ME 323: Mechanics of Materials — ***Homework Set 4***

Fall 2023 — ***Due: Friday, September 22***

Problem 4.1 (10 points)

Consider a three-member truss as shown. Members (1), (2) and (3) are pinned to the wall at H, D, and B, respectively, and pinned to each other at C. The orientations, lengths, Young's modulus and cross section areas are shown in the figure. A point force P is applied to joint C. It is desired to determine the axial load carried by the three members. Note that all members in the truss are two-force members.

a) Assuming all members are under tension, write down the equilibrium equations for joint C. Can the axial forces in the members be found from these equilibrium equations alone? Explain your reasoning.
b) Write down the force-elongation equations for the three members.
c) Write down the compatibility relations between elongation of members (1), (2), (3) and the horizontal and vertical displacements (u_C,v_C) of connector C.
d) Calculate the axial loads in the members, and horizontal and vertical displacements (u_C,v_C) of joint C. Indicate whether members are in tension or compression.

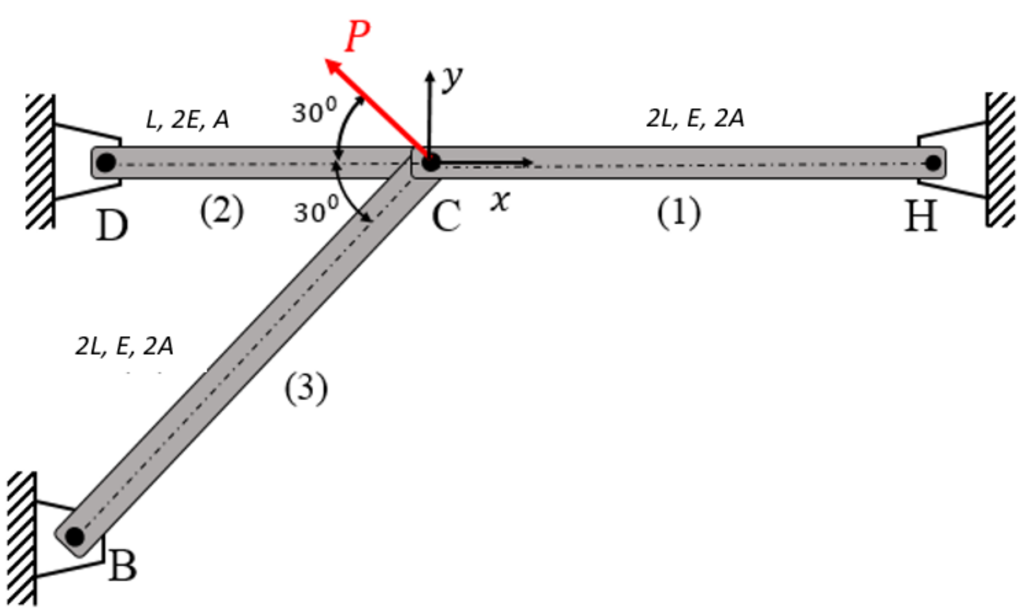
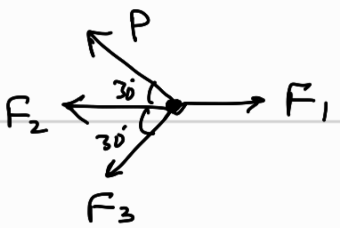


Fig. 4.1

Solution 1 :

(a) Free body diagram of joint C :



$\Sigma F_x = 0$: $$F_1 - F_2 - F_3 \frac{\sqrt{3}}{2} - \frac{P\sqrt{3}}{2} = 0 \quad — (1)$$

$\Sigma F_y = 0$: $$\frac{P}{2} - \frac{F_3}{2} = 0 \quad — (2)$$

2 equations , 3 unknown forces .

Cannot find forces from equilibrium alone .

(b) Member 1 : $$e_1 = \frac{F_1 \cdot 2L}{E \cdot 2A} = \frac{F_1 L}{EA} \quad — (3)$$

Member 2 : $$e_2 = \frac{F_2 \cdot L}{2E \cdot A} = \frac{F_2 L}{2EA} \quad — (4)$$

Member 3 : $$e_3 = \frac{F_3 \cdot 2L}{E \cdot 2A} = \frac{F_3 L}{EA} \quad — (5)$$

(c) $$e_1 = -u_c \quad — (6)$$

$$e_2 = u_c \quad — (7)$$

$$e_3 = u_c \cos 30^\circ + v_c \sin 30^\circ$$

$$e_3 = \frac{u_c \sqrt{3}}{2} + \frac{v_c}{2} \quad — (8)$$

(d) Solving the system of equations :

