1:
In the beam, the displacement at B is found by:

- The partial derivative of U with respect to P.
- The partial derivative of U with respect to M.
- The partial derivative of U with respect to a dummy load.
- The partial derivative of U with respect to a dummy moment.

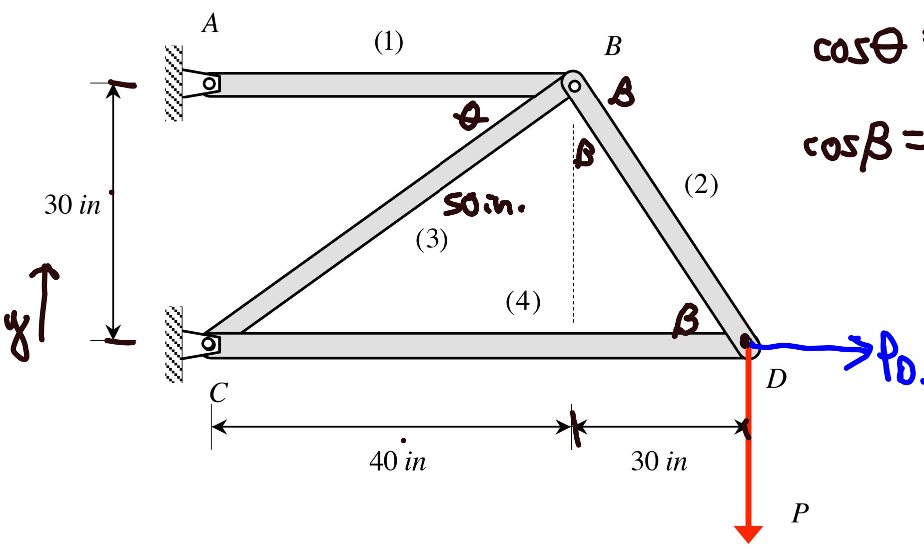


$\Delta = \frac{\partial U}{\partial P}$

$\Delta_B = \left[\frac{\partial U}{\partial P_D}\right]_{P_D = 0}$

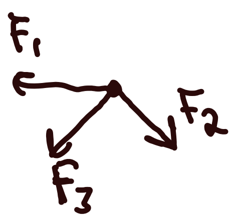
16.8

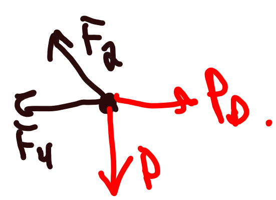
Problem D
Determine the vertical and horizontal deflection of the truss at joint D. All members of the truss have a cross-sectional area of A and are made of a material with a Young's modulus of E.



$\cos\theta = \frac{4}{5}$

$\cos\beta = \frac{30}{30\sqrt{2}} = \frac{1}{\sqrt{2}}$





$$\left.\begin{aligned}
(\Sigma F_x)_B &= -F_1 - F_3\left(\tfrac{4}{5}\right) + F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0 \;\Rightarrow\; F_1 = P + \left(\tfrac{4}{3}\right)P = \tfrac{7}{3}P \\
(\Sigma F_y)_B &= -F_3\left(\tfrac{3}{5}\right) - F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0 \;\Rightarrow\; F_3 = -\left(\tfrac{5}{3}\right)P \\
(\Sigma F_x)_D &= P_D - F_4 - F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0 \;\Rightarrow\; F_4 = P_D - P \\
(\Sigma F_y)_D &= -P + F_2\left(\tfrac{1}{\sqrt{2}}\right) = 0 \;\Rightarrow\; F_2 = \sqrt{2}P
\end{aligned}\right\}\text{ determinate}$$

Strain Energy.

$$U = U_1 + U_2 + U_3 + U_4$$

$$U_1 = U_{1A_1} + \cancel{U_{1Y}}^{0} + \underline{\underline{\cancel{U_{1M}}^{0}}} + \underline{\cancel{U_{1V}}^{0}}$$

$$U = \frac{F_1^2 L_1}{2EA} + \frac{F_2^2 L_2}{2EA} + \frac{F_3^2 L_3}{2EA} + \frac{F_4^2 L_4}{2EA}.$$

$$u = \left[\frac{\partial U}{\partial P_D}\right]_{P_D = 0}$$

$$\frac{\partial F_1}{\partial P_D} = 0 \qquad \frac{\partial F_2}{\partial P_D} = 0 \qquad \frac{\partial F_3}{\partial P_D} = 0 \qquad \frac{\partial F_4}{\partial P_D} = 1$$

$$u = \frac{F_4 L_4}{EA}\frac{\partial F_4}{\partial P_D} = \frac{(-P)L_4}{EA}(1)$$

