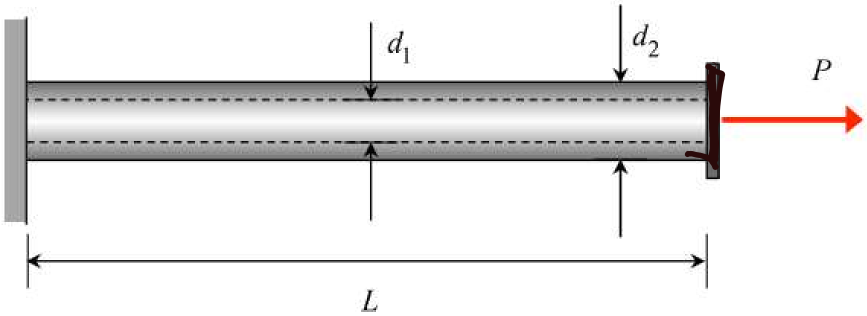
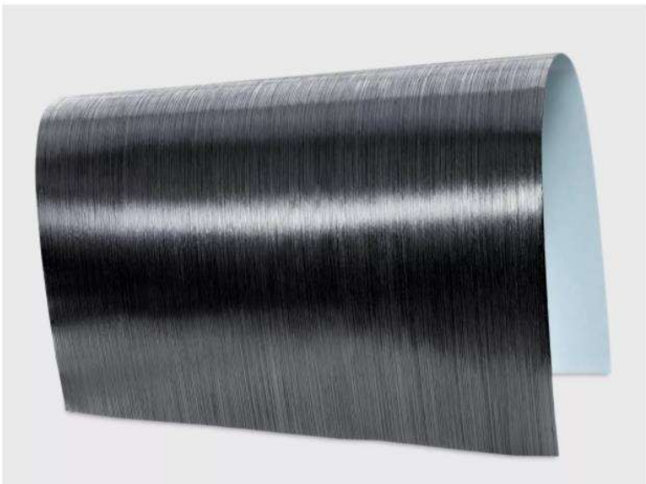
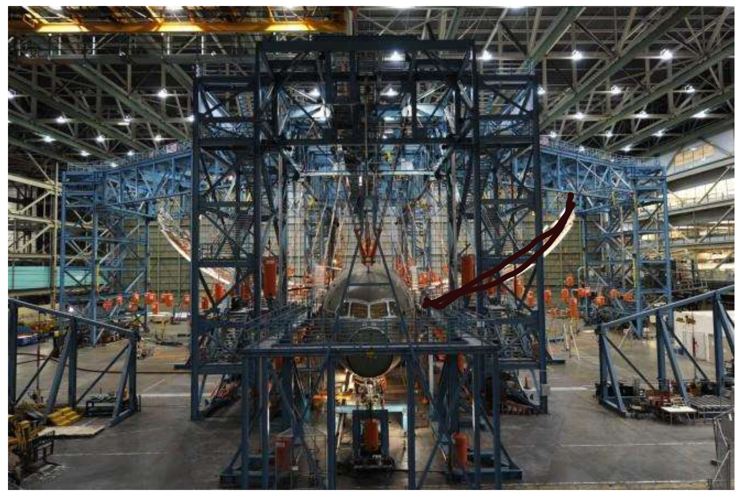


