MECHANICS OF MATERIALS

Fall 2023

ME 323- 005

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Lecture 6: Axial Deformation

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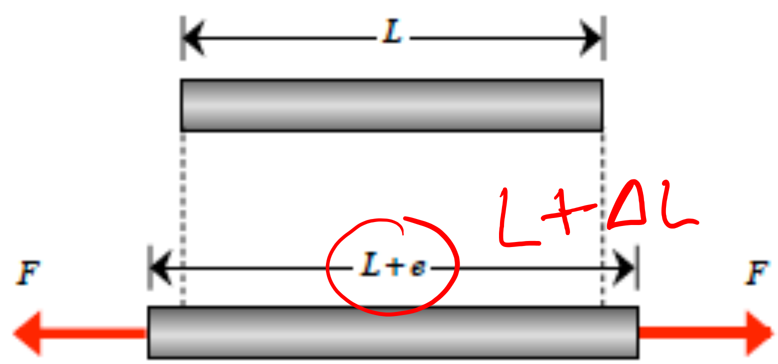
Axial Deformation- Determinate Systems

- *ELONGATION EQUATION:* $e=\int_0^L \frac{F(x)}{A(x)E(x)}dx$



- *SPECIAL CASE:* For homogeneous loading, geometry and properties: $e=\frac{FL}{EA}$

(ε) Strain $= \frac{\Delta L}{L} = \frac{e}{L}$

$\sigma = E\,\varepsilon$

$F/A = E\,\frac{e}{L} \Rightarrow \boxed{e = \frac{FL}{EA}}$

Example 6.1 from Lecturebook

Rigid beam AB (weighing $W_b = 200\ lb$) supports a crate weighing $W_c = 1000\ lb$. In turn, the beam is supported by rods (1) and (2) with lengths of $h = 6\ ft$, Young's moduli $E_1 = E_2 = E = 30 \times 10^3\ ksi$, and diameters $d_1 = d_2 = 0.4\ in$, respectively. What are the downward displacements u_A and u_B of ends A and B, respectively, of the beam? Assume small rotations in the beam.

