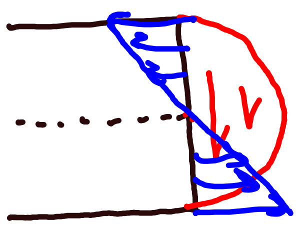
Midterm course evaluations are now open!

The whole class gets bonus HW points proportional to the number of people who complete the evals:

Bonus = 15*(proportion of class)

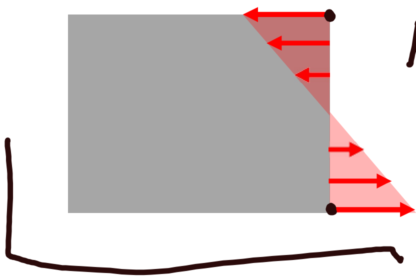
For a rectangular cross-section: at the neutral plane, the shear stress is _________ and the normal stress is _________.

- zero, zero
- zero, maximum
- maximum, zero
- maximum, maximum

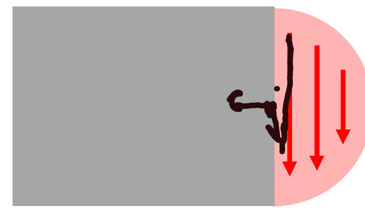


$$\sigma = -\frac{My}{I}$$

Failure of Beams

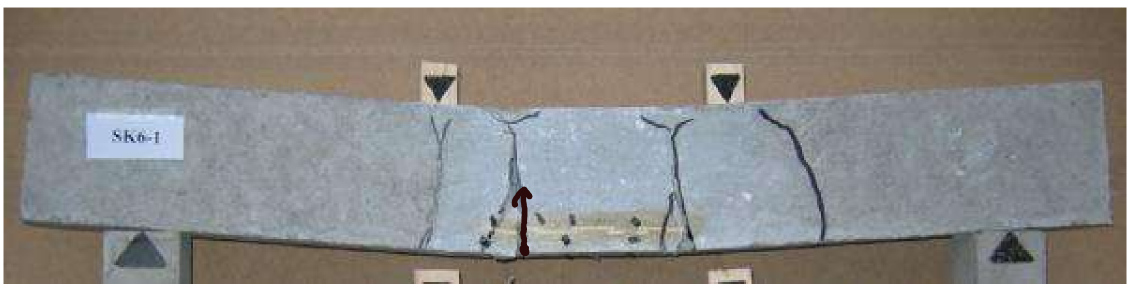


M



V

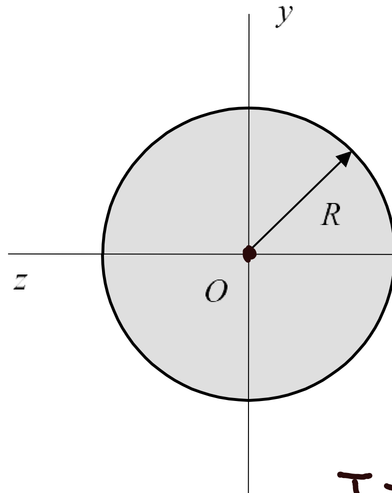
If a material fails due to normal stresses, how will it fail?

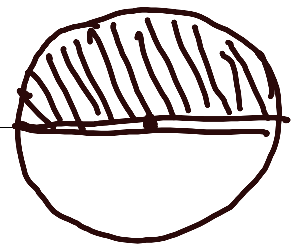


If a material fails due to shear stresses, how will it fail?

Example 10.11

Use the shear stress formula for a general shape cross section developed earlier in the chapter to determine an expression for the maximum shear stress along the symmetry axis *y* of the circular cross section beam shown below.





