Lecture 18 Review Questions

If you know the shear force along a beam (large V), how do you find the beam displacement in the y direction (small v):

- Integrate once
- Integrate three times
- Take derivative once
- Take derivative three times

$p(x)$

$V(x)$

$M(x)$

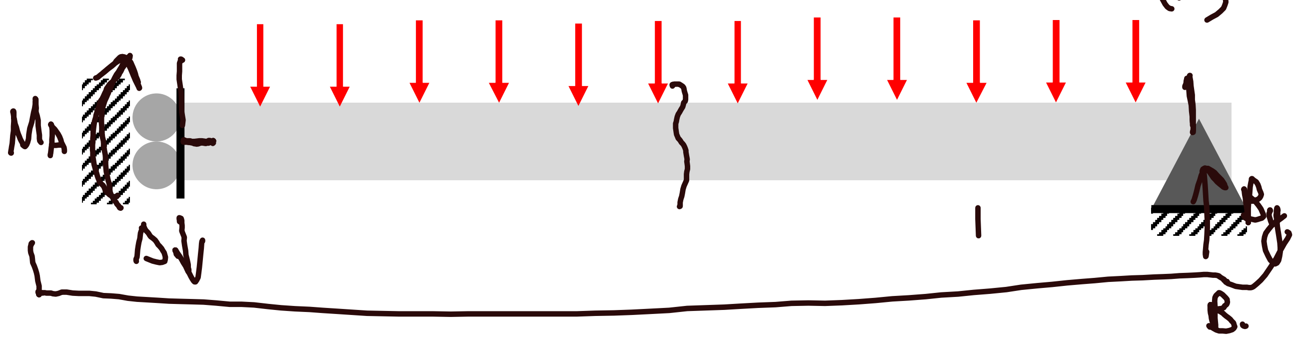
int

$\theta(x)$

$v(x)$

Next quiz: Oct 11

How would you do this question?



ΣM

ΣF

BC: $\theta(0)=0$

$v(L)=0$

$M(x)$.

$\theta(x) = \theta_A^{\,0} + f(x)$

$v(x) = v_A + f(x)$

$v(L) = v_A + f(L) = 0$

Discussion for indeterminate beams

Statically indeterminate beams are those for which we are unable to solve for reactions simply from the rigid body equilibrium analysis. This is the case since the number of unknowns exceeds the number of equilibrium equations available. As we found for axially-loaded rods and torque-loaded shafts, we need to solve deformation analysis equations simultaneously with the equilibrium equations in order to determine the reactions on the beams.

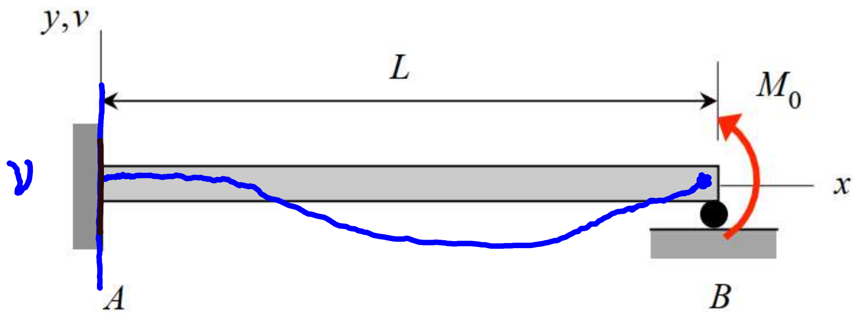
Reread the earlier discussion on the analysis of determinate beams. All of the points made there also apply to indeterminate beams, with the exception of point a). In point a), we need to revise the wording for indeterminate beams to state that we are not able to solve for reactions at the first step. Here, for indeterminate beams, we will leave the reactions as unknowns, and later enforce compatibility relations to produce the additional equations needed for a solution.

Note that reactions for determinate beams are independent of material and cross section properties of the beam since these reactions depend only on the distribution of applied loads. For indeterminate beams, material and cross section properties (EI) influence the values found for the reactions.