$W_b = 200 lb$

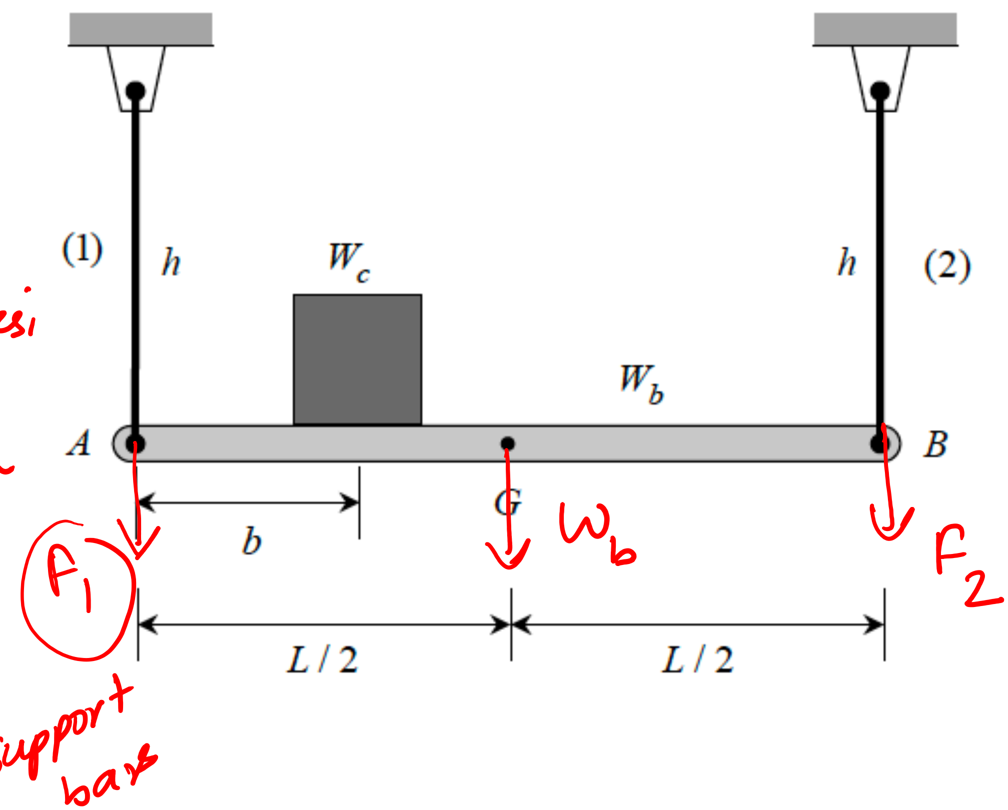
$W_c = 1000 lb$

$h = 6 ft$

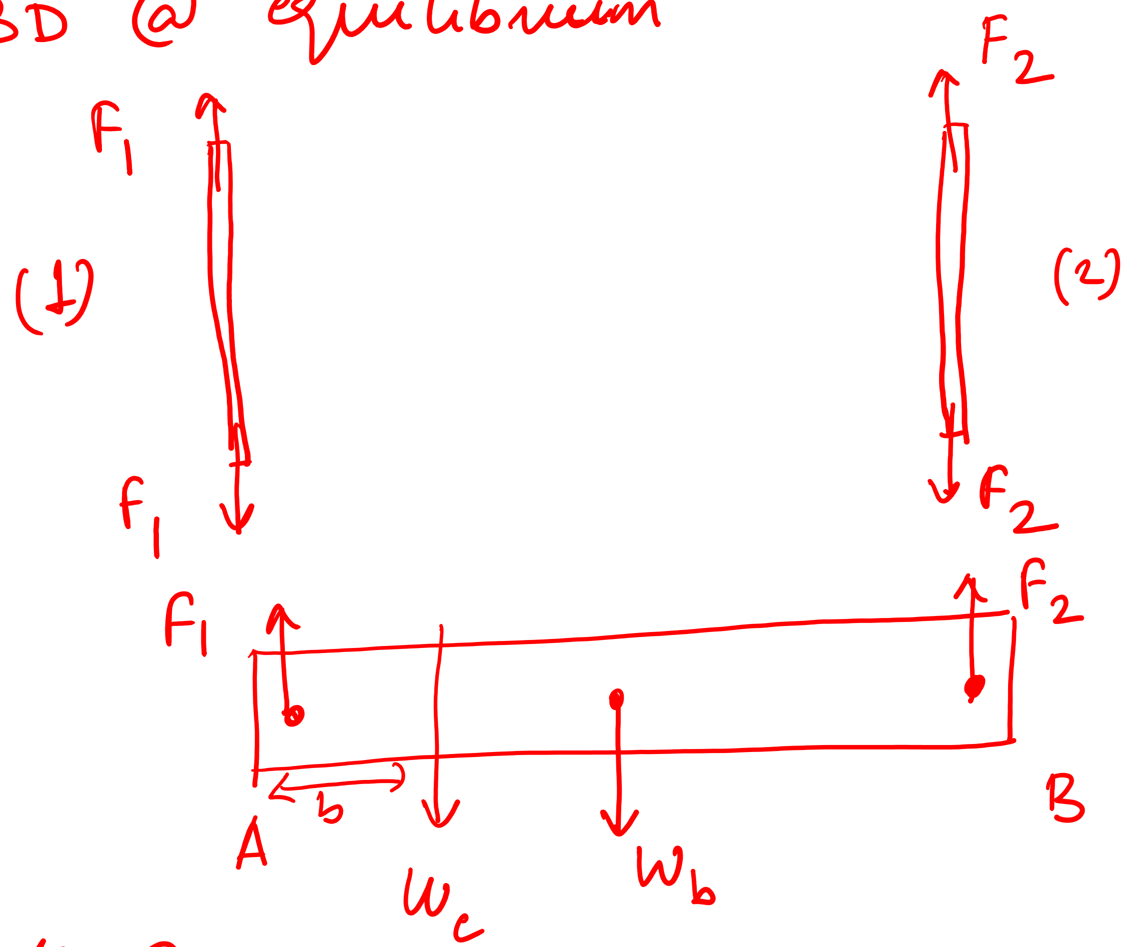
$E_1 = E_2 = E = 30\times10^3 ksi$

$d_1 = d_2 = d = 0.4$ inch

u_A, u_B



FBD @ equilibrium



moments @ point A –

$$(\Sigma M)_A = -W_c(b) - W_b\left(\frac{L}{2}\right) + F_2(L) = 0$$

$$F_2 = \frac{W_c b + W_b(L/2)}{L} = W_c\left(\frac{b}{L}\right) + \frac{W_b}{2}$$

$$\Sigma F_y = f_1 + f_2 - W_c - W_b = 0$$

$$f_1 = -f_2 + W_c + W_b$$

$$= -\left[W_c\left(\frac{b}{L}\right) + \frac{W_b}{2}\right] + W_c + W_b$$

$$= W_c\left[1 - b/L\right] + W_b/2$$

extension @ bar(1) $= e_1$, in bar(2) $= e_2$

$$e_1 = \frac{f_1 h}{E_1 A_1} = \frac{\left\{W_c\left[1 - b/L\right) + W_b/2\right\} h}{(E)\left(\pi\left(d/2\right)^2\right)}$$