$y = r\sin\theta$

$I = \iint y^2 \, dA.$

$$\tau(y) = \frac{VA^*\bar{y}^*}{It}$$

$$A^* = \frac{1}{2}\pi R^2$$

$$\bar{y}^* = \int_0^{\pi}\int_0^{R} y\, r\, dr\, d\theta$$

$$\bar{y}^* = \frac{4R}{3\pi}$$

$$\boxed{\tau_{max} = \frac{3}{2}\frac{V}{A}}$$

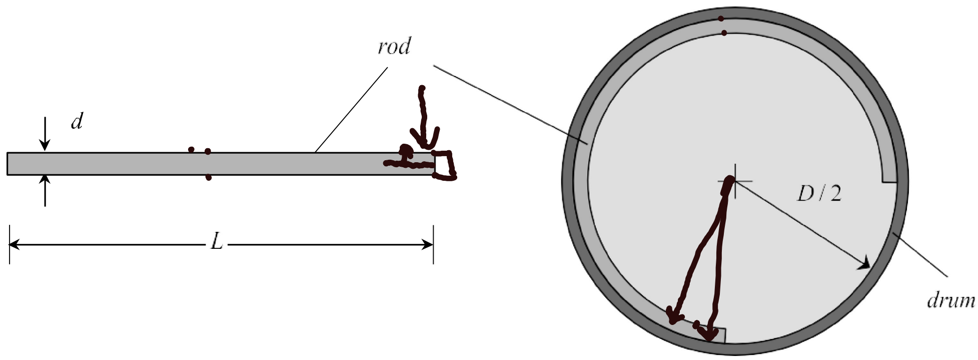
$$t = 2R$$

$$I = \frac{\pi}{4}R^4 \qquad I_p = \frac{\pi}{2}R^4$$

$$\tau_{max} = \frac{V\left(\frac{1}{2}\pi R^2\right)\left(\frac{4R}{3\pi}\right)}{\left(\frac{\pi}{4}R^4\right)(2R)} = \frac{4}{3}\frac{V}{\pi R^2} = \boxed{\frac{4}{3}\frac{V}{A}}$$

Example 10.4

A circular cross-sectioned, straight rod having a diameter of d, a length of L and of a material with a Young's modulus of E is stored by coiling the rod inside of drum with an inside diameter of D. Assuming that the yield strength of rod material is not exceeded, determine the maximum stress in the coiled rod, and the maximum bending moment in the rod.



$$\rho = \frac{D}{2} - \frac{d}{2}$$

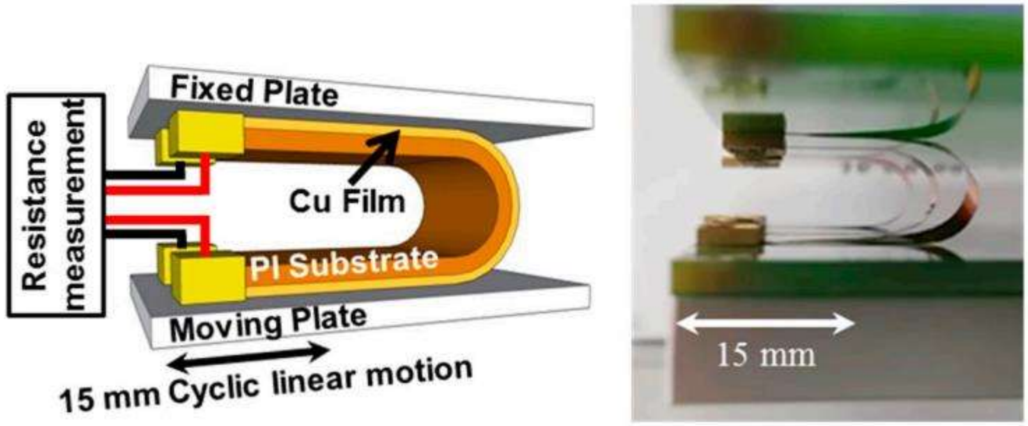
$$\varepsilon = -\frac{y}{\rho} \qquad \sigma = -\frac{yE}{\rho}$$

$$\varepsilon = \frac{-d}{D-d} \qquad \sigma = -\frac{(d/2)E}{\frac{D}{2} - \frac{d}{2}}$$