	Axial	Beams
1.)	ΣM ΣF	ΣM ΣF
2.)	$\underline{e} = \frac{FL}{EA}$	$v(x) = f(\text{reactions}, x)$ $\theta(x) = f(\text{reactions}, x)$
3.)	$e_1 = 2e_2$	$v(L) = 0 = f(\text{reactions})$
4.)	2 into 3	3 into 2.

Example 11.10

The beam is made up of a material with an elastic modulus E and has a cross-sectional second area moment I, both of which are constant along the length of the beam. Determine the reactions at A and B.



BCs: $\theta(0)=0$

$v(0)=0$

$v(L)=0$

Plan

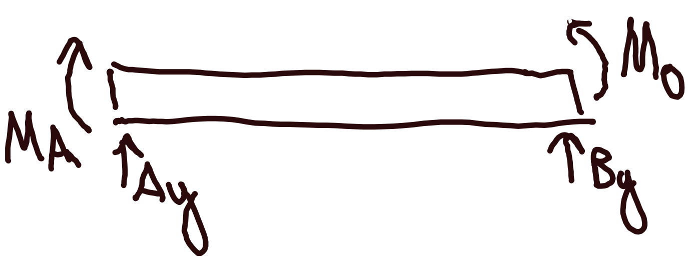
$M(x)$

$\theta(x) = \cancel{\theta_A}^{0} + f(\text{reactions}, x).$

$v(x) = \cancel{v_A}^{0} + f(\text{reactions}, x).$

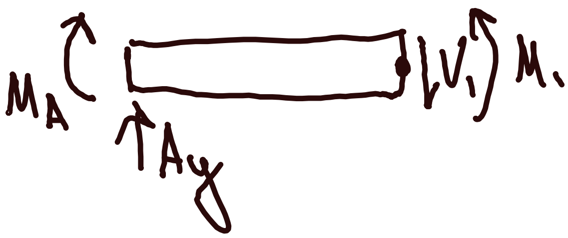
$v(L) = 0 = f(\text{reactions}).$ (3)

I.) Equilibrium



$$\left.\begin{aligned} \Sigma F_y &= A_y + B_y = 0 \quad (1) \\ (\Sigma M)_A &= -M_A + M_0 + B_y L = 0 \quad (2) \end{aligned}\right\} \begin{array}{l} 3 \text{ unk} \\ 2 \text{ eqn.} \end{array}$$

2.) Force-displacement.



$$(\Sigma M)_1 = M_1 - M_A - A_y x = 0$$

$$M_1(x) = M_A + A_y x.$$

$$\theta(x) = \cancelto{0}{\theta(0)} + \frac{1}{EI}\int_0^x M_A + A_y x \, dx.$$ 1st B.C.

$$\theta(x) = \frac{1}{EI}\left[M_A x + A_y \frac{x^2}{2}\right]$$

$$v(x) = \cancelto{0}{v(0)} + \frac{1}{EI}\int_0^x M_A x + A_y \frac{x^2}{2} \, dx.$$ 2nd B.C.

$$\underline{v(x)} = \frac{1}{EI}\left[M_A \frac{x^2}{2} + A_y \frac{x^3}{6}\right]$$

3.) Compatibility.

$$v(L) = 0 = M_A \frac{L^2}{2} + A_y \frac{L^3}{6} \qquad (3)$$

4.) Solve.