$$v = -\left[\frac{\partial U}{\partial P}\right]_{P_D = 0}$$

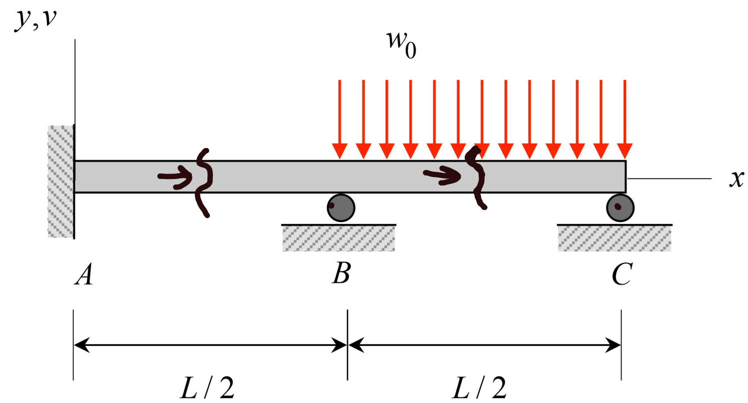
$$\frac{\partial F_1}{\partial P} = \frac{7}{3} \qquad \frac{\partial F_2}{\partial P} = \sqrt{2} \qquad \frac{\partial F_3}{\partial P} = -\left(\frac{5}{3}\right) \qquad \frac{\partial F_4}{\partial P} = -1$$

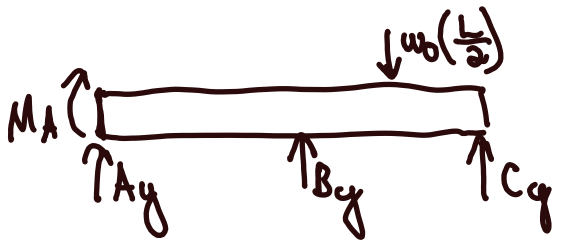
$$v = -\frac{L_1}{EA}\left(\frac{7}{3}\right)^2 P - \frac{L_2}{EA}(\sqrt{2})^2 P - \frac{L_3}{EA}\left(-\frac{5}{3}\right)^2 P - \frac{L_4}{EA}(-P)(-1)$$

$$\frac{F_1 L_1}{EA}\frac{\partial F_1}{\partial P} = \frac{\left(\frac{7}{3}P\right)L_1}{EA}\left(\frac{7}{3}\right)$$

Problem C

Determine the reactions at rollers B and C on the beam below. Let E and I be the Young's modulus and second area moment of the beam cross section, respectively, of the beam.



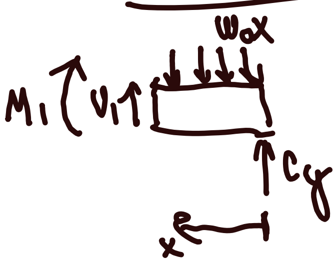


$$\sum F_y = A_y + B_y + C_y - w_0\left(\frac{L}{2}\right) = 0 \quad (1)$$

$$\left(\sum M\right)_A = -M_A + B_y\left(\frac{L}{2}\right) + C_y(L) - w_0\left(\frac{L}{2}\right)\left(\frac{3L}{4}\right) = 0 \quad (2)$$

4 unknowns
2 equs
2 redundant.

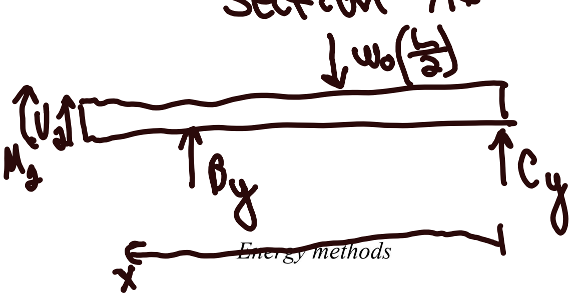
Section BC.



$$M_1(x) = C_y x - w_0 \frac{x^2}{2}.$$

$$V_1(x) = -C_y + w_0 x.$$

Section AB



$$M_2(x) = B_y\left(x - \frac{L}{2}\right) + C_y x - w_0\left(\frac{L}{2}\right)\left(x - \frac{L}{4}\right).$$

$$U = U_1 + U_2$$

$$U = \frac{1}{2EI}\int_0^{L/2} M_1^2\,dx + \frac{1}{2EI}\int_{L/2}^{L} M_2^2\,dx.$$