$d << D$

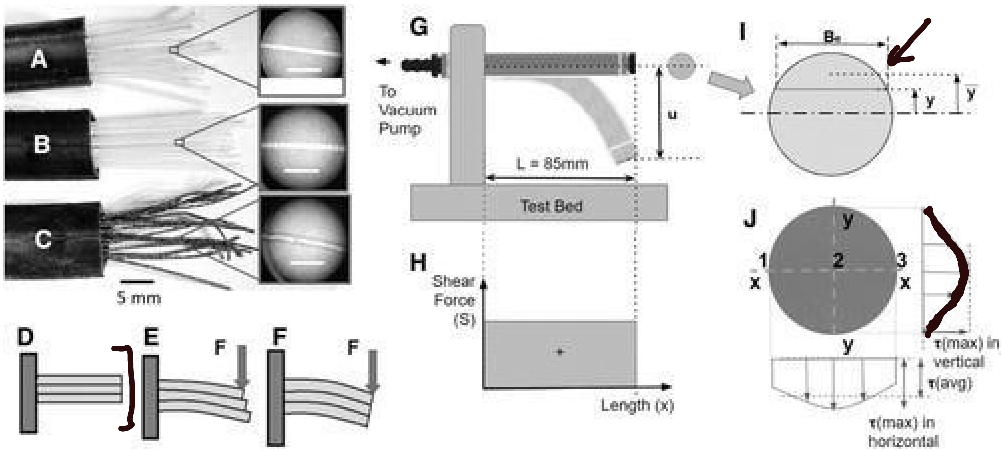
$\Rightarrow \varepsilon \sim \frac{d}{D}$

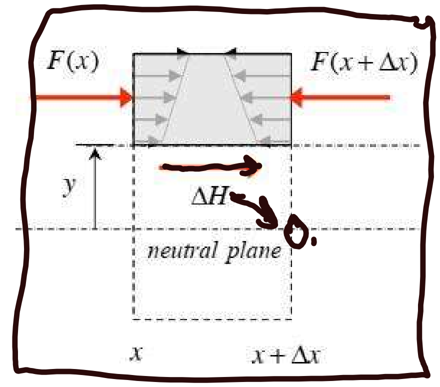
Kim et al, Materials, 12:2490, 2019.