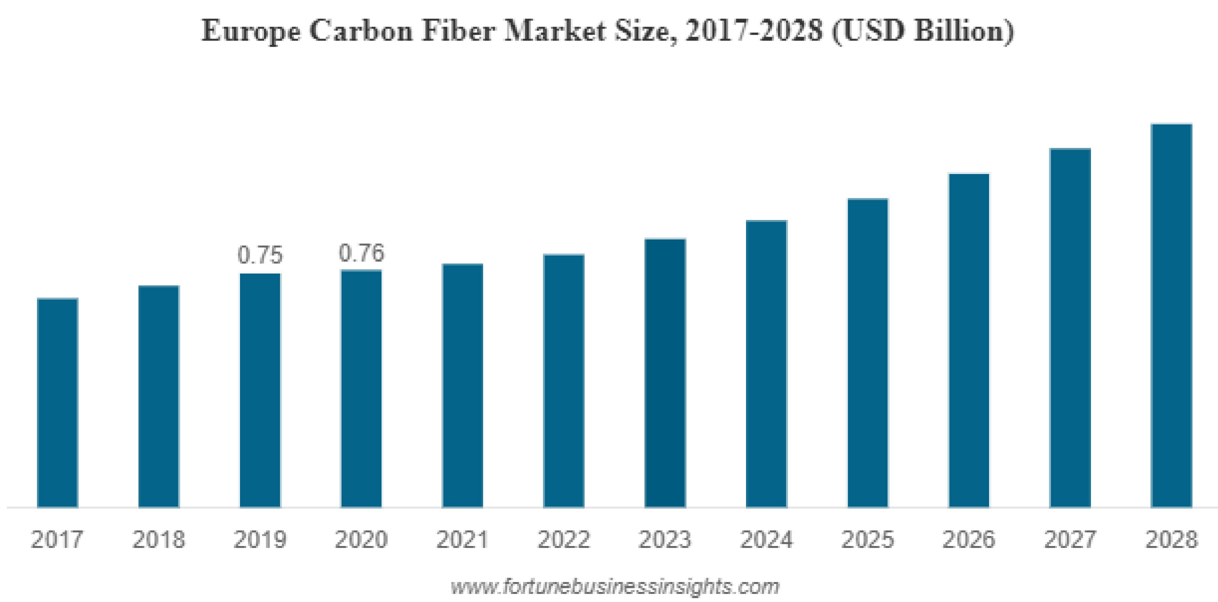



https://www.sglcarbon.com/en/markets-solutions/material/sigrapreg-pre-impregnated-materials/



https://www.wired.com/2010/03/boeing-787-passes-incredible-wing-flex-test/

Carbon fiber market will grow from $2.33B in 2021 to $4.01B in 2028



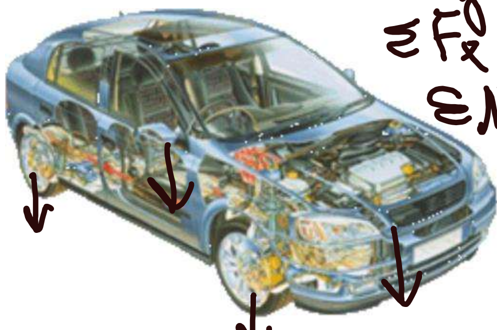
| | | | | |
|---|---|---|---|---|
| 4 M | 28-Aug | Stress – introduction to design of deformable bodies | Chap. 4 | |
| 5 W | 30-Aug | Stress and strain – general definitions | Chap. 5 | |
| 6 F | 1-Sep | Axial members – determinate structures | Chap. 6 | HW 1 |
| **M** | **4-Sep** | ***Labor Day – no class*** | | |
| 7 W | 6-Sep | Axial members – indeterminate structures | Chap. 6 | |
| 8 F | 8-Sep | Axial members – planar trusses | Chap. 6 | HW. 2 |
| 9 M | 11-Sep | Axial members – thermal effects | Chap. 7 | Quiz. 1. |
| 10 W | 13-Sep | Torsion members – stresses in circular bars | Chap. 8 | |
| 11 F | 15-Sep | Torsion members – statically determinate structures | Chap. 8 | HW 3 |

OH → PAXS 200 5:30pm. → ???

*c) <u>Stress analysis of statically indeterminate structures with externally applied loads</u>*

In some structures, the internal force resultants *cannot* be calculated from the static equilibrium of sections. One has to consider material properties and use compatibility relations for this

$\Sigma F_y$
$\Sigma F_x$
$\Sigma M$.



How much load does each tire carry on the automobile above? How much weight does each leg of the millipede support?

Recall that for a statically *determinate* structure:

*# of unknown forces = # of equations of static equilibrium*

For a statically *indeterminate* problem:

*# of unknown forces > # of equations of static equilibrium*

To identify determinate or indeterminate problems: *redundant supports* (if removed, structure remains equilibrium).

#unknowns − #equations = deg of indet.

