MECHANICS OF MATERIALS

Fall 2023

ME 323- 005

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Lecture 4: Introduction to Design of Deformable Bodies

Announcements

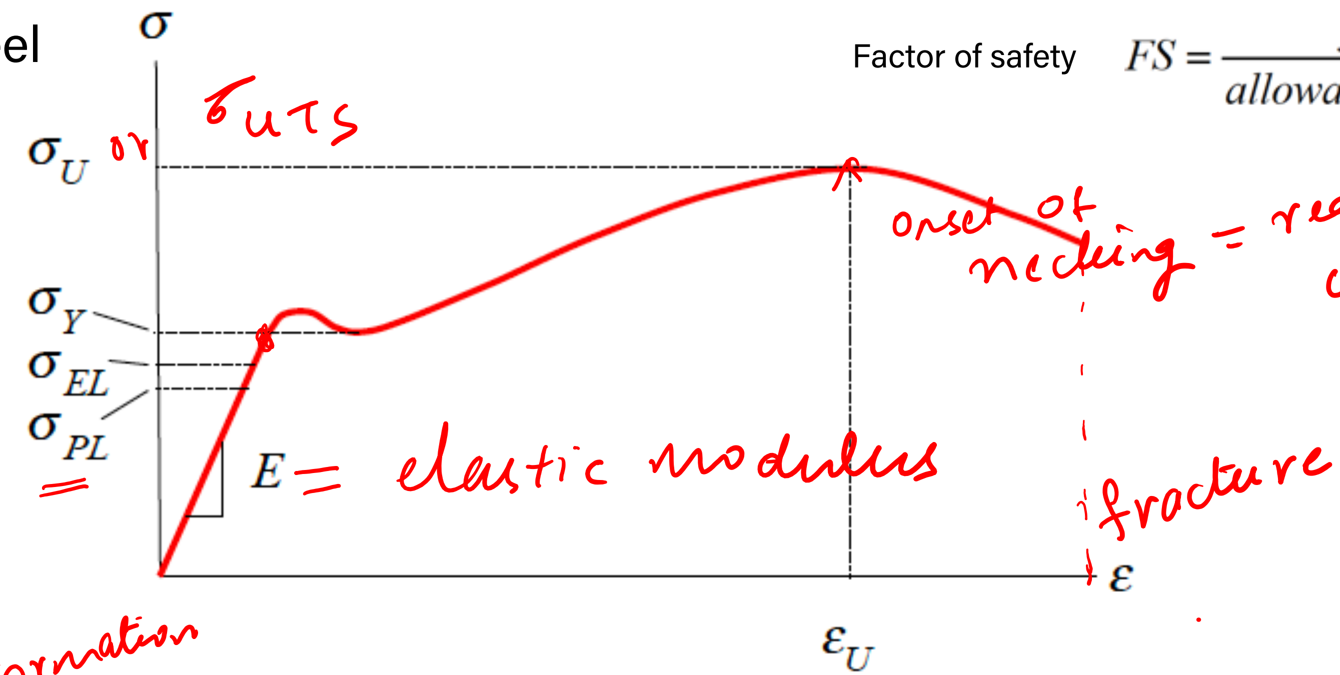
- Gradescope can be accessed through ME 323 freeform website
- Lecture notes are also available through ME 323 freeform website
- Course does not have Brightspace
- Homework 1 is now available in Gradescope
- No quizzes for 005 section

8EG4VW

Axial Stress-Strain Relationship

Steel

Factor of safety $FS = \dfrac{failure\ load}{allowable\,(applied)\,load} > 1$



elastic deformation

- σ_{PL} = Proportional limit of material. For $0 < \sigma < \sigma_{PL}$; linear relationship between normal stress and strain exists. After complete unloading $\varepsilon = 0, \sigma = 0$

- σ_{EL} = Elastic limit of material. For $\sigma_{PL} < \sigma < \sigma_{EL}$; non-linear relationship between normal stress and strain exists. Unloading follows reverse of loading curve. $\varepsilon = 0$, $\sigma = 0$