$$\boxed{e_1 = \frac{\left[W_c\left(1 - b/L\right) + W_b/2\right](4h)}{\pi d^2 E}}$$

$$e_2 = \frac{F_2 h}{E_2 A_2}$$

$$= \left[\frac{w_c (b/L) + wb/2}{E \pi (d/2)^2} \right] h$$

$$\boxed{e_2 = \frac{\left[w_c (b/L) + wb/2 \right] [4h]}{\pi E d^2}}$$

Example 6.3 from Lecturebook

Cu-Zn alloy

A magnesium-alloy rod ($E_1 = 8\times10^3\,ksi$), having a diameter of $d_1 = 1\ in$, is encased in a brass tube ($E_2 = 16\times10^3\,ksi$), having an outer diameter of $d_2 = 2\ in$. The rod and tube both have a length of $L = 30\ in$. An axial load $P = 20\ kips$ is applied to the free end, as shown below.

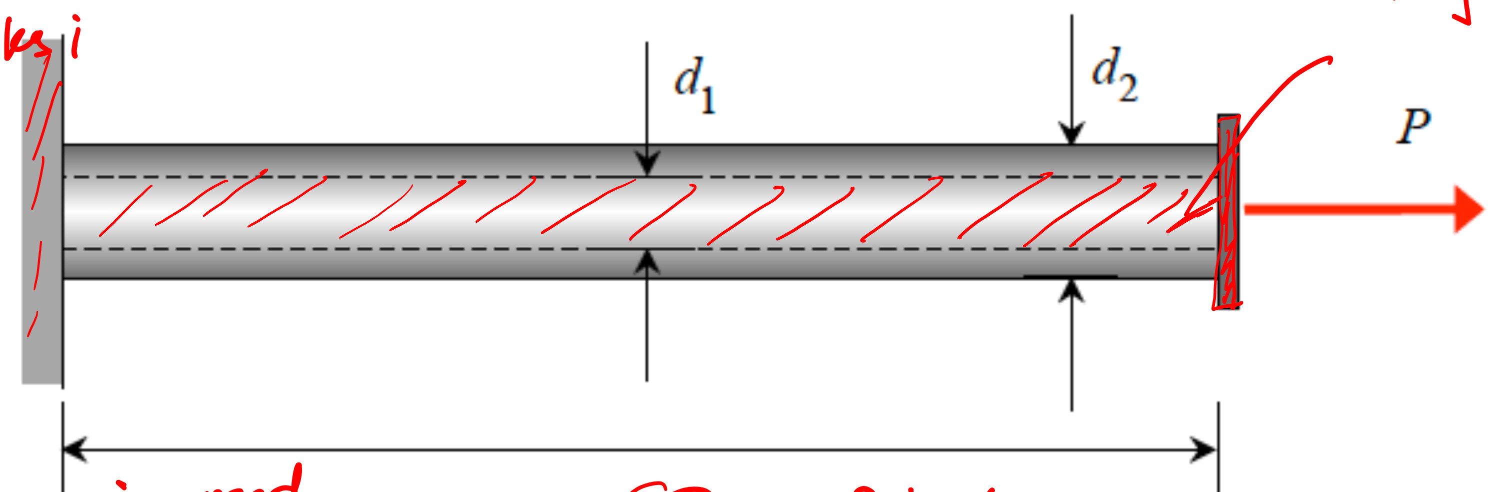
a) Determine the normal stresses σ_1 and σ_2 in the two materials.