General approach:

1) Force balance → tells us whether it's determinate ←
2) Force-elongation equations → $e = \frac{FL}{EA}$
3) Compatibility equations → relationship btw deformation
4) Solve

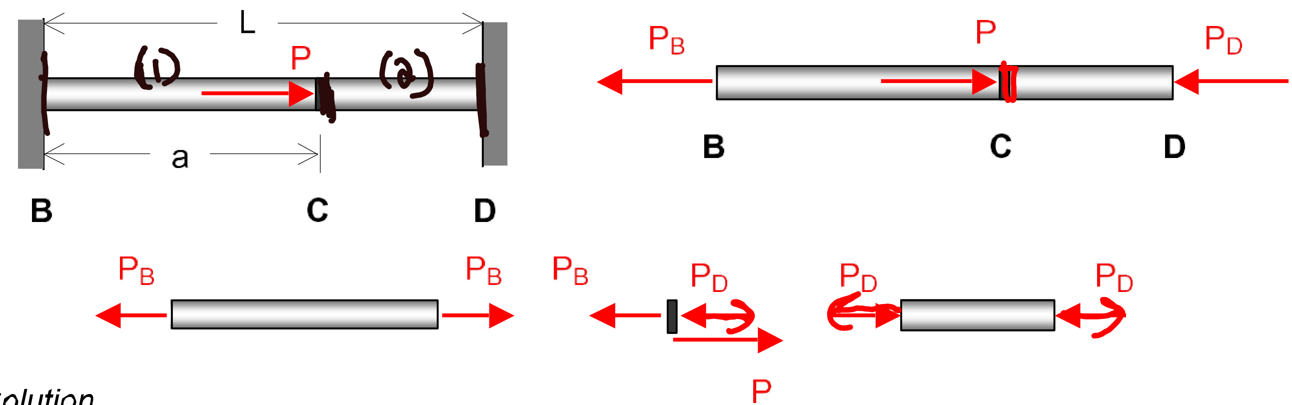
*Motivating Example*

An axial load P acts at point C on a rod, with $a > L/2$. Determine the reactions at the fixed ends B and D. The rod has a cross-sectional area of A with a Young's modulus of E. Answer the following questions:

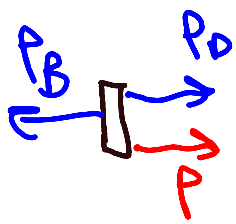
Q1: Why is this problem indeterminate if considering the rod as a rigid member?

Q2: How does a consideration of strain (deformation) allow you to solve for the reactions?

Q3: Which end (B or D) carries the largest reaction load? Defend your answer with a physical argument.




*Solution*




1) Force-Balance

$$\Sigma F_x = P + \underline{P_D} - \underline{P_B} = 0$$

2 unknowns
1 equation
1

3) $\underline{e_1 + e_2 = 0}$

2.) $e_1 = \dfrac{F_1 L_1}{E_1 A_1}$ $\quad e_2 = \dfrac{F_2 L_2}{E_2 A_2}$.

Lecture 6 Review Question:

Which statement best describes how to find the total elongation of a member?

- Elongation is calculated from the distribution of stresses over a cross section.
- Elongation is the (force x length)/(modulus x area)
- Elongation is the integral of the local strain over the length of the member.
- Elongation is the integral of the total distributed force (in N/m) divided by the average area.

in this class. $e = \frac{FL}{EA}$

$\int \epsilon(x) \, dL$

general

### *Example 6.7*

A three-segment rod that is initially stress-free is attached to rigid supports at ends A and D and is subjected to equal and opposite external loads P at nodes B and C as shown in the figure below. The rod is homogeneous and linearly elastic, having a Young's modulus of $E$. For this analysis, use the following: $A_1 = A_3 = A$, $A_2 = 2A$, $L_1 = 2L$ and $L_2 = L_3 = L$.

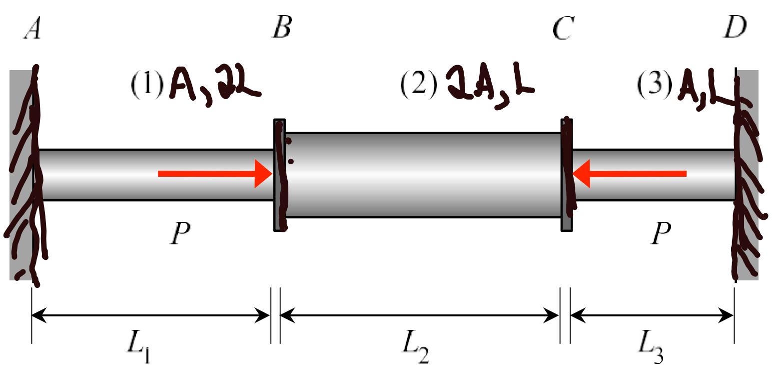
a) Determine the axial stresses in the three elements: $\sigma_1$, $\sigma_2$ and $\sigma_3$

