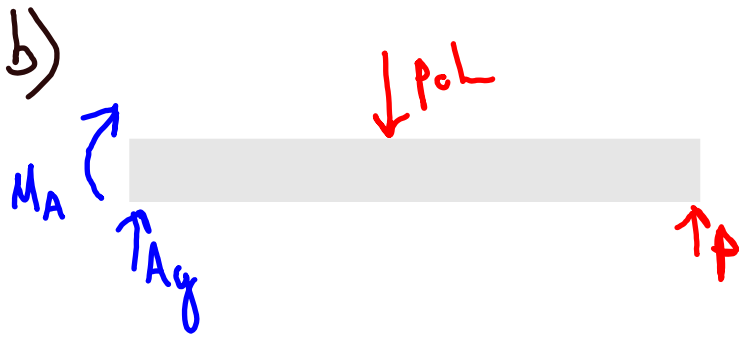


$$a) \quad v(0) = 0 \quad (1) \quad v(L) = -2 \text{ mm} \quad (1) \\ \theta(0) = 0 \quad (1)$$

Common mistakes:
Trying to come up with a B.C. at $\theta(L)$, e.g. $\tan(\theta(L)) = \frac{2}{80}$



$$\sum F_y = A_y + P - p_0 L = 0 \\ (\sum M)_A = -M_A - p_0 L \left(\frac{L}{2}\right) + PL = 0$$

Common mistake: Forgetting the reaction moment (M_A) at the left side.

c) Superposition approach

$$V_{p_0} = \frac{-p_0 x^2}{24EI} (6L^2 - 4Lx + x^2) \quad V_P = \frac{Px^2}{6EI} (3L - x)$$

$$V(x) = V_{p_0}(x) + V_P(x)$$

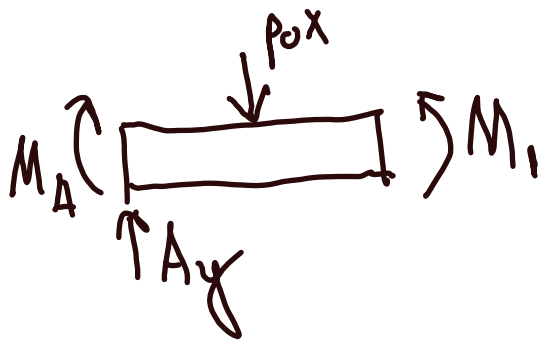
$$v(L) = -0.002 \text{ m} = -\frac{p_0 L^4}{8EI} + \frac{PL^3}{3EI}$$

$$p_0 = 0.689 \text{ N/m}$$

Common mistakes:

- Treat distributed load as point load.
- Use polar area moment ($\frac{\pi R^4}{2}$) instead of second area moment ($\frac{\pi R^4}{4}$)

Second order approach



$$(\sum M)_1 = M_1 - M_A - A_y x + p_0 x \left(\frac{x}{2} \right) = 0$$

$$M_1 = M_A + A_y x - p_0 \frac{x^2}{2}$$

$$\theta(x) = \theta(0) + \frac{1}{EI} \int_0^x M_1 dx$$

$$\theta(x) = \frac{1}{EI} \left[M_A x + A_y \frac{x^2}{2} - p_0 \frac{x^3}{6} \right]$$

$$v(x) = v(0) + \int_0^x \theta dx$$

$$v(x) = \frac{1}{EI} \left[M_A \frac{x^2}{2} + A_y \frac{x^3}{6} - p_0 \frac{x^4}{24} \right]$$

$$v(L) = -0.002 \text{ m} = \frac{1}{EI} \left[\frac{pL^3}{2} - \frac{pL^3}{6} - p_0 \frac{L^4}{4} + p_0 \frac{L^4}{6} - p_0 \frac{L^4}{24} \right]$$

$$\Rightarrow p_0 = 0.689 \text{ N/m}$$

$$\begin{aligned} A_y &= -P + p_0 L \\ M_A &= PL - p_0 \frac{L^2}{2} \end{aligned}$$