b) Determine the elongation of the bimetallic rod. = e

$E_1 = 8\times10^3$ ksi

$E_2 = 16\times10^3$ ksi

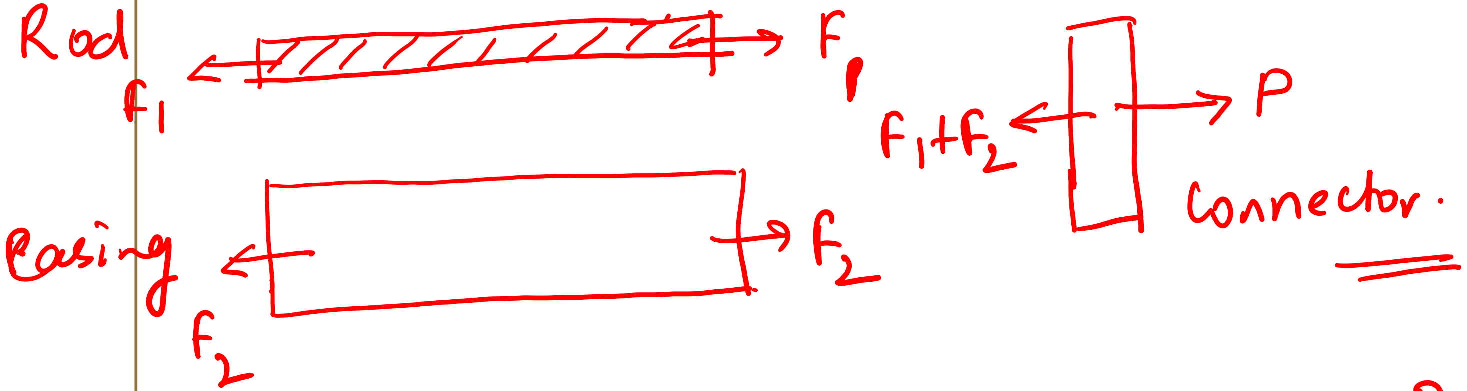
Mg alloy



L = 30 inch

σ_1 = stress in rod

σ_2 = stress in casing



@ equilibrium $\quad \Sigma F \Rightarrow \quad F_1 + F_2 - P = 0$

$$P = F_1 + F_2 \quad \text{———} \quad (1)$$

elongation for rod —

$$e_1 = e = \frac{F_1 L_1}{E_1 A_1} = \frac{F_1 L}{E_1 A_1}$$

L

$$F_1 = \frac{E_1 A_1 e}{L} \quad \text{———} \quad (2)$$

elongation of casing — $\quad e_2 = e = \dfrac{F_2 L_2}{E_2 A_2} = \dfrac{F_2 L}{E_2 A_2}$

$$F_2 = \frac{E_2 A_2 e}{L} \quad \text{———} \quad (3)$$

$$P = F_1 + F_2$$

$$P = \frac{E_1 A_1 e}{L} + \frac{E_2 A_2 e}{L}$$

$$\Rightarrow \quad e = \frac{PL}{(E_1 A_1 + E_2 A_2)}$$

$$e = \frac{(20\,\cancel{\text{kips}})(30\,\text{inch})}{\left(8\times10^3 \frac{\cancel{\text{kips}}}{\cancel{\text{in}^2}}\right)\left[\pi \frac{(1\,\cancel{\text{in}})^2}{4}\right] + \left(16\times10^3 \frac{\cancel{\text{kips}}}{\cancel{\text{in}^2}}\right)\left[\pi \frac{(2\,\cancel{\text{in}})^2}{4}\right]}$$