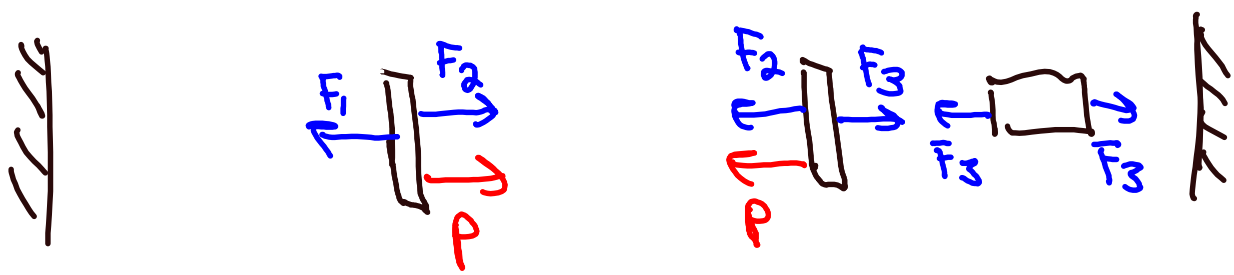
b) Determine the horizontal displacements $u_B$ and $u_C$ at nodes B and C, respectively.

$e_1 = u_B - \cancel{u_A}^{0}$ $\quad e_2 = u_C - u_B$ $\quad e_3 = \cancel{u_D}^{0} - u_C$

$\downarrow$

$e_1 + e_2 + e_3 = 0.$




1.) Equilibrium

$$\left.\begin{aligned}(\Sigma F_x)_B &= F_2 + P - F_1 = 0\\ (\Sigma F_x)_C &= F_3 - F_2 - P = 0\end{aligned}\right\}$$

3 unknowns
2 equations
1

$F_1 = F_2 + P$



$F_3 = F_2 + P$

2.) $e_1 = \dfrac{F_1 L_1}{E_1 A_1}$ $\quad e_2 = \dfrac{F_2 L_2}{E_3 A_2}$ $\quad e_3 = \dfrac{F_3 L_3}{E_3 A_3}$

3.) Compatility.

$$e_1 + e_2 + e_3 = 0$$

4.) Solve.

$$\frac{F_1 L_1}{E_1 A_1} + \frac{F_2 L_2}{E_2 A_2} + \frac{F_3 L_3}{E_3 A_3} = 0$$

$$\frac{2F_1 L}{EA} + \frac{F_2 L}{2EA} + \frac{F_3 L}{EA} = 0$$

$$2(P+F_2) + \frac{F_2}{2} + (P+F_2) = 0$$

$$\frac{7}{2}F_2 = -3P$$

$$F_2 = -\frac{6}{7}P$$

$$F_1 = \frac{1}{7}P$$

$$F_3 = \frac{1}{7}P.$$

a) $\sigma_1 = F_1/A \Rightarrow$ positive $\quad \sigma_3 =$ positive.

$\sigma_2 = F_2/2A \Rightarrow$ negative.

b) $e_1 = u_B - u_A = u_B = \dfrac{F_1 L_1}{E_1 A_1} = \dfrac{2PL}{7EA}$

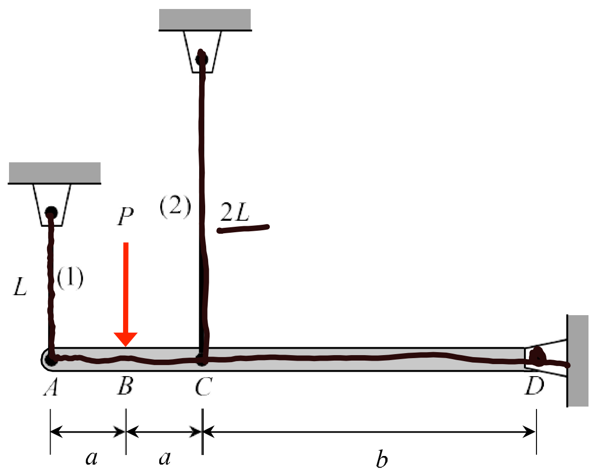
$u_C = u_B + e_2 = \dfrac{2PL}{7EA} - \dfrac{3PL}{7EA} = -\dfrac{PL}{7EA}.$