- σ_Y = Yield point of material. For $\sigma > \sigma_Y$, yielding occurs in the material. Unloading on this curve does NOT follow the reverse of the loading curve. $\varepsilon > 0, \sigma = 0$

- σ_U = Ultimate stress of material. For $\varepsilon > \varepsilon_U$, material begins to neck

Example 4.1 from Lecturebook

The critical components for the design of the frame shown below are assumed to be member BD and the pin at C.

a) Determine the required thickness t (into the page) of member BD (whose width is b) to avoid yielding failure with a factor of safety $FS = 3.0$.

b) Determine the required diameter d of pin C to avoid ultimate shear failure with a factor of safety $FS = 3.0$.

Use the following parameters in your analysis: $P = 2400 lb$, $L = 6\,ft$, $h = 4\,ft$, $b = 1 in$, $\sigma_Y = 36 ksi$ (for member BD) and $\tau_U = 60\ ksi$ (for pin C).

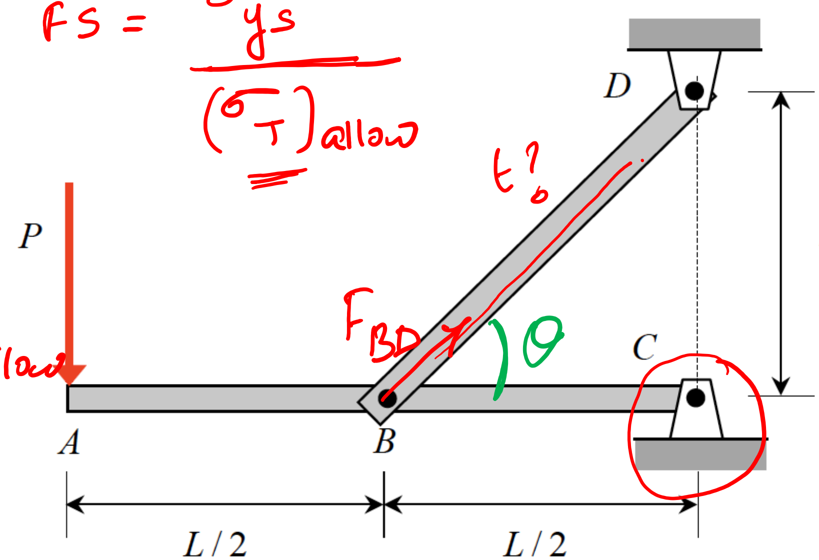
① forces

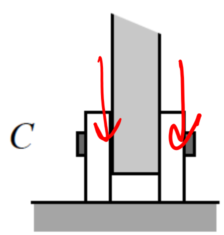
② stress = force/area

member BD, $FS = \frac{\sigma_{ys}}{(\sigma_T)_{allow}}$

Pin C

$FS = \frac{\tau_u}{(\tau_c)_{allow}}$





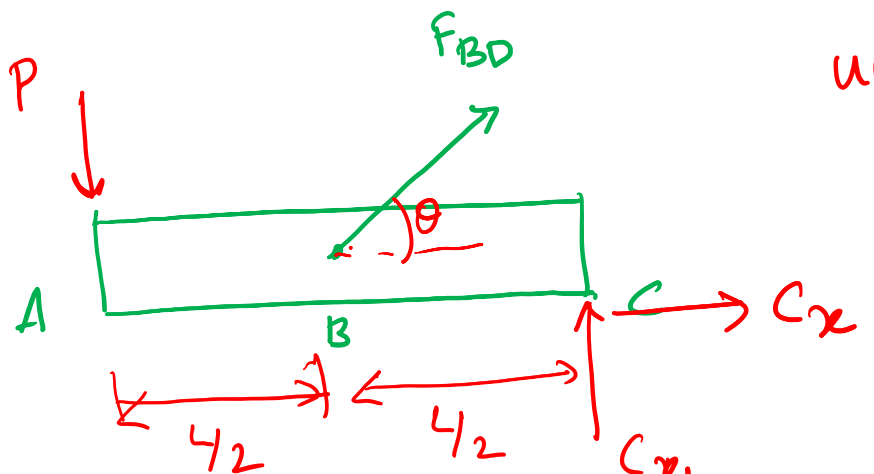
double-sided joint

EDGE view of joint C

$$\sin\theta = \frac{h}{\sqrt{h^2 + (L/2)^2}} = \frac{4}{\sqrt{(4)^2 + (6/2)^2}} = \frac{4}{5}$$

