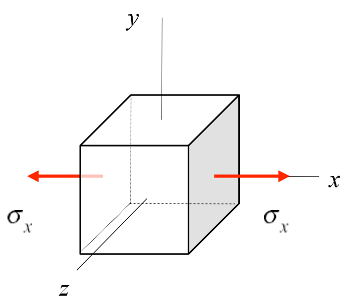
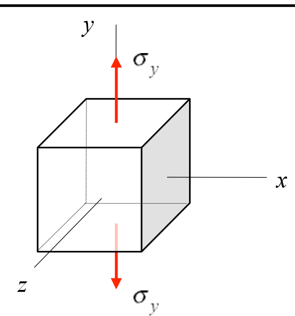
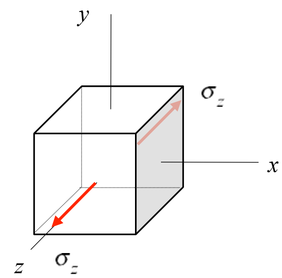
## *c) Generalized Hooke's law for normal stresses/strains*

Recall that for uni-axial loading along the x-axis, the normal strains in the $x$, $y$ and $z$ directions in the body were found to be:

$$\varepsilon_x = \sigma_x / E$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\sigma_x / E$$

where $E$ and $\nu$ are the Young's modulus and Poisson's ratio for the material. For a 3-D loading of a body, we have three normal stress components $\sigma_x$, $\sigma_y$ and $\sigma_z$ acting simultaneously. For this case, we will consider the strains due to each normal component of stress individually and add these together using linear superposition (along with the thermal strains) to determine the resulting three components of strain $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_z$. Consider the individual contributions of the three components of stress shown below:

| *Strains due to mechanical loading in the x-direction* | *Strains due to mechanical loading in the y-direction* | *Strains due to mechanical loading in the z-direction* |
|---|---|---|
|   |   |   |
| $\varepsilon_x = \sigma_x / E$ | $\varepsilon_x = -\nu\varepsilon_y = -\nu\sigma_y / E$ | $\varepsilon_x = -\nu\varepsilon_z = -\nu\sigma_z / E$ |
| $\varepsilon_y = -\nu\varepsilon_x = -\nu\sigma_x / E$ | $\varepsilon_y = \sigma_y / E$ | $\varepsilon_y = -\nu\varepsilon_z = -\nu\sigma_z / E$ |
| $\varepsilon_z = -\nu\varepsilon_x = -\nu\sigma_x / E$ | $\varepsilon_z = -\nu\varepsilon_y = -\nu\sigma_y / E$ | $\varepsilon_z = \sigma_z / E$ |

The total strain in each direction is found through superposition of the individual strains along with the thermal strains. Adding together these components (across each row of the above table) gives:

$$\varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_y - \frac{\nu}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha\Delta T$$

$$\varepsilon_z = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_y + \frac{1}{E}\sigma_z + \alpha\Delta T = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha\Delta T$$

The above are known as the generalized Hooke's law equations for normal stresses/strains due to 3-D loadings on a body.

## *Summary: Indeterminate axial problems (the four-step method)*

1. *Draw FBD's and write down equilibrium equations.*
   a. Break system into "elements". For problems in this course, an element:
      i. has forces acting only at its ends,
      ii. has a constant cross section, and
      iii. has constant material properties.
   b. It is recommended that you always draw end forces on element corresponding to "tension" (if the force actually corresponds to compression, you will get a negative value for the force in the end...trust the math...it works!).
   c. Be sure to abide by Newton's 3rd Law (reactions appear in equal and opposite pairs) when drawing your FBD's

2. *Write down the elemental force-deformation equations.*
   a. For the jth element:

$$\underline{e_j = \frac{P_j L_j}{E_j A_j}} + \alpha \Delta T L$$

   b. If you draw all element forces as in tension (as recommended in 1b) above), then $P_j$ has a positive sign in the equation above. If you choose to draw the elemental force as in compression, then $P_j$ has a negative sign.

3. *Write down appropriate compatibility equations*
   a. In this step you will write down the constraint equations that exists among the element elongations.
   b. This step is *problem-dependent* (and requires the most thought):
      i. For an axial system constrained between rigid supports, the compatibility equation needed is that the sum of all the elemental elongations is zero.
      ii. For elements attached to a rigid member, the motion of the rigid member dictates the relationship that exists among the elemental elongations.
      iii. For truss elements, a trigometric relationship must be used to relate the elemental elongations.

Draw Deformed Geometry!

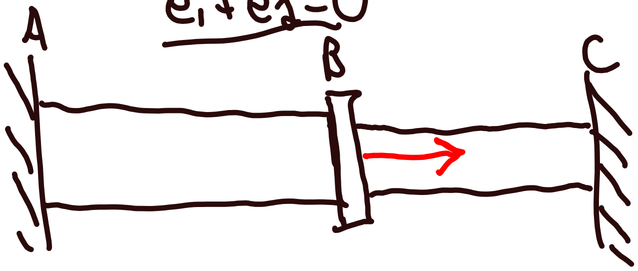
4. *Solve equations derived in Steps 1-3 for the elemental forces* $P_j$. Count your number of equations and number of unknowns. If you have sufficient equations to solve, then solve. If not, review the first three steps to see if you are missing needed equilibrium, force/deformation or compatibility equations. From these forces determine at this step, the elemental stresses, strains and elongations can be computed as needed.