From ②, $F_3 = P$

From ⑥, ⑦, $e_1 = -e_2$

From ③, ④ :

$$\frac{F_1 \cancel{L}}{\cancel{EA}} = -\frac{F_2 \cancel{L}}{2\cancel{EA}}$$

$$F_1 = \frac{-F_2}{2}$$

From ①,

$$3F_1 - \frac{P\sqrt{3}}{2} - \frac{P\sqrt{3}}{2} = 0$$

$$F_1 = \frac{P}{\sqrt{3}}$$

$$F_2 = \frac{-2P}{\sqrt{3}}$$

$$u_c = e_2 = \frac{F_2 L}{2EA} = -\frac{2P}{\sqrt{3}} \cdot \frac{L}{2EA} = \frac{-PL}{\sqrt{3}EA}$$

$$v_c = 2e_3 - u_c\sqrt{3} = \frac{2F_3 L}{EA} + \frac{PL}{\cancel{\sqrt{3}}EA} \cdot \cancel{\sqrt{3}}$$

$$v_c = \frac{3PL}{EA}$$

Thus,

$$F_1 = \frac{P}{\sqrt{3}}, \quad F_2 = \frac{-2P}{\sqrt{3}}, \quad F_3 = P$$

$$u_c = \frac{-PL}{\sqrt{3}EA}, \quad v_c = \frac{3PL}{EA}$$

Member 1 : Tension

Member 2 : Compression

Member 3 : Tension

Problem 4.2 (10 points)

A rigid block with a weight of 80×10^3lbs is supported by posts A, B, and C. The block is attached to the posts at all times. The Young's moduli for posts A and B are $E_A = E_B = 29 \times 10^3$ksi, and for post C is $E_C = 14.6 \times 10^3$ksi. All posts have the same initial length before loading with a cross-sectional area of 8sq in. After heating post C, its temperature is raised by 20°F while the temperature of A and B is held constant. The coefficient of thermal expansion for post C is $\alpha_C = 9.8 \times 10^{-6}$per °F. Determine the normal stresses in each post before and after heating post C.

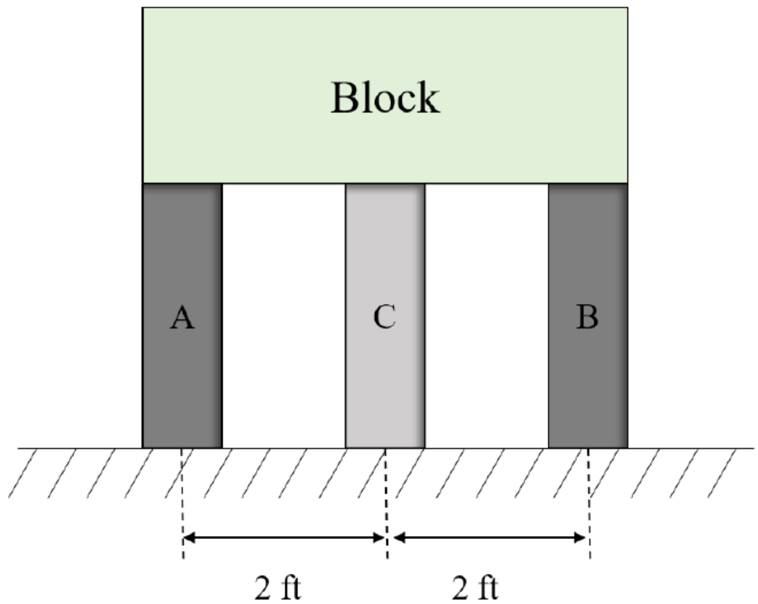
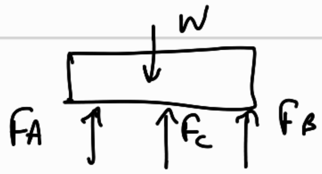


Fig. 4.2

Solution 2 :

FBD :



Before heating post C :

Assuming tension , equilibrium equations:

$$\Sigma M_c = 0 \quad : \quad F_A \cdot d - F_B \cdot d = 0$$

$$\Rightarrow F_A = F_B$$

$$\Sigma F_y = 0 \quad : \quad F_A + F_B + F_C + W = 0$$

$$2F_A + F_C + W = 0 \qquad -①$$

Force - elongation relations :

$$e_A = \frac{F_A L}{E_A A}, \quad e_B = \frac{F_B L}{E_B A}, \quad e_C = \frac{F_C L}{E_C A}$$