$$= \frac{100}{3\pi}\ \text{inch} \qquad \boxed{e = 10.61\ \text{inch}}$$

(a) $$\sigma_1 = F_1/A_1 = \frac{E_1 \cancel{A_1} e}{L(\cancel{A_1})} = \frac{E_1 e}{L}$$

$$\sigma_1 = \frac{(8\times10^3\ \text{ksi})(10.61\ \text{inch})}{(30\,\text{inch})} \qquad \boxed{\sigma_1 = 2.83\,\text{ksi}}$$

$$\sigma_2 = \frac{f_2}{A_2} = \frac{E_2 \cancel{A_2} e}{L(\cancel{A_2})} = \frac{E_2 e}{L}$$

$$\sigma_2 = \frac{(16 \times 10^3 \text{ ksi})(10.61 \text{ inch})}{(30 \text{ inch})}$$

$$\boxed{\sigma_2 = 5.66 \text{ ksi}}$$

Example 6.4 from Lecturebook

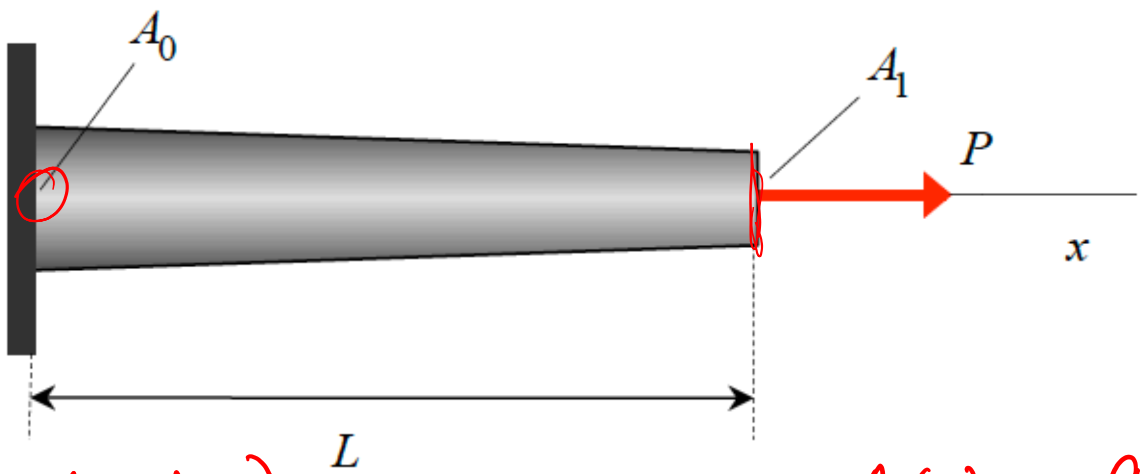
elastic modulus.

A tapered rod having a length of L and made up of a material with a Young's modulus of E is loaded with an axial load P, as shown below. The cross sectional area of the rod varies linearly from A_0 at $x = 0$ to A_1 at $x = L$. Determine the total axial elongation of the road as a result of the axial load P.