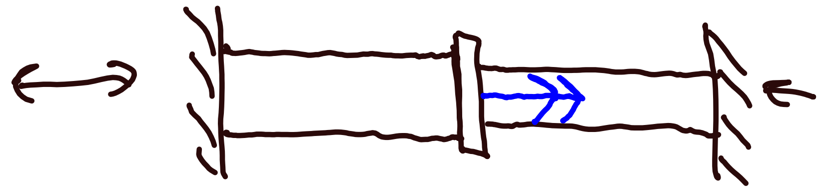
$u_B - u_A = e_1$ $\quad$ $u_C - u_B = e_2$

$\Delta\phi_1 = -\Delta\phi_2$

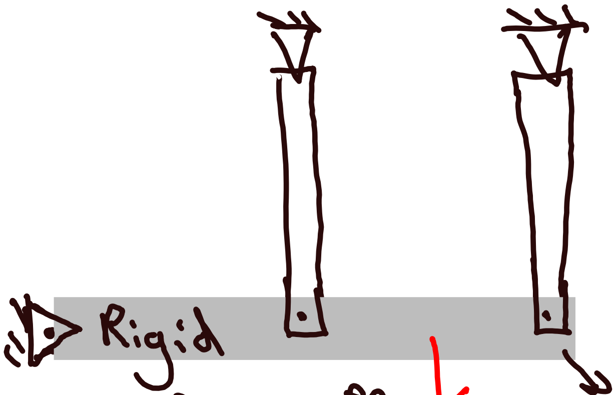
$e_1 + e_2 = 0$

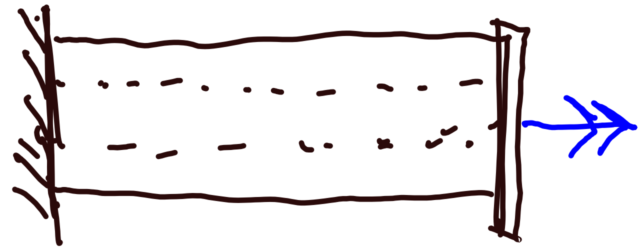
$\Delta\phi_1 + \Delta\phi_2 = 0$



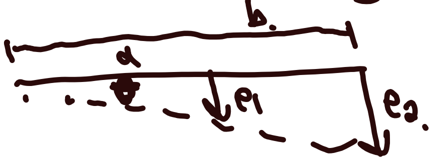



$\Delta\phi_1 = \Delta\phi_2$.

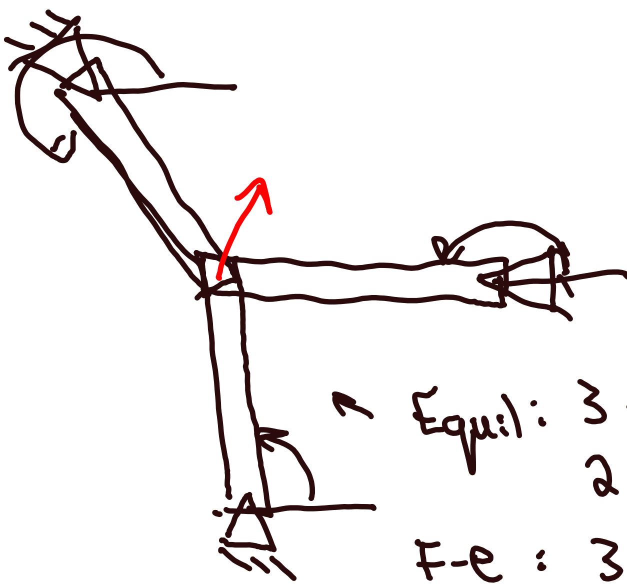


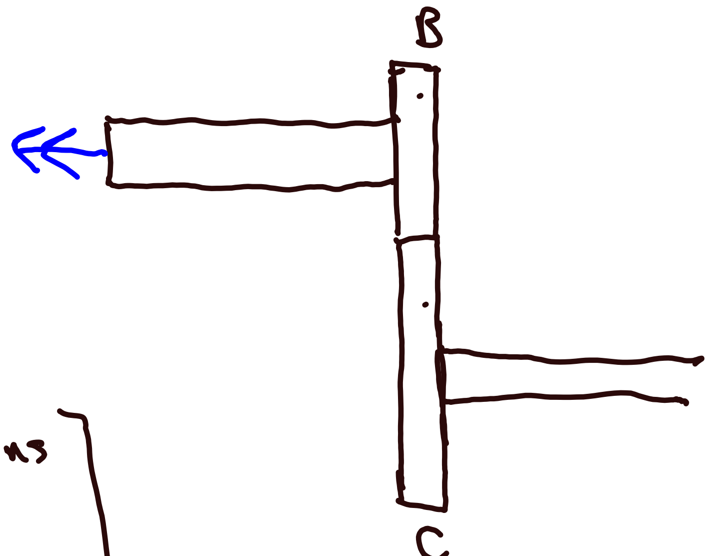



$\tan\theta \sim \theta = \frac{e_1}{a}$ $\quad$ $\theta = \frac{e_2}{b}$




$\frac{e_1}{a} = \frac{e_2}{b}$.






Equil: 3 unknowns
2 equ

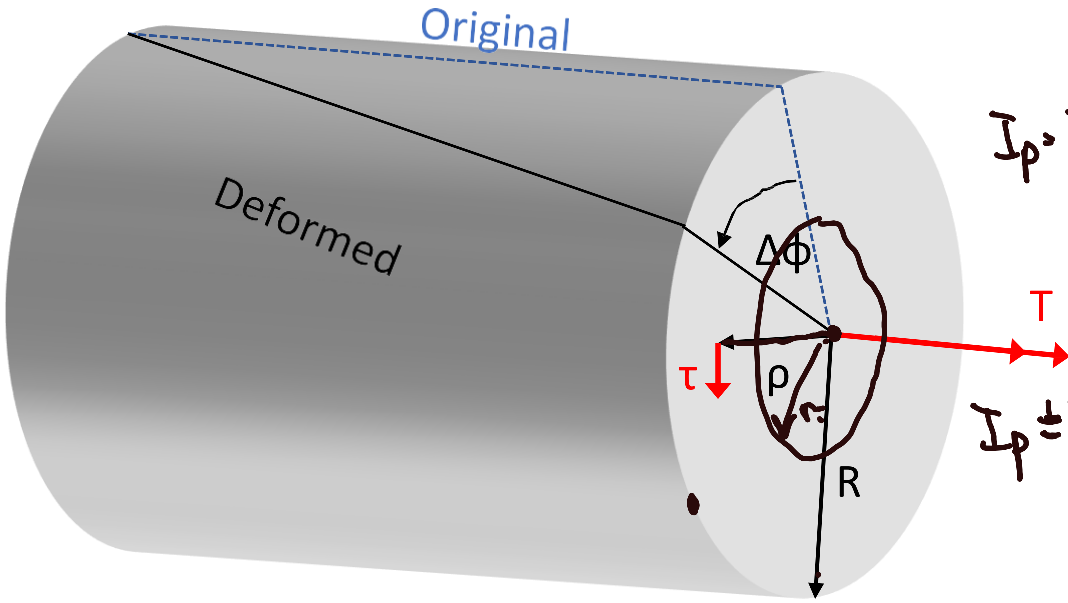
F-e: 3 unknowns
3 eqn.