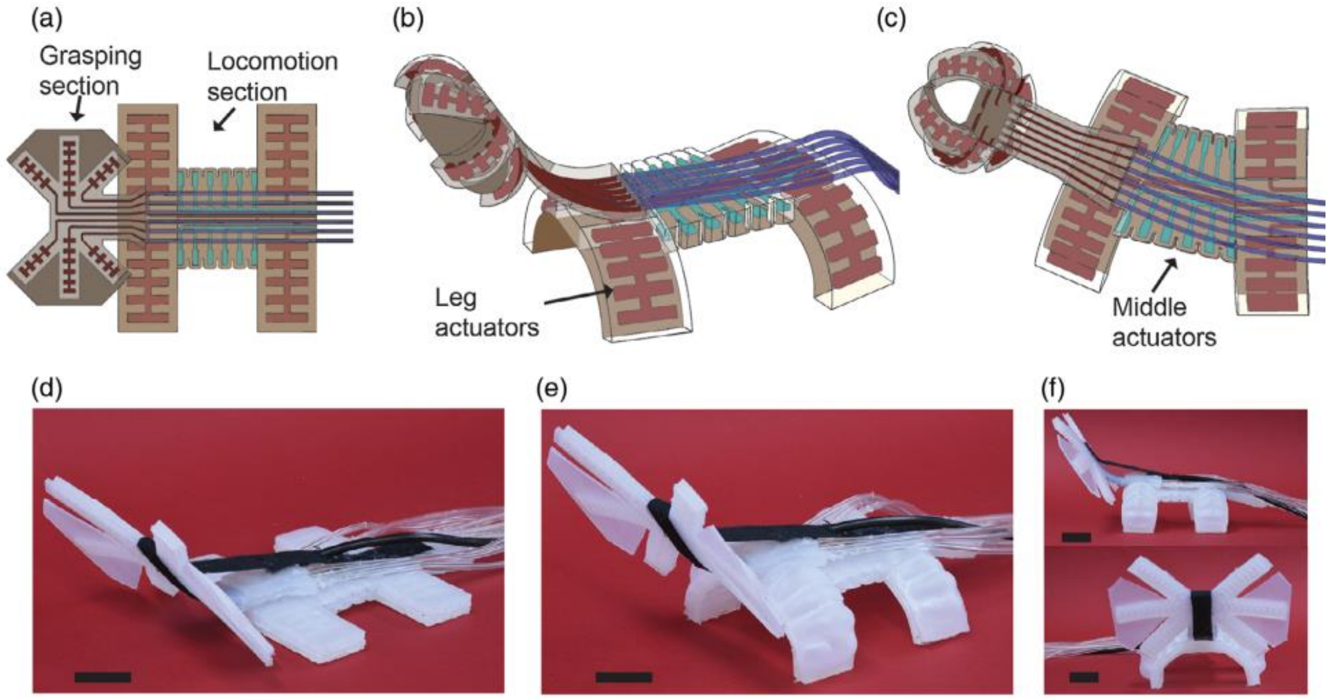
$$\frac{\partial U}{\partial B_y} = 0 = \frac{1}{EI}\int_0^{L/2} M_1 \frac{\partial M}{\partial B_y}\,dx + \frac{1}{EI}\int_{L/2}^{L} M_2 \frac{\partial M_2}{\partial B_y}\,dx.$$

$$\frac{\partial M_1}{\partial B_y} = 0 \qquad \frac{\partial M_2}{\partial B_y} = (x - L/2)$$

(3) $$0 = \frac{1}{EI}\int_{L/2}^{L}\left[B_y(x - L/2) + C_y - w_0\left(\frac{L}{2}\right)\left(x - L/4\right)\right]\left(x - \frac{L}{2}\right)dx.$$

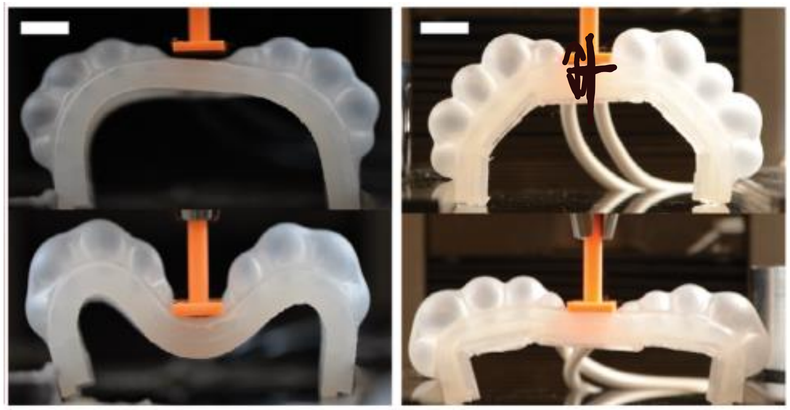
(4) $$\frac{\partial U}{\partial C_y} = 0$$