Shear and Normal Stresses in Practical Robots

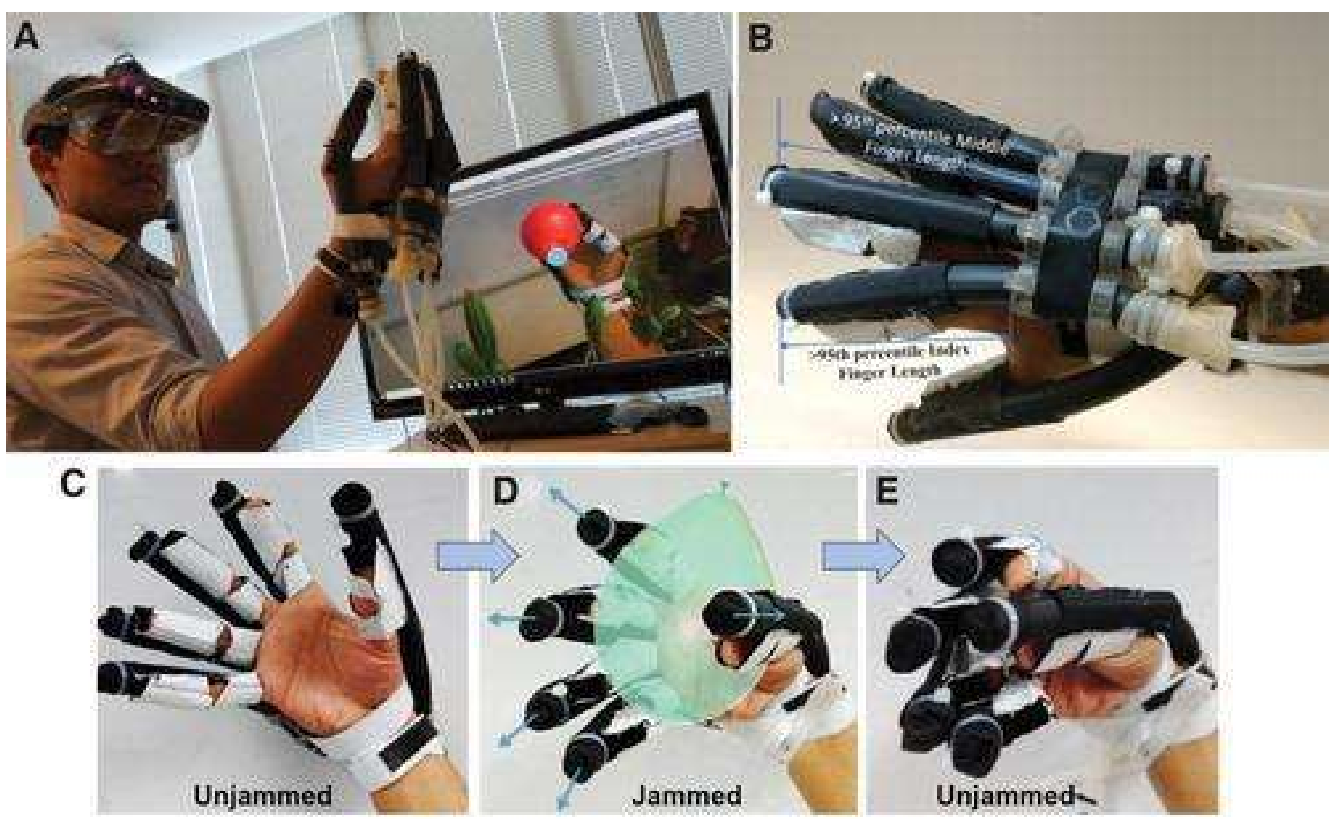
I





$$\boxed{\tau_{induced} = k_{correction} \times \frac{SA_e\hat{y}_e}{I_eB_e},} \ (1) = \frac{VA^*\bar{y}^*}{It.}$$

where A_e is the area of the cross section beyond the section at a distance of y away from the neutral axis, $\hat{y}_e$ is the centroid of that area away from the neutral axis, and B_e is the width of the section at a distance of y from the neutral axis. The accuracy of the above shear stress formula depends on the aspect ratio of the cross section of the ellipse. For very high



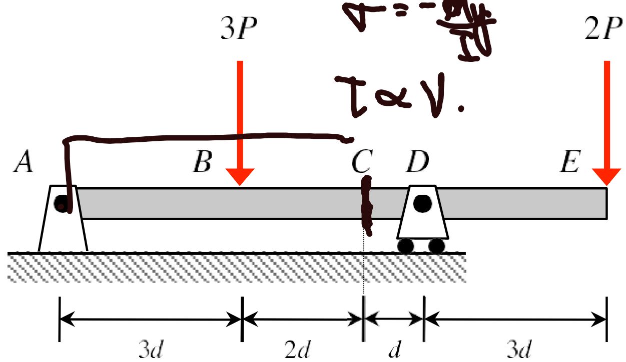
Jadhav et al, Soft Robotics, Vol. 9, pp. 173, (Feb 14, 2022).

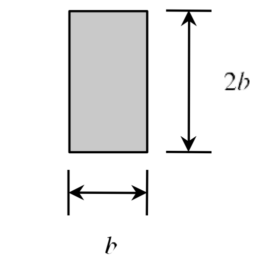
Example 10.7

A rectangular cross-section timber beam AE has dimensions and loading shown. Determine the normal and shear stress distributions at location C on the beam.