Compatibility condition :

$$e_A = e_C \quad \Rightarrow \quad \frac{F_A \cancel{L}}{E_A \cancel{A}} = \frac{F_C \cancel{L}}{E_C \cancel{A}}$$

$$\frac{F_A}{E_A} = \frac{F_C}{E_C} \qquad -②$$

Solving ① & ② ,

$$F_A = -32 \times 10^3 \text{ lbs} = -32 \text{ kip}$$

$$F_C = -16 \times 10^3 \text{ lbs} = -16 \text{ kip}$$

Normal stresses :

$$\sigma_A = \sigma_B = \frac{F_A}{A} = \frac{32}{8} = -4 \text{ ksi}$$

$$\sigma_C = \frac{F_C}{A} = \frac{16}{8} = -2 \text{ ksi}$$

(Negative indicates compression)

After heating post C :

Elongations:

$$e_A = \frac{F_A L}{E_A A}, \quad e_B = \frac{F_B L}{E_B A}, \quad e_C = \frac{F_C L}{E_C A} + \alpha \Delta T$$

Compatibility :

$$e_A = e_C .$$

$$\Rightarrow \frac{F_A L}{E_A A} = \frac{F_C L}{E_C A} + \alpha \Delta T L$$

$$\frac{F_A}{E_A} = \frac{F_C}{E_C} + A \alpha \Delta T \quad — (3)$$

Solving (1) & (3),

$$F_A = \frac{E_A(-W + A\alpha\Delta T \cdot E_C)}{2E_A + E_C}, \quad F_C = \frac{-E_C(W + 2A\alpha\Delta T E_A)}{2E_A + E_C}$$

Substituting the given values (taking care of units),

$$F_A = -22.811 \text{ kip}, \quad F_B = -22.811 \text{ kip}$$

$$F_C = -34.377 \text{ kip}$$

The corresponding stress are

$$\sigma_A = \sigma_B = \frac{F_A}{A} = \frac{-22.811}{8} = -2.85 \text{ ksi}$$

$$\sigma_C = \frac{F_C}{A} = \frac{-34.377}{8} = -4.30 \text{ ksi}$$

Problem 4.3 (10 points)

The gear-shaft system shown in Fig. 4.3 is supported by frictionless bearings and transmits the torque to the fixed end A using the rigid gears at B and C. Shafts 1 and 3 are made of a material with shear modulus $\boldsymbol{G_1 = G_3 = 100\ \mathrm{GPa}}$ and shaft 2 is made of a material with shear modulus $\boldsymbol{G_2 = 50\ \mathrm{GPa}}$. The diameters of the shafts are $\boldsymbol{d_1 = 32\ \mathrm{mm}}$, $\boldsymbol{d_2 = 64\ \mathrm{mm}}$, and $\boldsymbol{d_3 = 16\ \mathrm{mm}}$. Calculate the angle of twist generated at the free ends D and E.

Use $\boldsymbol{L = 1\ \mathrm{m}}$, $\boldsymbol{d_c = 150\ \mathrm{mm}}$, $\boldsymbol{d_b = 90\ \mathrm{mm}}$, and $\boldsymbol{T_0 = 5\ \mathrm{N \cdot m}}$