$$(3) \rightarrow 3M_A = -A_y L$$

$$-M_A + M_0 - A_y L = 0$$

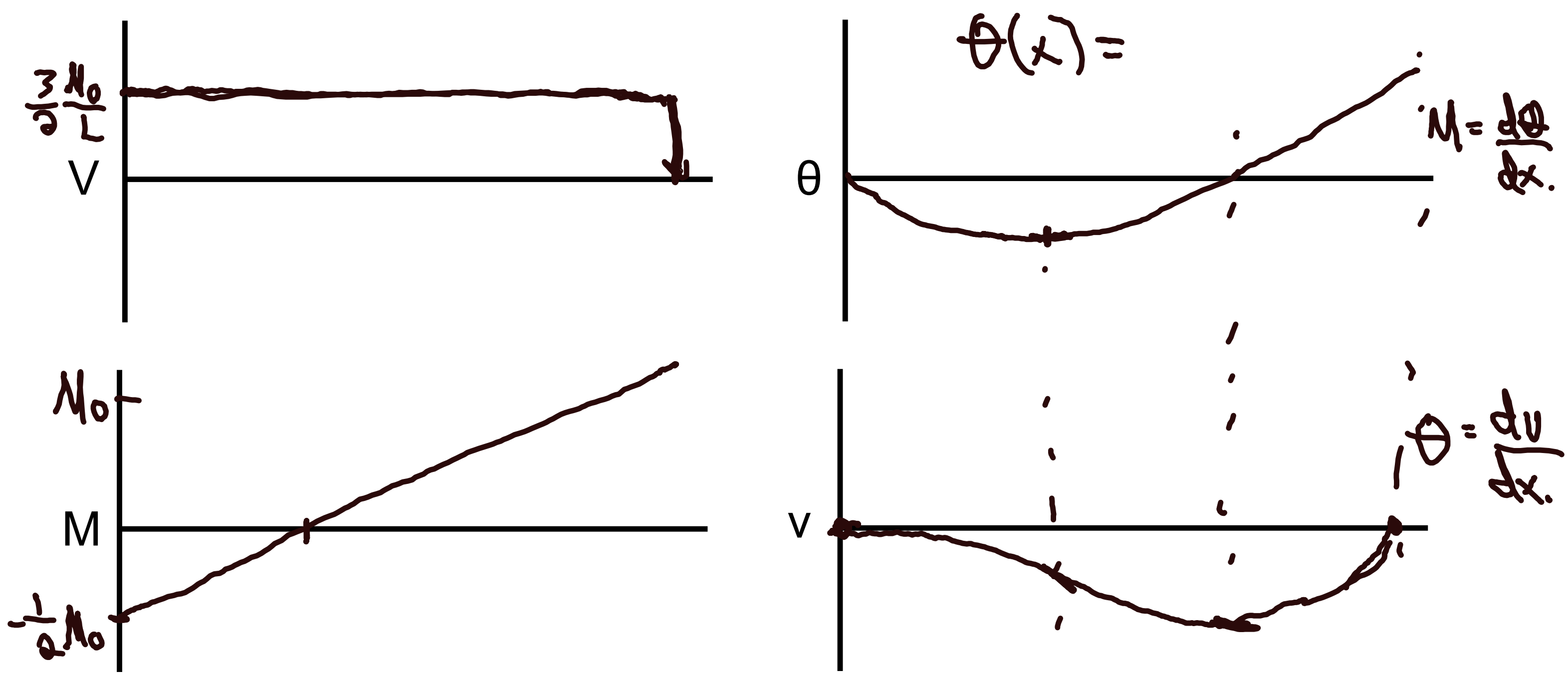
$$-M_A + M_0 + 3M_A = 0$$

$$M_A = -\frac{1}{2}M_0.$$

$$A_y = \frac{3}{2}\frac{M_0}{L}$$

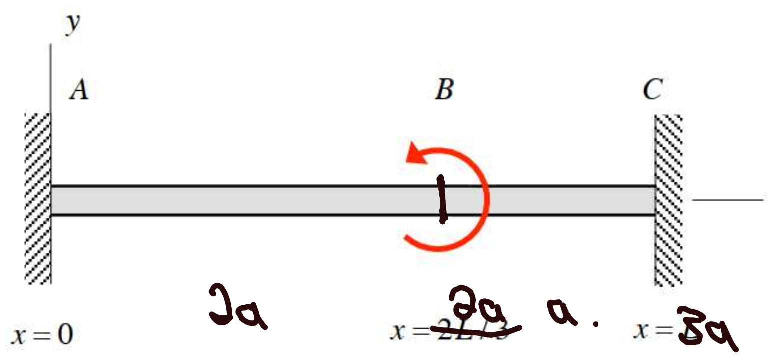
$$B_y = -\frac{3}{2}\frac{M_0}{L}$$

$$\left[\begin{aligned} \theta(x) &= \frac{1}{EI}\left[-\frac{1}{2}M_0 x + \frac{3}{4}\frac{M_0}{L}x^2\right] \\ v(x) &= \frac{1}{EI}\left[-\frac{1}{4}M_0 x^2 + \frac{1}{4}\frac{M_0}{L}x^3\right]. \end{aligned}\right.$$



Example 11.11

Determine the deflection curve for the beam shown below. The beam is made up of a material with an elastic modulus E and has a cross-sectional second area moment I, both of which are constant along the length of the beam.



4 unknows.

+ 4 unknows

B.C.s $\theta(0)=0$ $\quad$ $\theta(L)=0$

$v(0)=0$ $\quad$ $v(L)=0$

