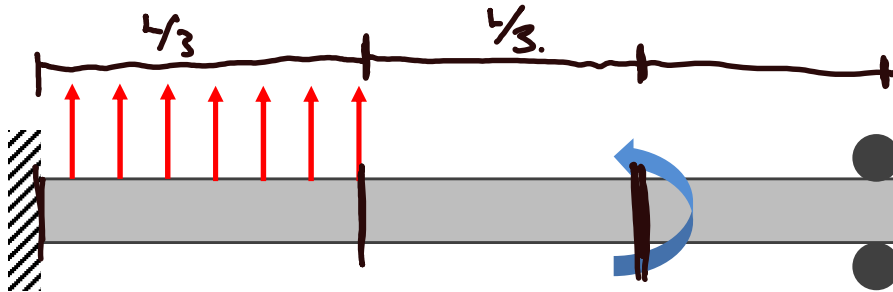


Lecture 19 Review

1. In the beam below, how many sections will we need to use?



2. This beam has ____ boundary conditions and ____ continuity conditions.

B.C.

$$\begin{aligned} \theta(0) &= 0 & v(L) &= 0 \\ v(0) &= 0 & M(L) &= 0 \end{aligned}$$

$M(x)$

$$\begin{aligned} v_1\left(\frac{4}{3}\right) &= v_2\left(\frac{4}{3}\right) \\ \theta_1\left(\frac{4}{3}\right) &= \theta_2\left(\frac{4}{3}\right) \end{aligned}$$

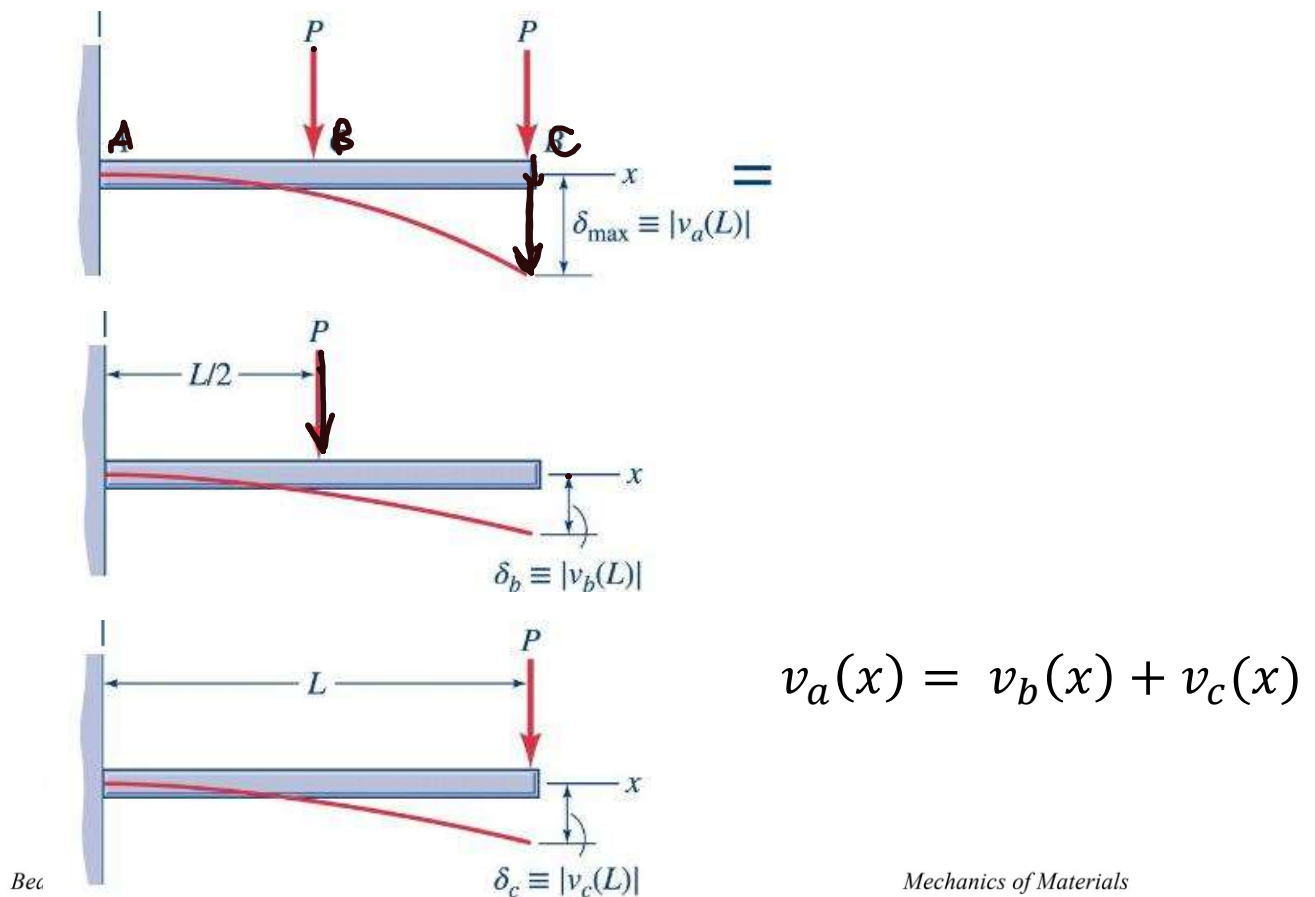
Calculation of beam deflections using superposition