$$\cos\theta = \frac{L/2}{\sqrt{h^2 + (L/2)^2}} = \frac{3}{5}$$

at equilibrium –



unknowns F_{BD}, C_x, C_y

$$(\Sigma M)_B = P\left(\frac{L}{2}\right) + C_y\left(\frac{L}{2}\right) = 0$$

$$\boxed{C_y = -P}$$

$$\Sigma F_y = -P + C_y + F_{BS}\sin\theta = 0 \Rightarrow F_{BD} = \frac{2P}{\sin\theta}$$

$$F_{BD} = \frac{2P}{(4/5)} = \frac{5}{2}P \qquad \boxed{F_{BD} = \frac{5}{2}P}$$

$$\Sigma F_x = F_{BD}\cos\theta + C_x = 0 \Rightarrow C_x = -\frac{5}{2}P\cos\theta$$

$$C_x = -\frac{5}{2}P\left(\frac{3}{5}\right) = -\frac{3}{2}P$$

$$\boxed{C_x = -\frac{3}{2}P}$$

(a) $$\sigma_{BD} = \frac{F_{BD}}{A_{BD}} = \frac{\left(\frac{5}{2}P\right)}{(b)(t)} = \frac{5P}{2bt}$$

$$FS = 3 \Rightarrow \frac{\sigma_y}{\sigma_{BD}} = 3$$

$$\frac{\sigma_y}{\left(\frac{5}{2}P/bt\right)} = 3 \Rightarrow t = \frac{15}{2}\frac{P}{b\sigma_y}$$

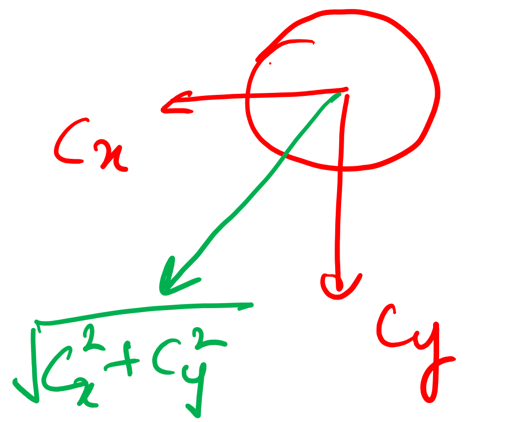
$$t = \frac{15}{2}\frac{(2400)(b)}{(1\text{ inch})(36 \times 1000\ lb/in^2)}$$

$$\boxed{t = 0.5\ \text{inch}}$$ ✓

(b) pin C

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{\left(\frac{-3P}{2}\right)^2 + (-P)^2}$$



$$C = \frac{\sqrt{13}}{2} P \Rightarrow \text{double-sided shear}$$

$$\tau = \frac{C/2}{\pi\left(\frac{d}{2}\right)^2} = \frac{\frac{\sqrt{13}}{4} P}{\pi (d/2)^2}$$

$$FS = \frac{\tau_u}{\tau} = 3 \Rightarrow \frac{\tau_u}{\left[\frac{\sqrt{13}/\cancel{4}\, P}{\pi d^2/\cancel{4}}\right]} = 3$$

$$d = \sqrt{\frac{(3)(\sqrt{13})(2400 lb)}{(\pi)(60)}}$$

$$\boxed{d = 11.74 \text{ inch}}$$

Example 4.2 from Lecturebook

The members of the truss shown below are designed for an allowable stress in tension of $(\sigma_T)_{allow} = 140\,MPa$ and an allowable stress in compression of $\left|(\sigma_C)_{allow}\right| = 85\,MPa$. If the truss is to support a maximum load of $P_{allow} = 50\,kN$, what are the required cross-sectional areas of the three truss members?

Use the following in your analysis: $b = 1200\,mm$, $h = 600\,mm$ and $d = 450\,mm$.