$\theta_1(2a)=\theta_2(2a)$

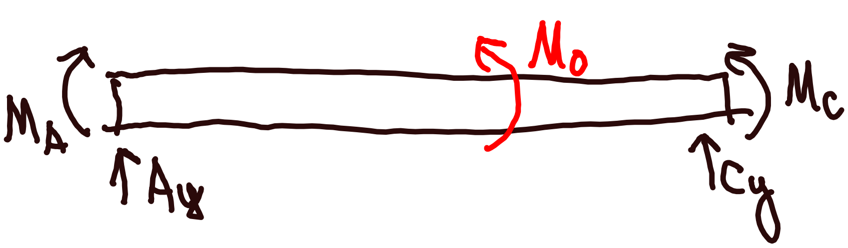
$v_1(2a)=v_2(2a)$.

ΣF

ΣM

+6 B.C.s.

I.) Equilibrium



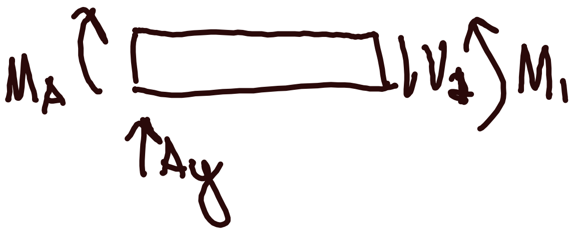
$(\Sigma M)_A = -M_A + M_0 + M_c + C_y(3a) = 0$

$\Sigma F = A_y + C_y = 0$

4 unkowns

2 eqn

2.) Force-displacement



$$(\Sigma M)_1 = M_1 - A_y x - M_A = 0$$

$$M_1(x) = M_A + A_y x.$$

$$\theta_1(x) = \cancel{\theta(0)}^{\,0} + \frac{1}{EI}\int_0^x M_1(x)\,dx.$$

$$v_1(x) = \cancel{v(0)}^{\,0} + \frac{1}{EI}\int_0^x\int_0^x M_1(x)\,dx.$$

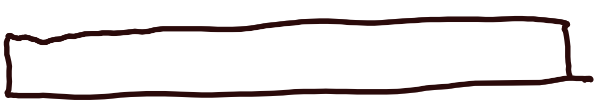
$$\theta_1(x) = \frac{1}{EI}\left[M_A x + A_y \frac{x^2}{2}\right]$$

$$v_1(x) = \frac{1}{EI}\left[M_A \frac{x^2}{2} + A_y \frac{x^3}{6}\right].$$

$$\theta_1(2a) = \theta_B$$

$$v_1(2a) = v_B.$$

Section BC



$$\theta(x) = \theta_B + \int_{2a}^{x} M_2(x)\,dx.$$