In many situations, the loadings on beams can be recognized as the combination of simple loadings. You might discover that the beam deflection due to each of the simple loadings individually have already been computed by others before you, with some of these results appearing in the Appendix of the textbook (with these results repeated here in these notes). Since the Euler-Bernoulli theory used for the deflection of beams here is *linear*, the deflection of the beam due the combination of simpler loads is a “superposition” of the deflections due of the beam as a result of each simpler load acting individually.

For determinate beams, we need only to find the deflection due to each loading individually, and then add together these deflections.

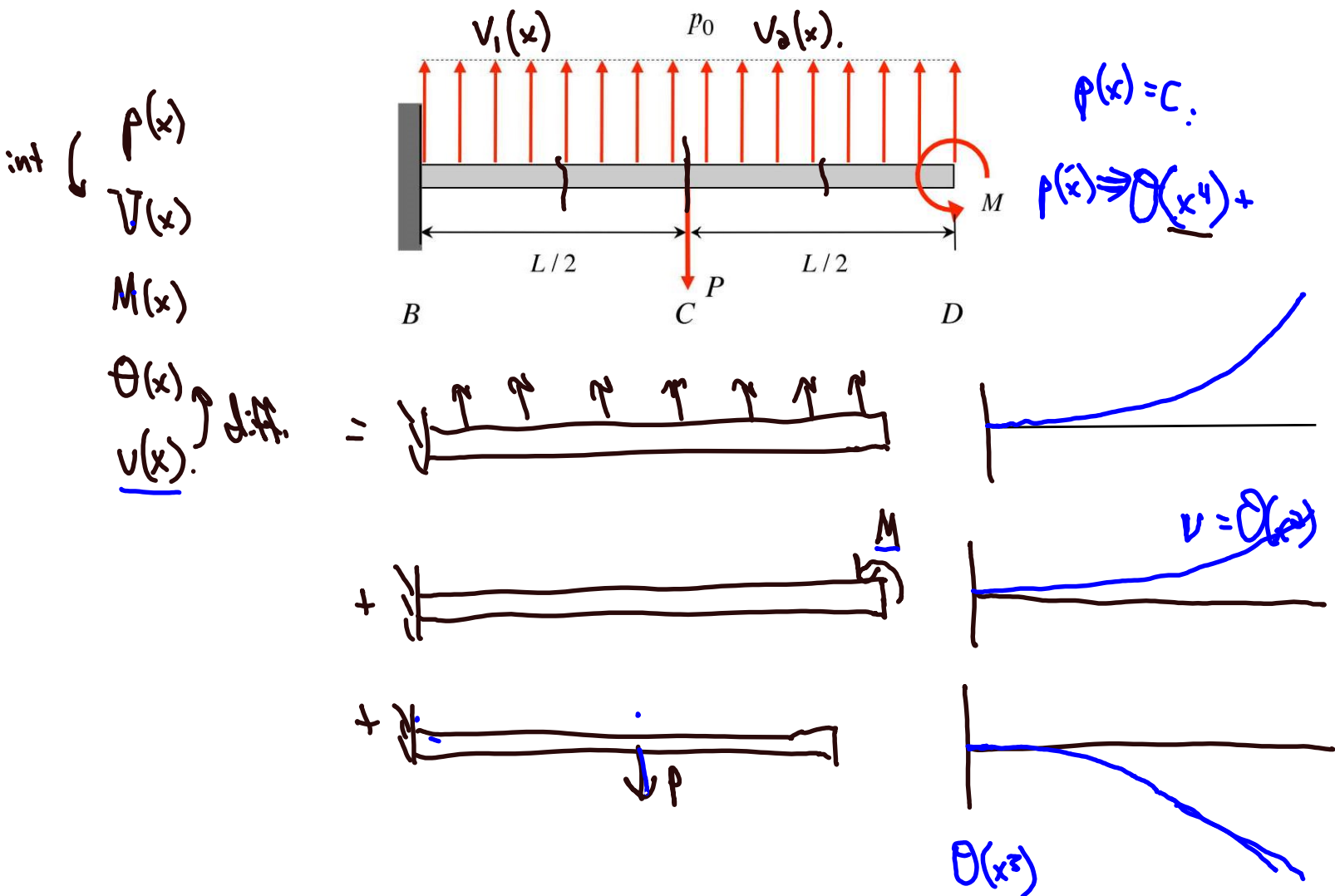
For indeterminate beams, we will first need to recognize which loadings (both applied and reactive) combine together to give the indeterminate problem. We next find the deflections due to the simple loadings individually, and add together through superposition. Prescribed displacements/rotations are then imposed on the combined deflection relations to determine the unknown reactions.