2 unknowns
3 eqn.

$-r_C\phi_C = r_B\phi_B$

$-\phi_C = \frac{r_B}{r_C}\phi_B$.

$$\begin{cases} e_1 = u_C\cos\theta_1 + v_C\sin\theta_1 \\ e_2 \\ e_3 \end{cases}$$




$I_p = \frac{\pi}{2} R^4$

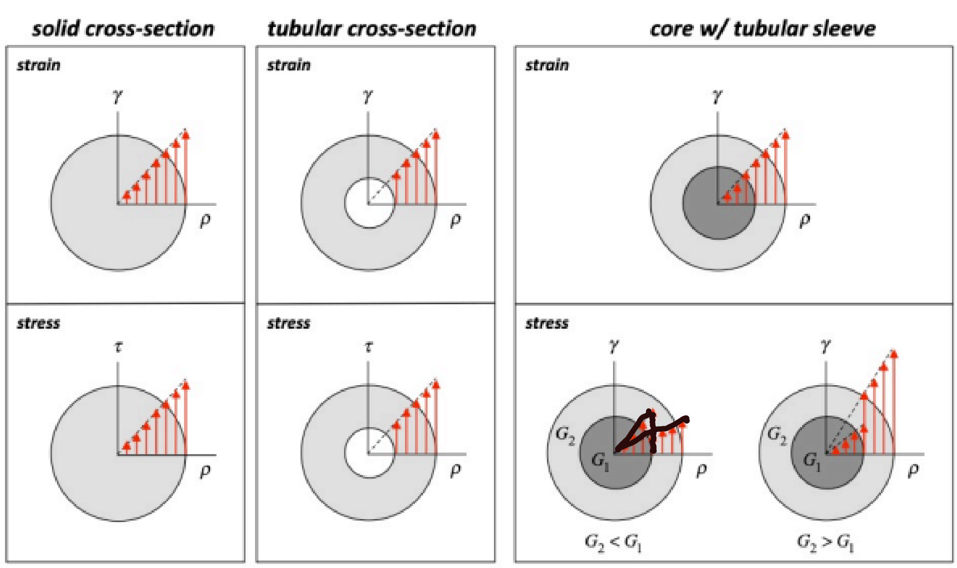
$I_p = \frac{\pi}{2}(R^4 - r_i^4)$

$$\tau_{max} = \frac{T\rho}{I_p} \qquad \Delta\Phi = \frac{TL}{GI_p} \qquad I_p = \frac{\pi}{2} r_0^4$$

# *Summary*: torsion stresses in shafts

Consider an axial torque T acting on a shaft with a circular cross section.

- *STRAIN*: The shear strain, $\gamma$, varies linearly with radius, $\rho$, through the cross-section of the shaft, regardless of the material makeup of the cross-section.
- *STRESS*: Across annular regions on the cross-section where the material makeup is a constant, the shear stress, $\tau$, varies linearly with radius, $\rho$, through the cross-section of the shaft: $\tau = G\gamma = T\rho / I_P$ where $I_P$ is the polar area moment of the cross section.
- *STRAIN/STRESS DISTRIBUTIONS*:



# Spring 2019 Exam 1

**ME 323 Examination # 1**

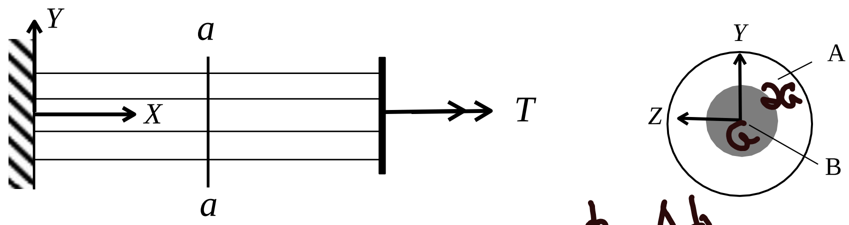
**Name ________________________________**
**(Print) (Last) (First)**

**February 15, 2017**

**Instructor ______________________________**

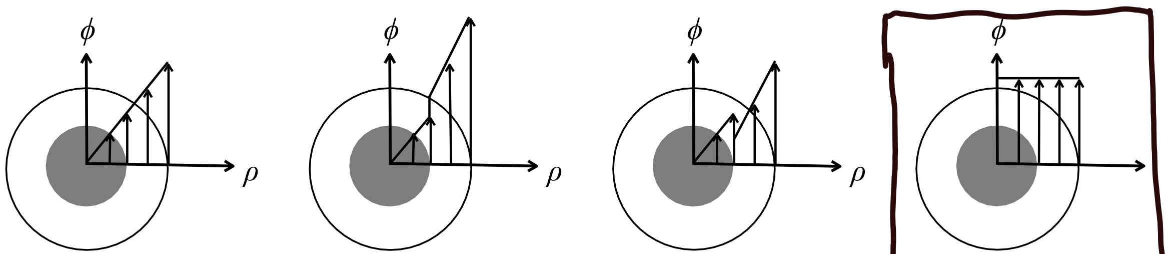
**4.2. (6 Points)**

A bimetallic bar with circular cross section consists of a shell A and a core B. The bimetallic bar is subjected to a torque $T$. The shear moduli of the core and shell are known to be $G_A = 2G_B$, and polar moment of inertia $I_{PA} = 0.5\, I_{PB}$.