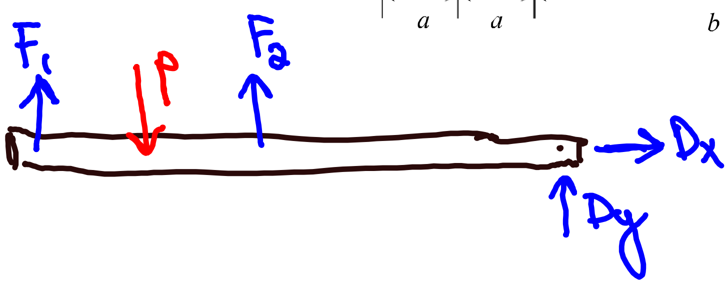
### *Example 6.8*

Load $P$ is applied to rigid beam AD. Members (1) and (2) are made up of a material having a Young's modulus of $E$, and each have a cross-sectional area of $A$.

a) Determine the axial stresses in support rods (1) and (2) after the load $P$ is applied.

b) Determine the resulting rotation angle $\theta$ in beam AD. Assume $\theta$ to be small.




1 1.) Equilibrium

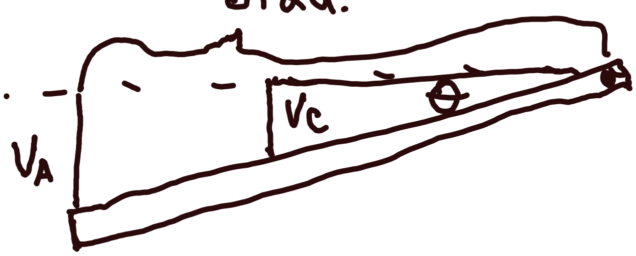
$$(\Sigma M)_D = -F_2(b) + P(b+a) - F_1(b+2a) = 0 \Big\}$$ 2 unknowns 1 equation

$$\cancel{\Sigma F_y = D_y + F_1 + F_2 - P = 0}$$

2.) Force-elongation.

$$e_1 = \frac{F_1 L_1}{E_1 A_1} \qquad e_2 = \frac{F_2 L_2}{E_2 A_2}.$$

3.) Compatibility.




$V_A = e_1$

$V_c = e_2$.

$$\tan\theta \sim \theta = \frac{e_2}{b} = \frac{e_1}{b+2a}$$

4.) Solve

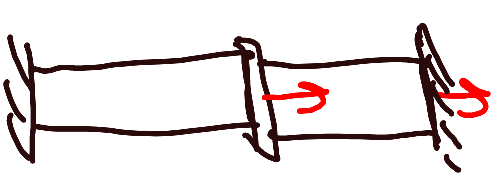
$$e_2 = \left(\frac{b}{b+2a}\right)e_1$$

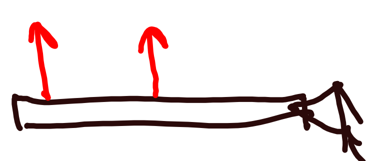
$$2F_2 = F_1\left(\frac{b}{b+2a}\right)$$

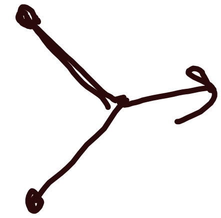
$$F_1(b+2a) + \frac{F_1}{2}\left(\frac{b^2}{b+2a}\right) = P(a+b)$$