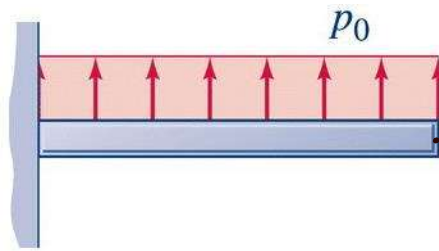
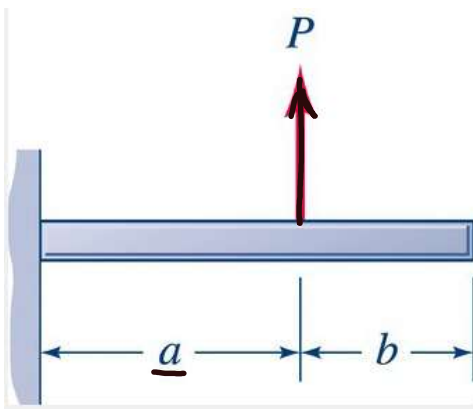
Please note that linear superposition is a generic and widely applicable method in linear elasticity. This method originates from the assumed linear relationship between applied loads and deformations. Although we will use linear superposition here in combining known deflection relationships of simple problems in determining the deflection of more complicated loadings, the concept of “superposition” extends well beyond this limited application.



Example 11.17

Determine the deflection of the beam loaded as below using superposition.





$$\begin{aligned} v &= \frac{Px^2}{6EI}(3a-x) & v' &= \frac{Px}{2EI}(2a-x) & 0 \leq x \leq a \\ v &= \frac{Pa^2}{6EI}(3x-a) & v' &= \frac{Pa^2}{2EI} & a \leq x \leq L \\ \delta_B &= \frac{Pa^2}{6EI}(3L-a) & \theta_B &= \frac{Pa^2}{2EI} \end{aligned}$$

$$\begin{aligned} v &= \frac{p_0 x^2}{24EI}(6L^2 - 4Lx + x^2) \\ v' &= \frac{p_0 x}{6EI}(3L^2 - 3Lx + x^2) \\ \delta_B &= \frac{p_0 L^4}{8EI} & \theta_B &= \frac{p_0 L^3}{6EI} \end{aligned}$$

$$\begin{aligned} v &= \frac{M_0 x^2}{2EI} & v' &= \frac{M_0 x}{EI} \\ \delta_B &= \frac{M_0 L^2}{2EI} & \theta_B &= \frac{M_0 L}{EI} \end{aligned}$$

$$v_{p_0}(x) = \frac{p_0 x^2}{24EI} \left[x^2 - 4Lx + 6L^2 \right] \quad 0 < x < L$$