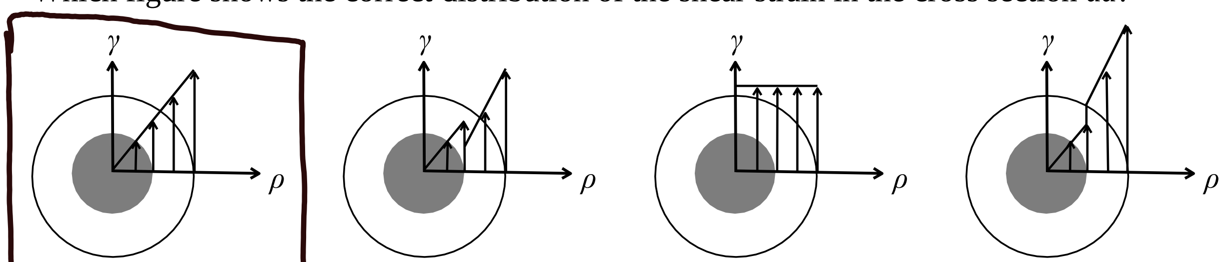

ϕ Δϕ

Which figure shows the correct distribution of the twist angle in the cross section *aa*?



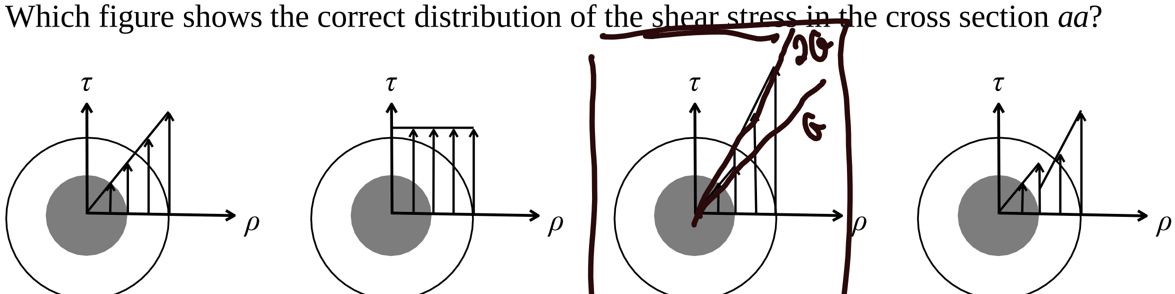

(a) (b) (c) (d)

Which figure shows the correct distribution of the shear strain in the cross section *aa*?




(a) (b) (c) (d)

Which figure shows the correct distribution of the shear stress in the cross section *aa*?




(a) (b) (c) (d)

Fall 2019 Exam 1

**ME 323 Examination # 1**

**Name** ______________________________
**(Print) (Last) (First)**

**PROBLEM #4 (25 Points):**
***PART A – 6 points***

For each state of plane stress shown below, i.e., for configurations (a) and (b), indicate whether each component of the state of strain is:

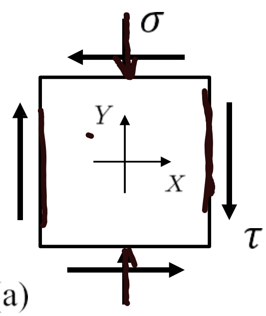
- = 0 (equal to zero)
- > 0 (greater than zero)
- < 0 (less than zero)

The material is linear elastic with Poisson's ratio $\nu$ $(0 < \nu < 0.5)$, and the deformations are small.

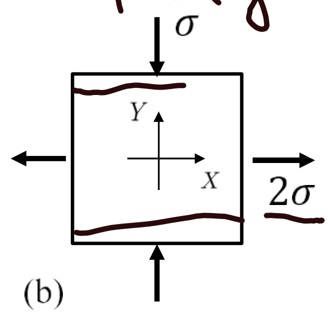
$\epsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z))$

$\tau_{ij}$

$\tau_{xy} = <0$




(a)




(b)

$2\sigma \quad -\sigma$

$\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))$

$\epsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y))$

| | (a) | (b) |
|---|---|---|
| $\epsilon_x$ | >0 | >0 |
| $\epsilon_y$ | <0 | <0 |
| $\epsilon_z$ | >0 | <0 |
| $\gamma_{xy}$ | <0 | 0 |
| $\gamma_{xz}$ | 0 | 0 |
| $\gamma_{yz}$ | 0 | 0 |

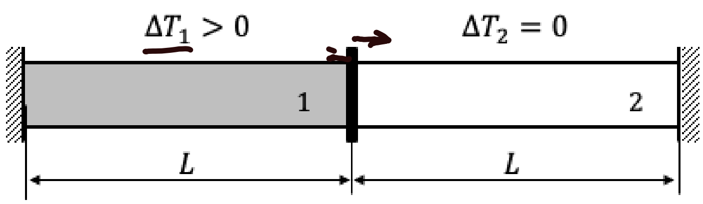
$\gamma_{xz} = \frac{\tau_{xz}}{G}$

*Fill in with '= 0', '> 0', or '< 0'.*

**PROBLEM #4 (cont.):**

***PART C – 4 points***

A rod is made up of elastic elements 1 and 2, each having a length $L$ and cross-sectional area $A$. Elements 1 and 2 have the same Young's modulus $E_1 = E_2$ and coefficient of thermal expansion $\alpha_1 = \alpha_2$. Let $F_1$ and $F_2$ represent the axial load carried by elements 1 and 2, respectively, when the temperature of element 1 is increased by $\Delta T_1 > 0$ — while the temperature of element 2 is kept constant $\Delta T_2 = 0$.