$A_1 = ?$

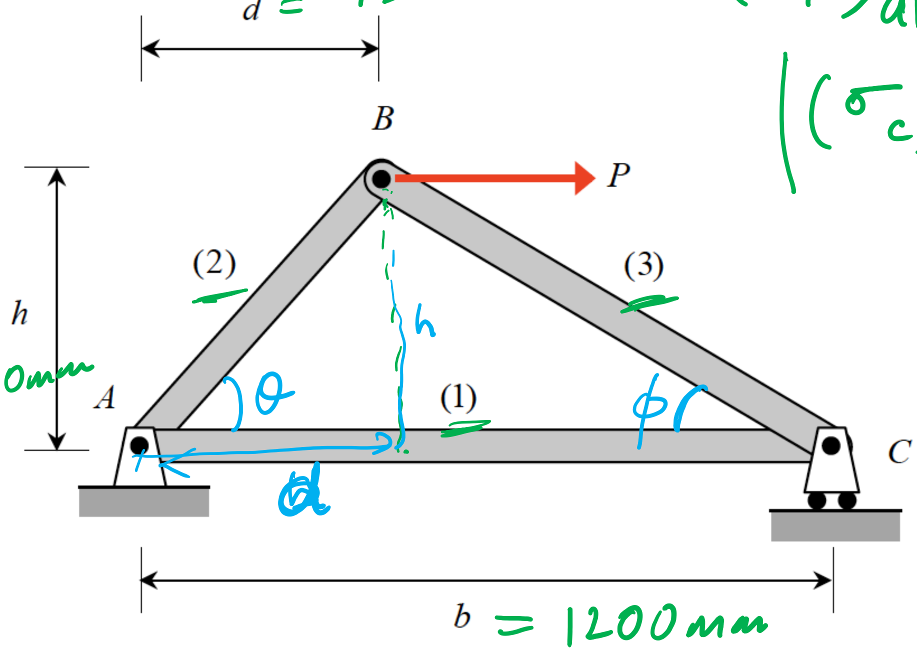
$A_2 = ?$

$A_3 = ?$

$(\sigma_T)_{allow} = 140\,MPa$

$\left|(\sigma_c)_{allow}\right| = 85\,MPa$

$P_{allow} = 50\,kN$



$\theta = \tan^{-1}\left(\frac{h}{d}\right)$

$\theta = \tan^{-1}\left(\frac{600}{450}\right)$

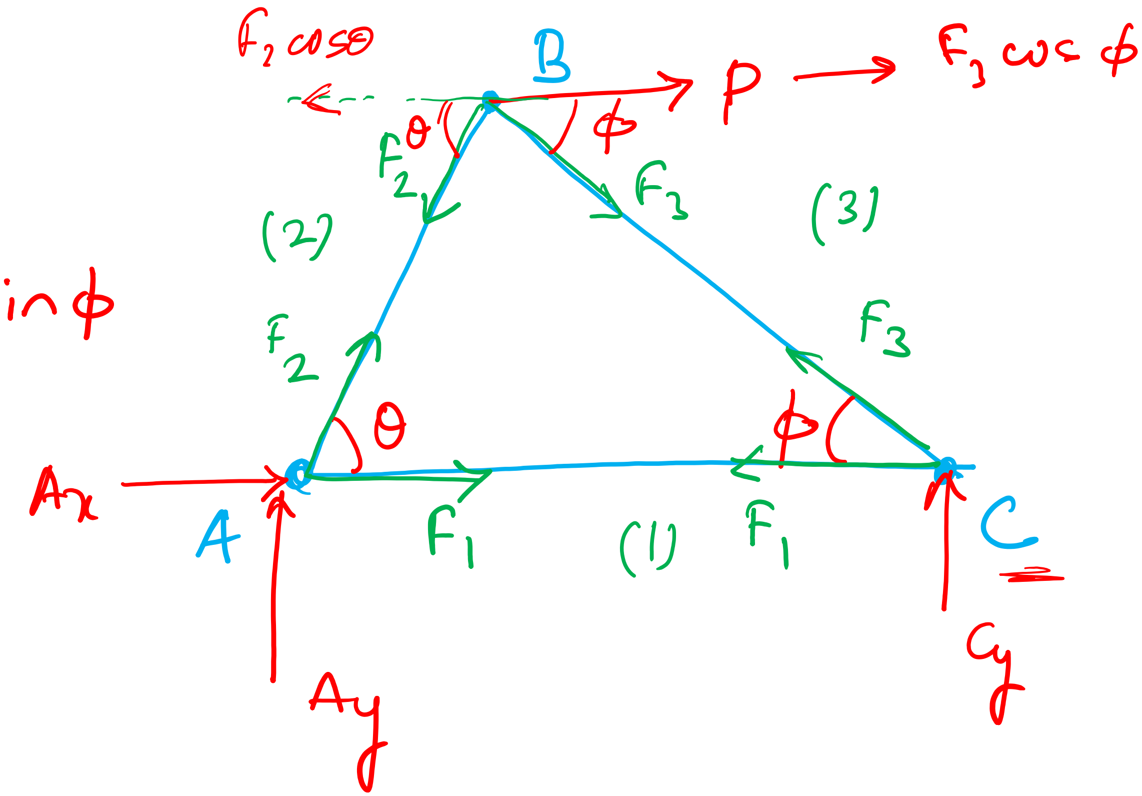
$= 53.13\ deg$

$\phi = \tan^{-1}\left(\frac{600}{750}\right)$

$\phi = 38.66\ deg$

y, x

equilibrium @ B



$$\Sigma F_y = -F_2 \sin\theta - F_3 \sin\phi = 0$$

$$F_2 = -F_3 \frac{\sin\phi}{\sin\theta}$$

$$\Sigma F_x = P - F_2 \cos\theta + F_3 \cos\phi = 0$$

$$F_3 = \frac{F_2 \cos\theta - P}{\cos\phi}$$

$$F_3 = \left[- \frac{P \sin\theta}{[\sin\phi \cos\theta + \cos\phi \cdot \sin\theta]} \right] = \frac{-P \sin\theta}{\sin(\theta+\phi)}$$

$$F_2 = \left[\frac{P \sin\phi}{\sin\phi \cdot \cos\theta + \cos\phi \cdot \sin\theta} \right] = \frac{P \sin\phi}{\sin(\theta+\phi)}$$