$$v_M(x) = \frac{M_0 x^2}{2EI} \quad 0 < x < L$$

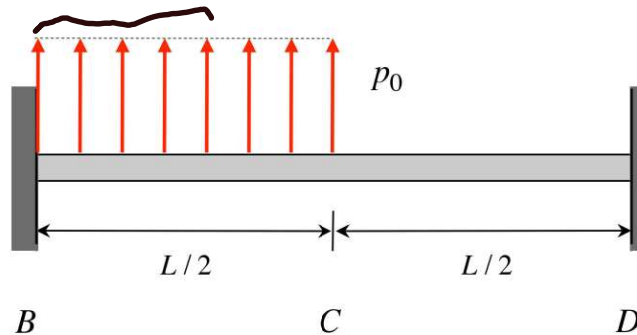
$$v_P(x) = -\frac{P}{EI} \left(\frac{1}{6} \right) \left[x^2 \left(\frac{3L}{2} - x \right) \right] \quad 0 < x < \frac{L}{2}$$

$$v_P(x) = -\frac{P}{EI} \left(\frac{1}{6} \right) \left[\frac{L^2}{4} \left(3x - \frac{L}{2} \right) \right] \quad \frac{L}{2} < x < L$$

Indeterminate: treat redundant reactions as applied loads.

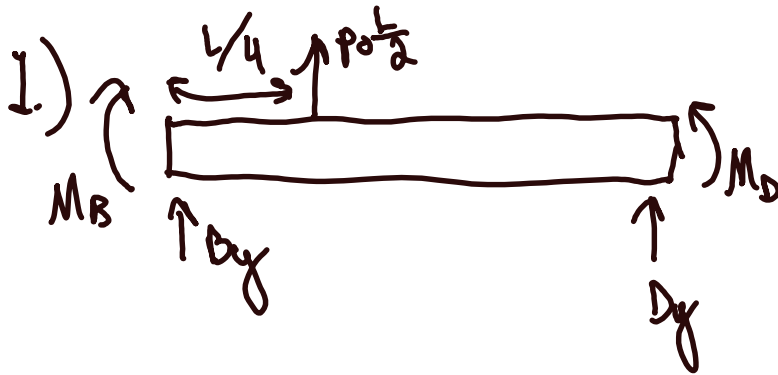
Example 11.20

Determine the reactions acting on the beam loaded as below using superposition.



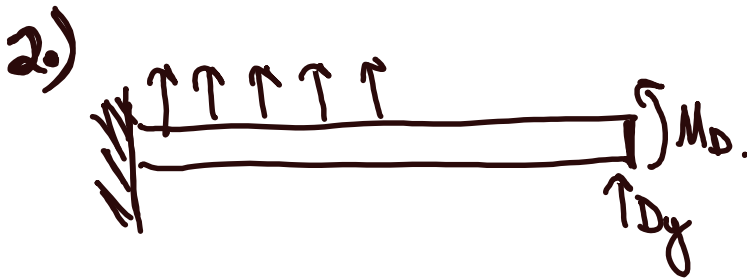
B.C.S.

$$\begin{cases} v(0)=0 & v(L)=0 \\ \theta(0)=0 & \theta(L)=0 \end{cases}$$



$$D_y + P_0 \frac{L}{2} + D_y = 0 \quad (1)$$

$$-M_B + M_D + P_0 \left(\frac{L}{2}\right)\left(\frac{L}{4}\right) + D_y L = 0 \quad (2)$$



BCS

$$V_2(L) = \frac{P_0}{24EI} \left(\frac{L}{2}\right)^3 \left(4L - \frac{L}{2}\right) + \frac{D_y L^3}{3EI} + \frac{M_D L^2}{2EI} = 0 \quad (3)$$

BC4.

$$\theta_2(L) = \frac{P_0}{6EI} \left(\frac{L}{2}\right)^3 + \frac{D_y L^2}{2EI} + \frac{M_D L}{EI} = 0 \quad (4)$$



$$v(0) \checkmark = 0 \quad v(L) \checkmark = 0$$

$$\theta(0) = 0 \quad \theta(L) = 0$$

$$\underline{\theta_1(0) = 0}$$

$$\underline{\theta_2(L) = 0}$$