Circle the correct answer:

$e_1 + e_2 = 0$

a) $|e_1| > |e_2|$

b) $|e_1| = |e_2|$

c) $|e_1| < |e_2|$

Circle the correct answer:

a) $|F_1| > |F_2|$

b) $|F_1| = |F_2|$

c) $|F_1| < |F_2|$

Circle the correct answer:

*a) Elastic element 1 is under tension*

*b) Elastic element 1 is under compression*

*c) Elastic element 1 is stress-free*

Circle the correct answer:

*a) Elastic element 2 is under tension*

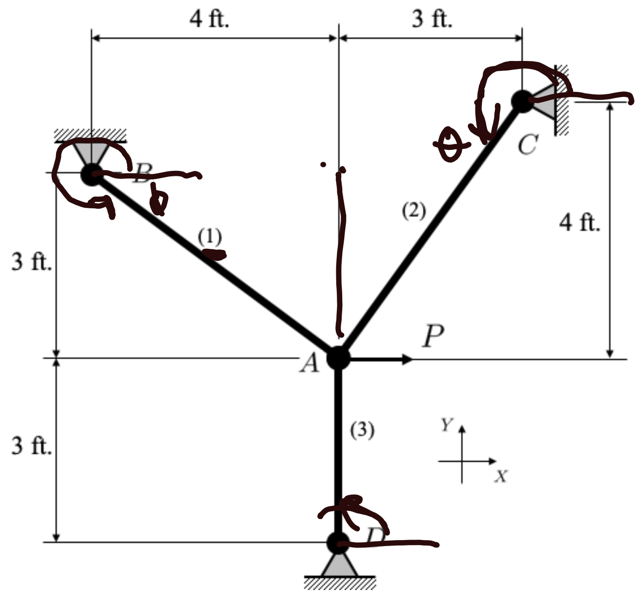
*b) Elastic element 2 is under compression*

*c) Elastic element 2 is stress-free*

**PROBLEM # 4 (cont.):**

***PART E – 7 points***

Use only the compatibility condition for the truss structure shown in the figure to find the value of the elongation of member 1 ($e_1$) in terms of the elongation of member 2 ($e_2$) and the elongation of member 3 ($e_3$).




a) Determine an expression for the **compatibility condition** at A:

$$e_1 = \boldsymbol{a}\, e_2 + \boldsymbol{b}\, e_3$$

where ***a*** and ***b*** are numbers.

**Note: Please notice that you are not required to solve for the internal axial forces at equilibrium.**

b) Circle the correct answer.

$\boxed{\textit{TRUE}}$ or ***FALSE***: The elongation of element 1 depends on the material makeup of elements 2 and 3.

$$e_1 = u_A \cos\theta_1 + V_A \sin\theta_1$$

$$e_2 = u_A \cos\theta_2 + V_A \sin\theta_2$$

$$e_3 = u_A \cos\theta_3 + V_A \sin\theta_3$$

$$\theta_1 = 2\pi - \tan^{-1}\left(\frac{3}{4}\right)$$

$$\theta_2 = \pi + \tan^{-1}\left(\frac{4}{3}\right)$$

$$\theta_3 = \frac{\pi}{2}.$$

# Fall 2019 Exam 1

**ME 323 Examination # 1**

**Name** ________________________________
**(Print) (Last) (First)**

**PROBLEM #3 (25 points)**

A torque $T$ is applied to a gear-shaft system and is transmitted through rigid gears B and C to a fixed end E as shown in Fig. 3(a). The shafts (1) and (2) are tightly fit to each other. Frictionless bearings are used to support the shafts. The geometry and material property of the shafts and gears are listed in the following table.

| | Size | Length | Shear modulus |
|---|---|---|---|
| Shaft (1) | Outer diameter = $2d$<br>Inner diameter = $d$ | $L$ | $2G$ |
| Shaft (2) | Diameter = $d$ | $L$ | $G$ |
| Shaft (3) | Diameter = $d$ | $L/2$ | $G$ |
| Shaft (4) | Diameter = $d$ | $L$ | $G$ |
| Gear B | Diameter = $1.5d$ | Negligible | Rigid |
| Gear C | Diameter = $3d$ | Negligible | Rigid |

(a) Determine the torque carried by each shaft.
(b) Determine the angle of twist at the free ends A and D.
(c) Consider the cross section *aa*' for the shafts (1) and (2), show the magnitude of the shear stress as a function of the distance from the center on Fig. 3(b). Mark the critical values in the diagram.
(d) Consider the points M and N on the cross section *aa*', shown in Fig. 3(c). Sketch the stress states at M and N on the stress elements on Fig. 3(d).

Express all your answers in terms of $d, L, G, T, \pi$.

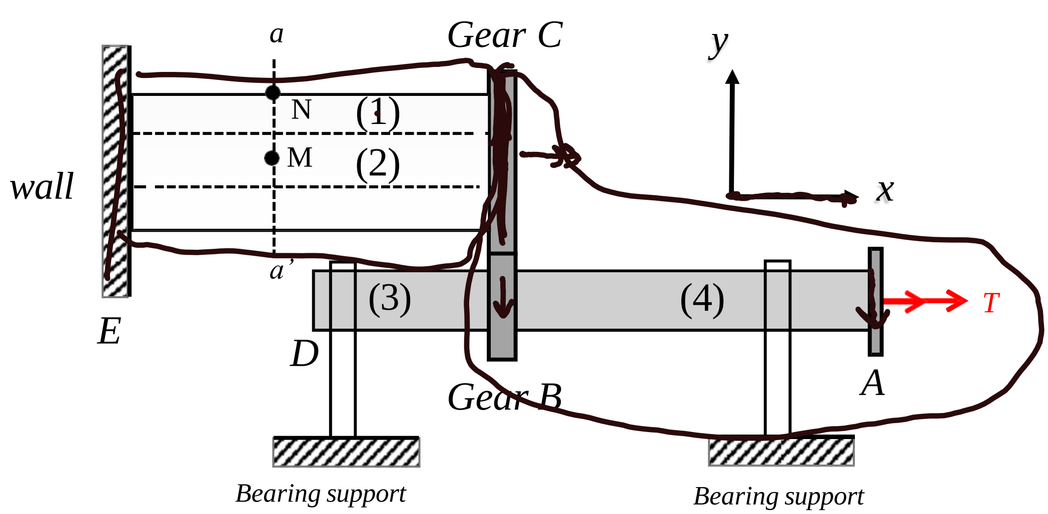



Fig. 3(a)