L

E

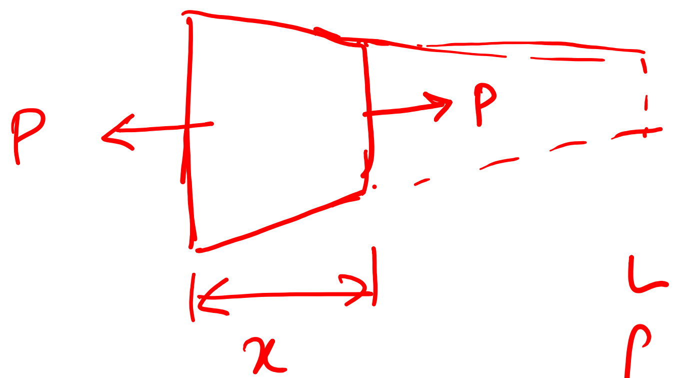
$y = mx + C$



$$A(x) = \underbrace{\left(\frac{A_1 - A_0}{L}\right)}_{b} x + \underbrace{A_0}_{a} \Rightarrow A(x) = a + bx.$$

$a = A_0$

$b = \left(\frac{A_1 - A_0}{L}\right)$



elongation $e = \int_0^L \frac{F(x)}{E(x)\,A(x)} \cdot dx$

$F(x) \to P$, $E(x) \to E$

$$e = \frac{P}{E} \int_0^L \frac{1}{A(x)} \cdot dx = \frac{P}{E} \int_0^L \frac{dx}{(a+bx)}$$

$$e = \frac{P}{Eb} \ln(a+bx) \Big|_0^L = \left(\frac{P}{Eb}\right)\left[\ln(a+bL) - \ln(a)\right]$$

$$e = \frac{P}{Eb} \ln\left(\frac{a+bL}{a}\right)$$

slip

$$e = \frac{P}{Eb} \ln\left(\frac{a+bL}{a}\right)$$

$$a = A_0$$

$$b = \left(\frac{A_1 - A_0}{L}\right)$$

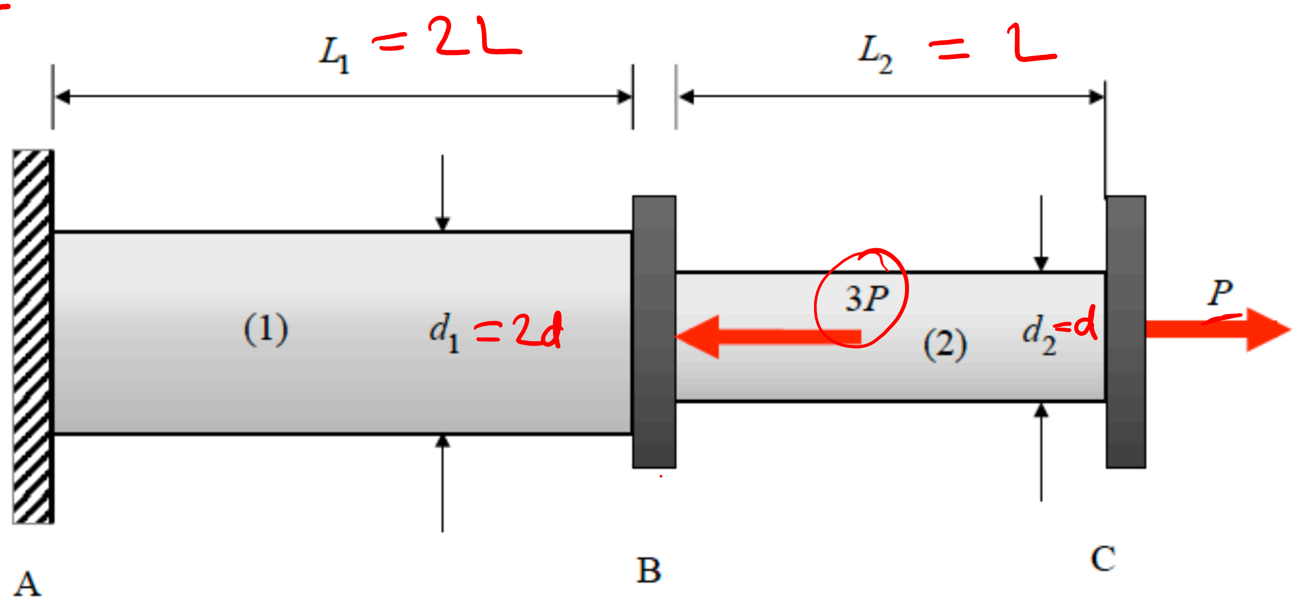
$$= \frac{P}{E\left(\frac{A_1-A_0}{L}\right)} \ln\left[\frac{\cancel{A_0} + \frac{(A_1 - \cancel{A_0})}{\cancel{L}}\cancel{L}}{A_0}\right]$$