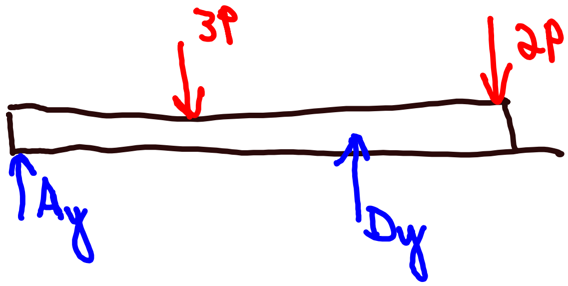
$\sigma = -\frac{My}{I}$

$\tau \propto V$.





beam cross section



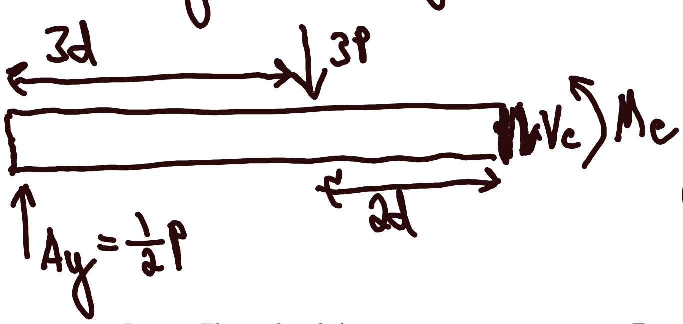
$$(\Sigma M)_A = -3P(3d) + D_y(6d) - 2P(9d) = 0$$

$$\Sigma F_y = A_y + D_y - 2P - 3P = 0$$

$$D_y(6d) = 27Pd$$

$$D_y = \frac{9}{2}P$$

$$A_y = 5P - D_y = \frac{1}{2}P$$



$$\Sigma F_y = A_y - 3P - V_c = 0$$

$$V_c = A_y - 3P = \underline{-\frac{5}{2}P}$$

$$(\Sigma M_c) = 3P(2d) - A_y(5d) + M_c = 0$$

$$\frac{12}{2}Pd - \frac{5}{2}Pd + M_c = 0$$

$$M_c = -\frac{7}{2}Pd.$$

Normal stresses

$$I = \frac{bh^3}{12} = \frac{b(2b)^3}{12} = \frac{8b^4}{12} = \frac{2b^4}{3}$$

$$\sigma = -\frac{My}{I} = -\left(-\frac{7}{2}Pd\right)y\left(\frac{3}{2b^4}\right) = \frac{21}{4}P\frac{d}{b^4}y$$