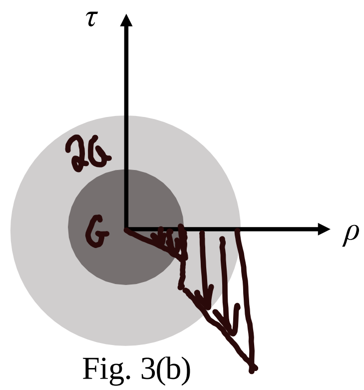



Fig. 3(b)

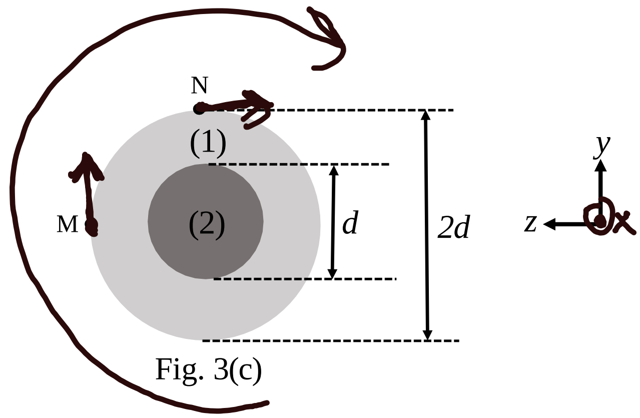



Fig. 3(c)

$\tau_{xy} = \tau_{yx}.$

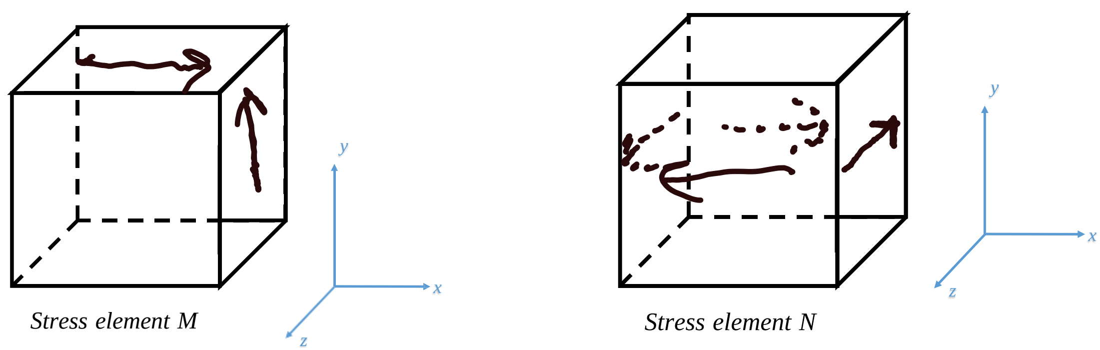
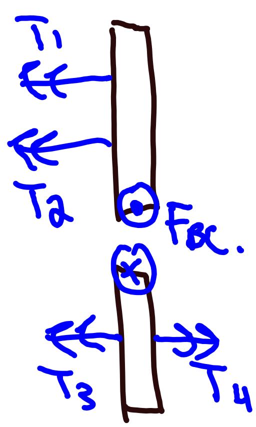
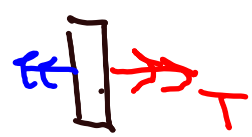



Fig. 3(d)

1.)




$$(\Sigma M)_A = T - T_4 = 0 \Rightarrow T_4 = T \qquad T_3 = 0$$

$$(\Sigma M)_B = T_4 - F_{BC} r_B = 0 \qquad F_{BC} = \frac{T}{r_B}$$

$$\underline{(\Sigma M)_C = -T_1 - T_2 - F_{BC} r_C = 0}$$

$\Big\}$ $T_1, T_2, T_4, F_{BC}$

HW 4.3   Example 8.4

ME 323 Examination # 1

Name ______________________________
(Print)   (Last)   (First)

2.) Force-displacement.

$$\Delta\phi_1 = \frac{T_1 L_1}{G_1 I_{p1}} \qquad \Delta\phi_2 = \frac{T_2 L_2}{G_2 I_{p2}}.$$

3.) Compatibility.

$$\Delta\phi_1 = \Delta\phi_2.$$

4.) Solve.