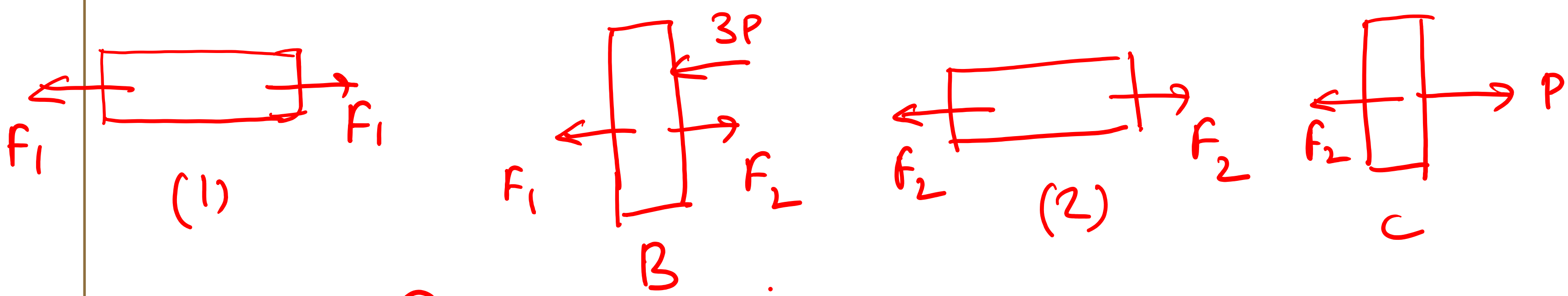
$$\boxed{e = \frac{PL}{E(A_1 - A_0)} \ln\left(\frac{A_1}{A_0}\right)}$$

Example 6.6 from Lecturebook

A rod is constructed from elements (1) and (2), with these elements being made up of materials having Young's moduli of E_1 and E_2, respectively. Elements (1) and (2) have lengths of $L_1 = 2L$ and $L_2 = L$, and diameters of $d_1 = 2d$ and $d_2 = d$, respectively. Elements (1) and (2) are joined by a rigid connector at B, with a rigid connector being attached to element (2) at C. The rod is loaded on connectors B and C, as shown in the figure below. Determine the displacement of C.

$e = e_1 + e_2$





connector B @ equilibrium

$$-F_1 - 3P + F_2 = 0 \quad \text{——} \quad (1)$$

connector C @ equi-

$$-F_2 + P = 0 \Rightarrow \boxed{F_2 = P} \quad \text{——} \quad (2)$$

from (1). $F_1 = -3P + P = -2P$ $\quad \boxed{F_1 = -2P}$

$$e_1 = \frac{F_1 L_1}{E_1 A_1} \qquad A_1 = \pi\left(\frac{d_1}{2}\right)^2$$

$$e_2 = \frac{F_2 L_2}{E_2 A_2} \qquad A_2 = \pi\left(\frac{d_2}{2}\right)^2$$

elongation @ C $u_c = e_1 + e_2$

$L_1 = 2L$

$L_2 = L$

$d_1 = 2d$

$d_2 = d$

$$u_c = \frac{F_1 L_1}{E_1 (\pi d_1^2/4)} + \frac{F_2 L_2}{E_2 (\pi d_2^2/4)}$$

$$= \frac{(-2P)(2L)(\cancel{4})}{E_1 \pi (\cancel{4} d^2)} + \frac{(P)(L)(4)}{E_2 (\pi d^2)}$$

$$= -\frac{4PL}{\pi E_1 d^2} + \frac{4PL}{\pi E_2 d^2}$$

$$\boxed{u_c = \frac{4PL}{\pi d^2}\left[\frac{1}{E_2} - \frac{1}{E_1}\right]}$$

THANK YOU