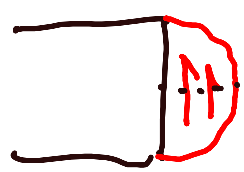
Shear stresses

$$\underline{\tau} = \frac{VA^*\bar{y}^*}{It}$$

$$A^* = b\left(\frac{h}{2} - y\right)$$

$$\bar{y}^* = \frac{1}{2}\left(\frac{h}{2} + y\right)$$

$$t = b$$

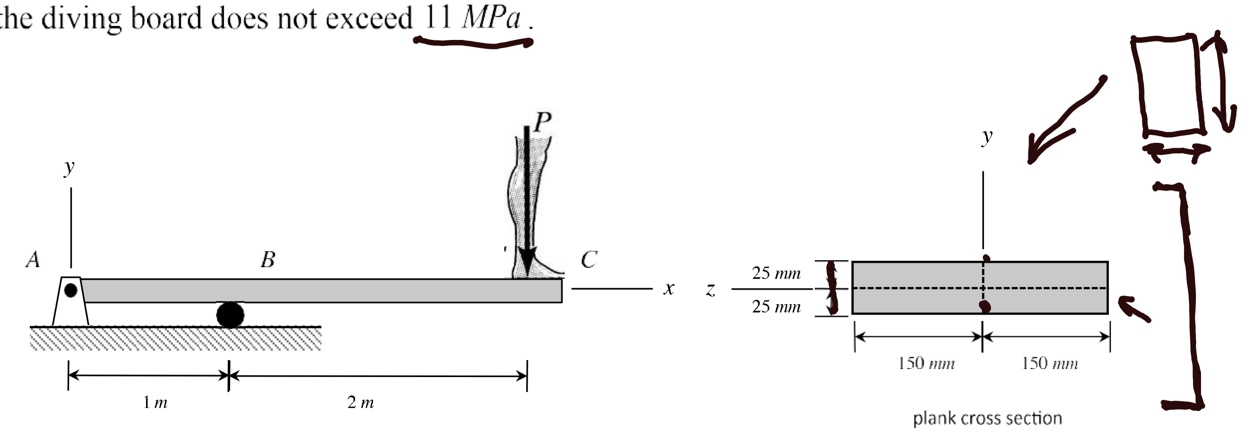


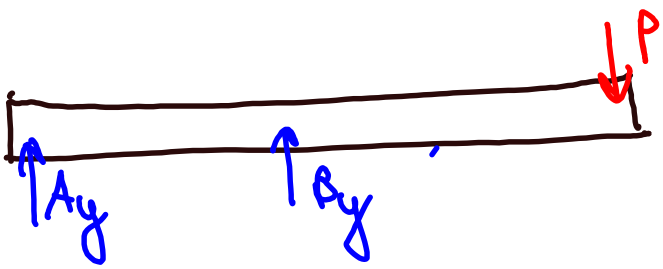
$$\tau = \frac{6}{Ah^2}\left(\frac{h^2}{4} - y^2\right)V$$

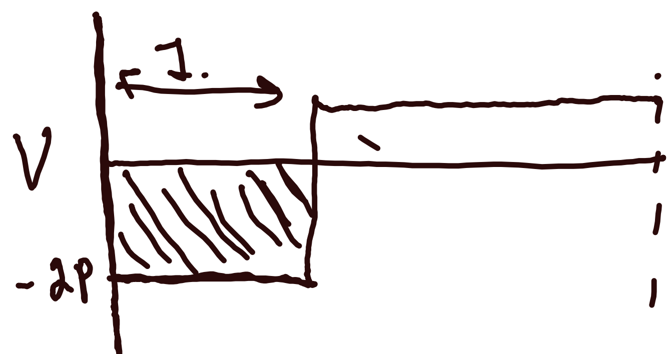
$$\underline{\tau_{max}} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{5}{2}P\right)\left(\frac{1}{b(2b)}\right) = -\frac{15}{8}\frac{P}{b^2}$$

Example 10.10

A timber plank is to be used as a diving board. The diving board is held down at end A by a steel strap that is secured by anchor bolts and rests on a roller at location B. Calculate the maximum permissible load P_{max} such that the maximum normal stress in the diving board does not exceed $11\ MPa$.



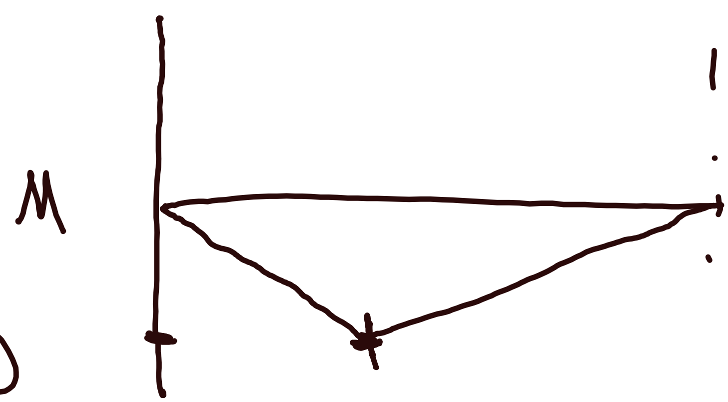




$$(\Sigma M)_A = B_y(1) - P(3) = 0$$

$$\Sigma F_y = A_y + B_y - P = 0$$

$$\Rightarrow B_y = 3P \qquad A_y = -2P$$



$$M(1) = -2P(1)$$

$$\sigma_{max} = \frac{-My}{I} = \frac{-2P\left(\frac{h}{2}\right)}{\left(\frac{bh^3}{12}\right)} = -\frac{12P}{bh^2}$$

$$P = \frac{bh^2}{12}\left|\sigma_{max}\right| = \frac{(0.3)(0.05)^2}{12}\left(11\times10^6\right) = 687\,N = 154 \text{ lbs.}$$

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shear strength of wood ~ 8 MPa.

$$\tau = \frac{3V}{2A} = \frac{3(-2P)}{2(0.05\times0.3)} = 137 \text{ kPa.}$$

$\Rightarrow$ long beams have low shear stress.