ME 323 - Mechanics of Materials
Midterm #1
February 23rd, 2022

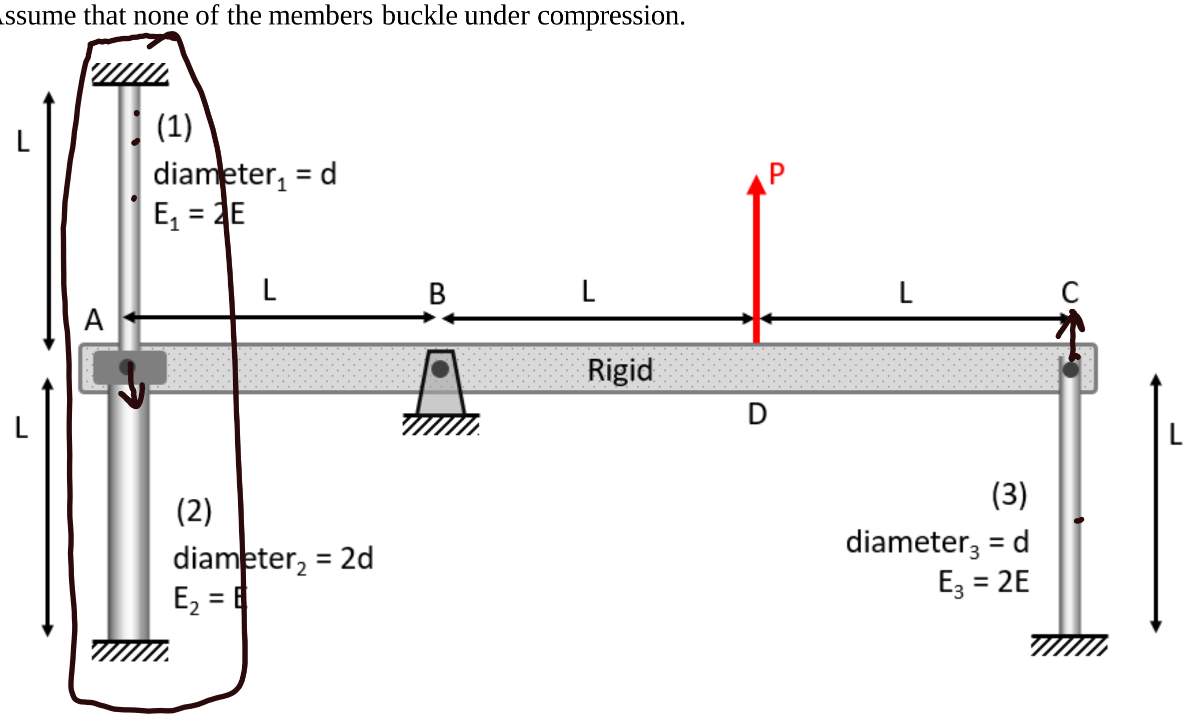
**Name (Print)** ______________________________

**(Last)** **(First)**

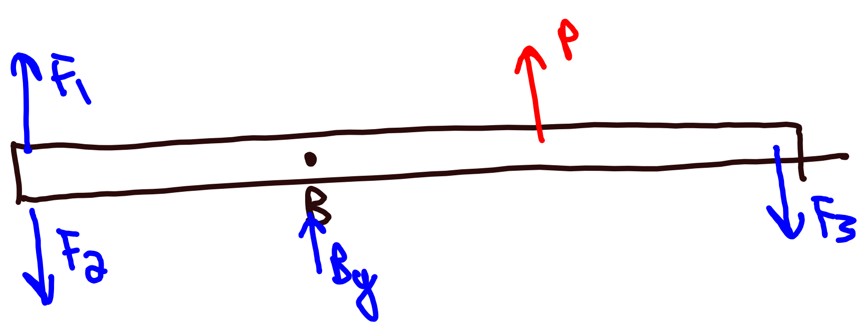
## PROBLEM # 3 (25 points)

Rigid bar ABC is held by a pin joint at B and connected to three deformable rods. Two deformable rods are connected at A and one deformable rod is connected at C. A force *P* is applied at point D. The diameters and elastic moduli of the deformable rods are listed in the figure.

a) Determine whether the assembly is determinate or indeterminate.
b) What are the axial stresses on each of the deformable members? Leave your answers in terms of *P*, *L*, *E*, and/or *d*.
c) If the yield stress ($a_{-Y}$) is 200 MPa, what is the maximum value of *P* that can be applied to maintain a factor of safety (FoS) of 2? Use values of **L = 1 m**, **E = 70 GPa**, and **d = 0.02 m**.
Assume that none of the members buckle under compression.




I.)




$$(\Sigma M)_B = 0 = -F_1 L + F_2 L + PL - F_3(2L)$$ } 3 unknowns, 1 equation

2.) Force-elongation

3.). Compatibility.

$$e_1 + e_2 = 0$$

$$\boxed{\frac{e_1}{L} = \frac{e_3}{2L}}$$

**ME 323 Examination # 1**

**Name ______________________________**
**(Print) (Last) (First)**

**February 15, 2017**

**Instructor ____________________________**

**PROBLEM #2 (28 points)**

A shaft is made up of three components: solid circular shafts (1) of length $2L$ and diameter $d$; solid circular shaft (2) of length $L$ and diameter $2d$; and *tubular* shaft (3) of length $L$, outer diameter $3d$ and inner diameter $2d$. All three components are made of the same material having a shear modulus of $G$. Shafts (1) and (2) are connected by a thin, rigid connector C, whereas shafts (2) and (3) are connected by a rigid connector D. Shaft (1) is rigidly attached to ground at B. Shaft (3) is rigidly attached to ground at H. Torques $T$ and $2T$ are applied to C and D, respectively, as shown in the Fig. 2 (a).

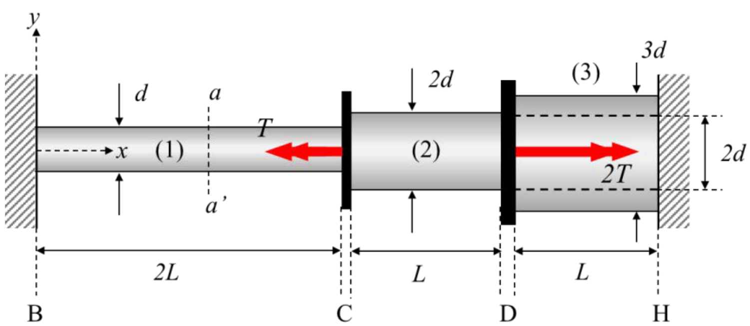



Fig. 2 (a)

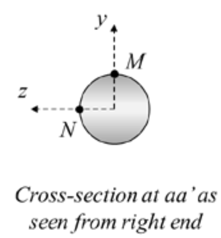



Fig. 2 (b)


Fig. 2 (c)

1) Determine the torque carried by each of the shaft components.
2) Determine the angle of rotation at connector C.
3)Show the stress element for the points M ($x$, $0.5d$, 0) and N ($x$, 0, $0.5d$), whose locations on shaft (1) are also shown on Fig. 2 (b). Indicate both magnitudes and axes corresponding to the state of stress on Fig. 2 (c).
Express your final answers in terms of $T$, $L$, $d$ and $G$.