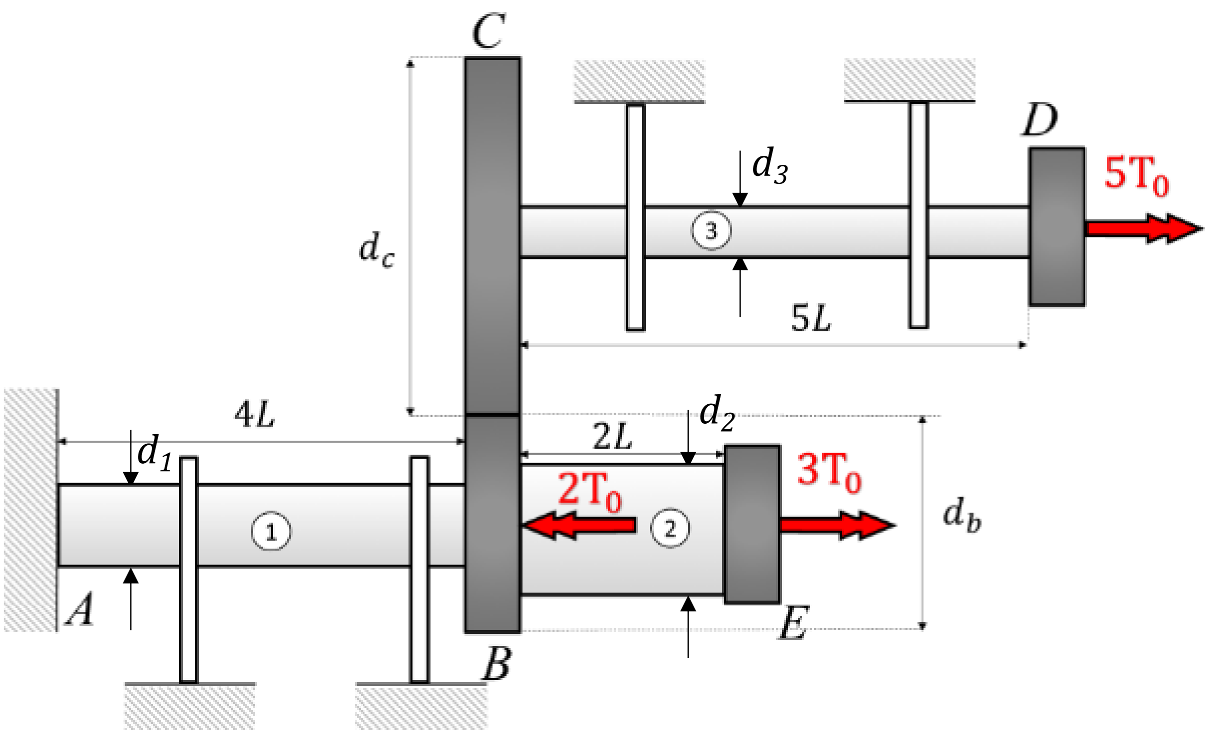


Fig. 4.3

Solution

FBD of the system

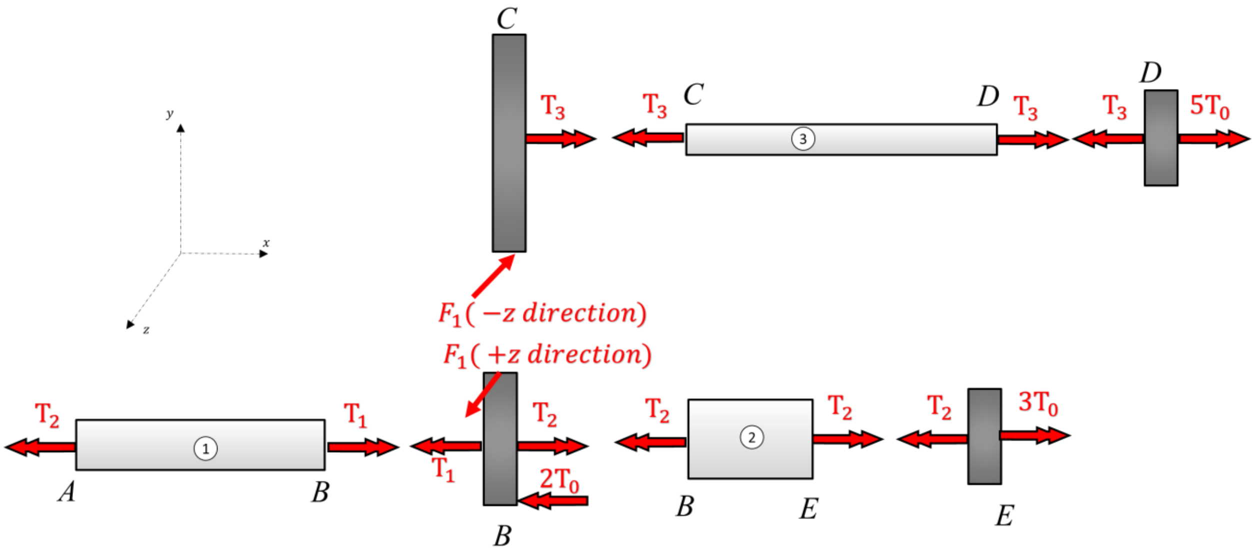


Figure S3: FBD

Equilibrium

Torque Balance at connector D

$$\Sigma T_D = 0 \;\; => \;\; -T_3 + 5T_0 = 0$$

$$T_3 = 5T_0 \qquad [1.1]$$

Torque Balance at connector C

$$\Sigma T_C = 0 \;\; => \;\; T_3 + r_c F_1 = 0$$

$$F_1 = -\frac{T_3}{r_c} \qquad [1.2]$$

Torque Balance at connector E

$$\Sigma T_E = 0 \;\; => \;\; 3T_0 - T_2 = 0$$

$$T_2 = 3T_0 \qquad [1.3]$$

Torque Balance at connector B

$$\Sigma T_C = 0 \;\; => \;\; T_2 - 2T_0 - T_1 + r_b F_1 = 0$$

From Eqs. [4.2] and [4.3]