$$F_1 = \frac{P(a+b)}{\left(b+2a+\frac{1}{2}\frac{b^2}{b+2a}\right)}$$

b) $\theta = \dfrac{e_2}{b} = \dfrac{F_2 L_2}{b E_2 A_2}$





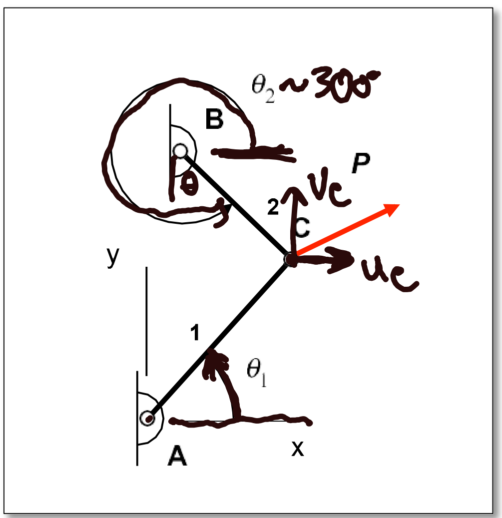


*d) Stress and deformation analysis of one-node planar trusses*

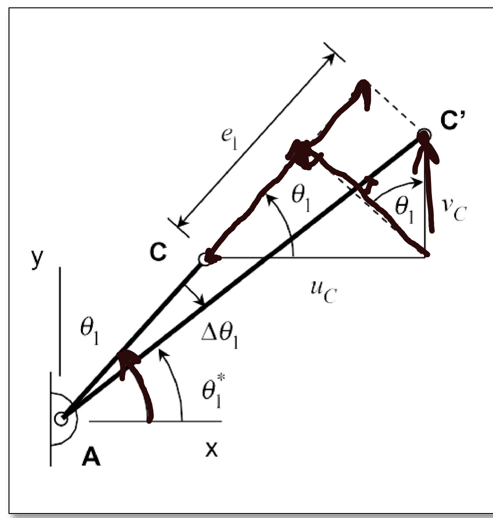
Up to this point in this course and in your earlier mechanics courses, we have performed force analyses of members in planar trusses using some combination of the method of sections and the method of joints. From these results, we know:

- The component of stress normal to the cross section of a member is found from $\sigma = P / A$ where P is the load carried by the member and A is the cross-sectional area of the member.
- The total elongation of the member is: $e = PL / AE$, where L is the length of the member, $E$ is the Young's modulus of the material and $A$ is the cross sectional area of the member.

***BEFORE*** **deformation**




***AFTER*** **deformation (exaggerated)**




$\cos\theta_1 = \frac{e_{1,x}}{u_c}$

$\sin\theta_1 = \frac{e_{1,y}}{V_c}.$

Now let's determine the deformation of the members in a truss due to the loading. Consider the simple truss shown above loaded with a force $P$ at joint C. Since this is a determinate truss, we can determine the loadings carried by the two members 1 and 2, $F_1$ and $F_2$, respectively, from standard equilibrium analysis. From these, we can calculate the elongations of member 1 and 2 as,

$$e_1 = \frac{F_1 L_1}{E A_1} \tag{7}$$

$$e_2 = \frac{F_2 L_2}{E A_2} \tag{8}$$

respectively. Based on these results, what are the horizontal and vertical components of displacement of the node from C to C' ($u_C$ and $v_C$, respectively) as a result of this deformation?

To answer this question, first note that the total movement of C takes into account both movement causing strain (the elongation $e_1$ along axis of member) and movement causing no axial strain (rigid rotation $\Delta\theta_1$ of the member). From the preceding figure, we see that for a small rotation angle $\Delta\theta_1$, the components $(u_C, v_C)$ of the displacement of C that contribute to the elongation of member 1, $e_1$, are those along the axis of member 1; that is, from the figure we have:

$$\underline{e_1} = \underline{u_C}\, \underline{cos\theta_1} + \underline{v_C}\, \underline{sin\theta_1} \qquad (9)$$

e1 ⇒ uc, vc.

Similarly, for member 2, we can write:

e2 ⇒

$$e_2 = u_C\, cos\theta_2 + v_C\, sin\theta_2 \qquad (10)$$

e3

Substituting equations (7) and (8) into equations (9) and (10) provides us with the two equations needed to solve for $u_C$ and $v_C$:

$$\underline{u_C}\, cos\theta_1 + v_C\, sin\theta_{\underline{1}} = \frac{F_1 L_1}{EA_1}$$

$$u_C\, cos\theta_2 + v_C\, sin\theta_2 = \frac{F_2 L_2}{EA_2}$$

Please note that the above elongation-displacement equations rely on the following angle definitions of the truss elements.

***Definition of truss member angles***

Let $\theta_j$ be the angle of the jth truss member:

- $\theta_j$ is measured counterclockwise with respect to the positive $x$-axis, and
- with the origin of the $x$-axis placed at the point on the element that is pinned to ground

(as demonstrated by angles $\theta_1$ and $\theta_2$ in the preceding figure).

Only the angles defined as above are valid for the above compatibility equations. Before starting your analysis, you should clearly identify these truss member angles.

As we will see in some of the following examples, we need to use the above deformation analysis in order to do stress analysis of indeterminate trusses.