Name ______________________
(Print) (Last) (First)

Instructor ______________________

Name ______________________
(Print) (Last) (First)

Instructor ______________________

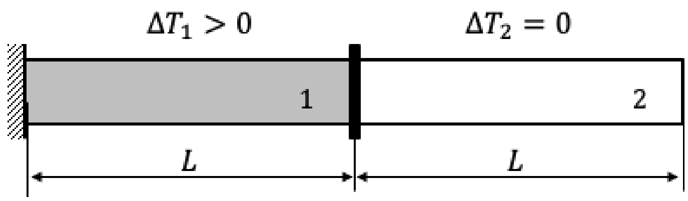
**ME 323 Examination # 1**

**Name ________________________________**
**(Print) (Last) (First)**

**PROBLEM #4 (cont.)**:

***PART B – 4 points***

A rod is made up of elastic elements 1 and 2, each having a length $L$ and cross-sectional area $A$. Elements 1 and 2 have the same Young's modulus $E_1 = E_2$ and coefficient of thermal expansion $\alpha_1 = \alpha_2$. Let $F_1$ and $F_2$ represent the axial load carried by elements 1 and 2, respectively, when the temperature of element 1 is increased by $\Delta T_1 > 0$ — while the temperature of element 2 is kept constant $\Delta T_2 = 0$.




Circle the correct answer:

a) $|e_1| > |e_2|$

b) $|e_1| = |e_2|$

c) $|e_1| < |e_2|$

Circle the correct answer:

a) $|F_1| > |F_2|$

b) $|F_1| = |F_2|$

c) $|F_1| < |F_2|$

Circle the correct answer:

*a) Elastic element 1 is under tension*

*b) Elastic element 1 is under compression*

*c) Elastic element 1 is stress-free*

Circle the correct answer:

*a) Elastic element 2 is under tension*

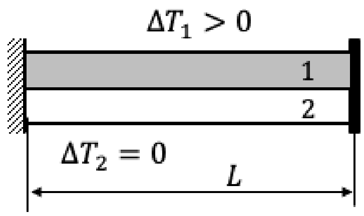
*b) Elastic element 2 is under compression*

*c) Elastic element 2 is stress-free*

**PROBLEM #4 (cont.)**:

***PART D – 4 points***

A rod is made up of elastic elements 1 and 2, each having a length $L$ and cross-sectional area $A$. Elements 1 and 2 have the same Young's modulus $E_1=E_2$ and coefficient of thermal expansion $\alpha_1=\alpha_2$. Let $F_1$ and $F_2$ represent the axial load carried by elements 1 and 2, respectively, when the temperature of element 1 is increased by $\Delta T_1 > 0$ — while the temperature of element 2 is kept constant $\Delta T_2 = 0$.




Circle the correct answer:

a) $|e_1| > |e_2|$
b) $|e_1| = |e_2|$
c) $|e_1| < |e_2|$

Circle the correct answer:

a) $|F_1| > |F_2|$
b) $|F_1| = |F_2|$
c) $|F_1| < |F_2|$

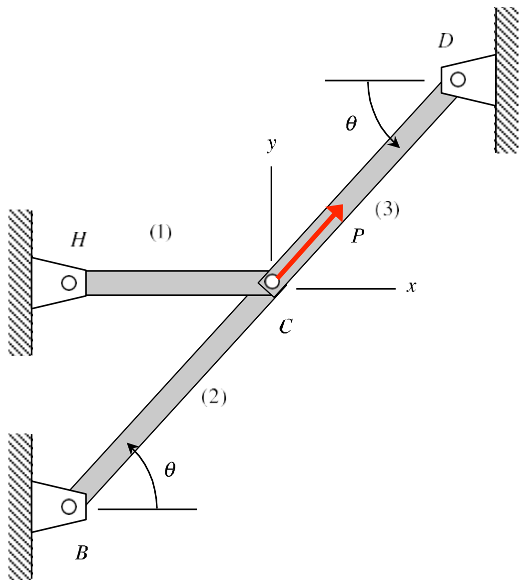
Circle the correct answer:

*a) Elastic element 1 is under tension*
*b) Elastic element 1 is under compression*
*c) Elastic element 1 is stress-free*

Circle the correct answer:

*a) Elastic element 2 is under tension*
*b) Elastic element 2 is under compression*
*c) Elastic element 2 is stress-free*

***PROBLEM NO. 4 - PART C – 6 points max.***




In the truss shown above, member (1) is horizontal, with members (2) and (3) aligned and at an angle of $\theta$ with respect to the horizontal. A load P is applied to joint C in a direction that is aligned with members (2) and (3). Simultaneously, the temperature of member (2) is *increased*, with the temperatures of the remaining members being held constant. Let $e_1$ be the elongation of member (1), and $(u_C, v_C)$ being the x- and y-components of displacement of joint C due to the load P.

For this loading on the truss, the axial stress in member (1) is (circle the correct response):

a) compressive.

b) tensile.

c) zero.

*HINT*: consider an FBD of joint C.

Also, for this loading the *displacement* of joint C is (circle the correct response):

a) up and to the right ($u_C > 0$ and $v_C > 0$)

b) directly to the right ($u_C > 0$ and $v_C = 0$)

c) directly up ($u_C = 0$ and $v_C > 0$)

d) zero ($u_C = 0$ and $v_C = 0$)

You are NOT asked to provide explanations for your answers.