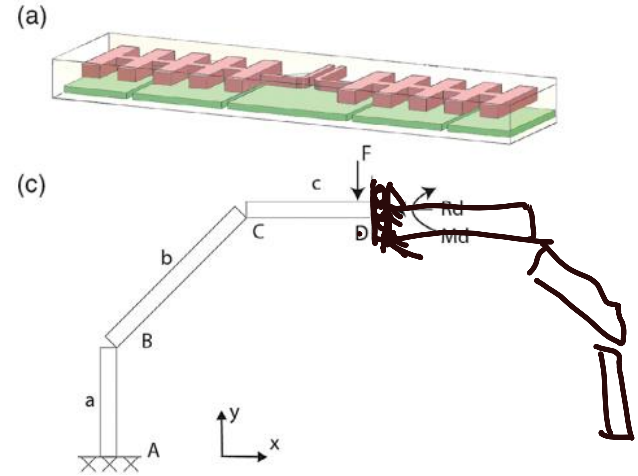
Demogorgon Soft Robot



Yin et al, Adv Intell Syst, 1:1900089, 2019.

Demogorgon Soft Robot





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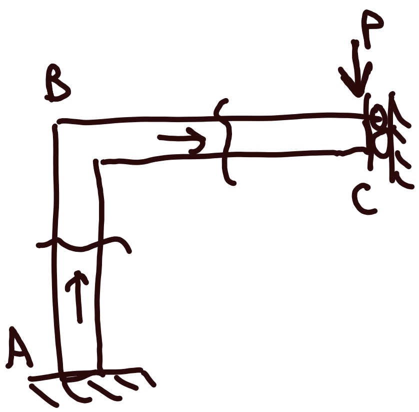
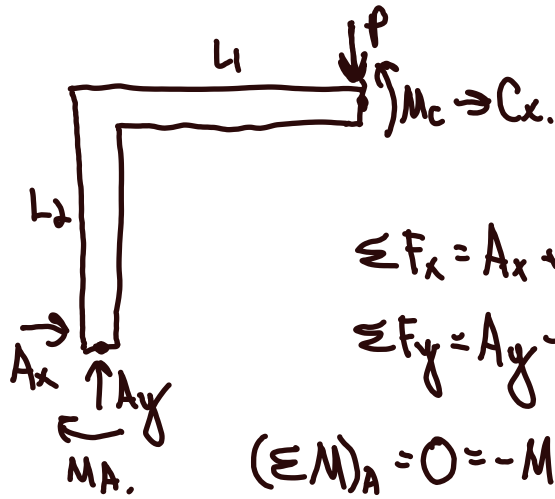




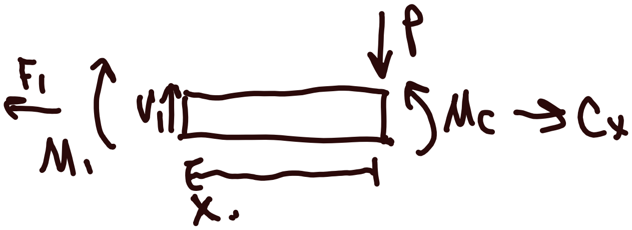
$$\Sigma F_x = A_x + C_x = 0$$

$$\Sigma F_y = A_y - P = 0$$

$$(\Sigma M)_A = 0 = -M_A + M_c - PL_1 - C_x L_2$$

5 unknowns
3 eqns
2 redundant.

Section BC

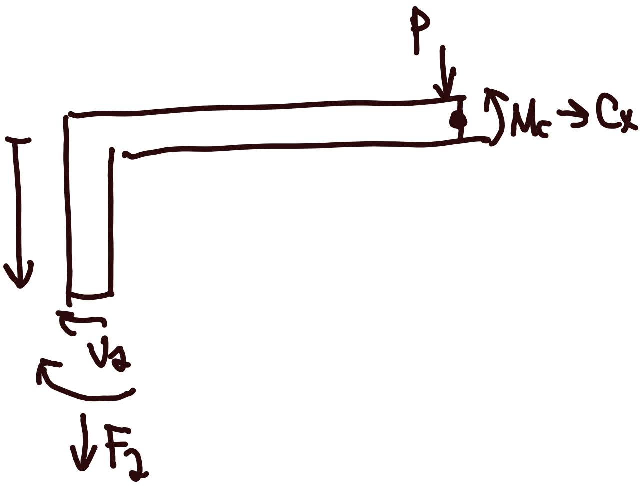


$F_1 = C_x$

$V_1 = P$

$M_1 = M_c - Px$

$$U = U_A + U_T + U_M + U_V$$



$F_2 = -P$

$V_2 = C_x$

$M_2 = M_c - PL_1 - yC_x$

Find Reactions

$$\frac{\partial U}{\partial C_x} = 0 \qquad \frac{\partial U}{\partial M_c} = \theta_c = 0$$