$$3T_0 - 2T_0 - T_1 + r_b\left(-\frac{T_3}{r_c}\right) = 0$$

$$T_0 - T_1 - \frac{r_b}{r_c}T_3 = 0$$

From Eq. [4.1] and $_c = 75$ mm, $_b = 45$ mm,

$$T_1 = T_0 - \frac{45}{75}(5T_0) = -2T_0 \qquad [1.4]$$

Angle of twist at E

$$\phi_E = \phi_E - \phi_A = \phi_E - \phi_B + \phi_B - \phi_A = (\phi_E - \phi_B) + (\phi_B - \phi_A)$$

$$= \Delta\phi_2 + \Delta\phi_1 = \frac{T_2 L_2}{G_2 I_{P_2}} + \frac{T_1 L_1}{G_1 I_{P_1}}$$

$$=> \;\; \phi_E = \frac{(3T_0)(2L)}{G_2\left[\frac{\pi}{32}d_2^4\right]} + \frac{(-2\,T_0\,)(4L)}{G_1\left[\frac{\pi}{32}d_1^4\right]}$$

$$\phi_E = -0.00352 \text{ rad} \qquad [1.5]$$

Angle of twist at D

$$\phi_D = (\phi_D - \phi_C) + \phi_C = \Delta\phi_3 + \phi_C$$

From the gear combination at BC we have

$$\phi_C = -\frac{d_b}{d_c}\phi_B$$

Where $\phi_B = (\phi_B - \phi_A) = \Delta\phi_1$, which gives,

$$\phi_D = \Delta\phi_3 - \frac{d_b}{d_c}\Delta\phi_1$$

$$= \frac{T_3 L_3}{G_3 I_{P_3}} - \frac{d_b}{d_c}\frac{T_1 L_1}{G_1 I_{P_1}}$$

$$=> \ \phi_D = \frac{(5T_0)(5L)}{G_3\left[\frac{\pi}{32}d_3^4\right]} - \frac{d_b}{d_c}\frac{(-2\,T_0\,)(4L)}{G_1\left[\frac{\pi}{32}d_1^4\right]}$$

Substituting the values for each parameter we have,

$$\phi_D = \ 0.19661 \text{ rad}$$

[1.6]

Problem 4.4 - Conceptual (5 points)

Consider a circular shaft of radius R and length L composed of a material with shear modulus G fixed at one end. The shaft is subjected to a torque T at the other end. Determine the effect the following changes would have on the shear stress and angle of rotation.

(1) If the shaft radius R is increased, the maximum shear stress in the rod (a) increases (b) decreases (c) does not change.

(2) If the shaft length L is increased, the maximum shear stress in the rod (a) increases (b) decreases (c) does not change.

(3) If the shaft shear modulus G is increased, the maximum shear stress in the rod (a) increases (b) decreases (c) does not change.

(4) If the shaft radius R is increased, the maximum angle of rotation in the rod (a) increases (b) decreases (c) does not change.

(5) If the shaft radius L is increased, the maximum angle of rotation in the rod (a) increases (b) decreases (c) does not change.

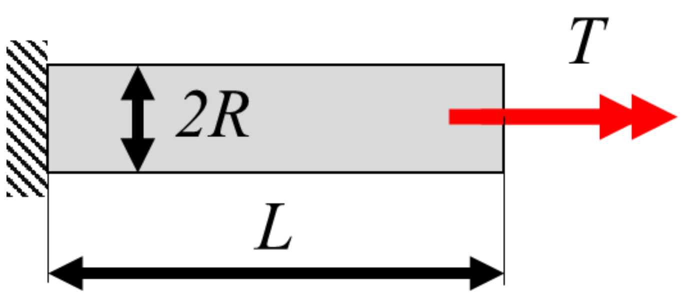


Fig. 4.4

Solution:

Maximum shear stress in the rod is $\tau = 2T/\pi R^3$

Maximum angle of rotation in the rod is $\theta = \frac{2TL}{G\pi R^4}$

(1) (b) decreases
(2) (c) does not change
(3) (c) does not change
(4) (b) decreases
(5) (a) increases