$$\boxed{F_2 = \frac{P\sin\phi}{\sin(\theta+\phi)}} \quad ; \quad \boxed{F_3 = \frac{-P\sin\theta}{\sin(\theta+\phi)}}$$

at joint c

$$\Sigma F_x = -F_1 - F_3\cos\phi = 0$$

$$F_1 = -F_3\cos\phi$$

$$\boxed{F_1 = \frac{P\sin\theta.\cos\phi}{\sin(\theta+\phi)}}$$

(1), (2) $\rightarrow$ tension $< (\sigma_T)_{allow} = 140\text{ MPa}$

(3) $\rightarrow$ compression $< \left|(\sigma_c)_{allow}\right| = 85\text{ MPa}$

$$\sigma_1 = \frac{F_1}{A_1} \Rightarrow A_1 = \frac{F_1}{\sigma_1} = \frac{F_1}{(\sigma_T)_{allow}}$$

$$A_1 = \left[\frac{P\sin\theta\cdot\cos\phi}{\sin(\theta+\phi)}\right]\frac{1}{(\sigma_T)_{allow}}$$

$$A_1 = \left[\frac{(50\,kN)\,[\sin(53.13)][\cos(38.66)]}{\sin(53.13 + 38.66)}\right]\left[\frac{1}{140\,MPa}\right]$$

$$A_2 = \frac{f_2}{(\sigma_T)_{allow}} = \left[\frac{P\sin\phi}{\sin(\theta+\phi)}\right]\left[\frac{1}{140\,MPa}\right]$$

$$A_2 = \left\{\frac{(50\,kN)\,[\sin(38.66)]}{\sin\{53.13 + 38.66)}\right]\left[\frac{1}{140\,MPa}\right]$$

$$A_3 = \frac{|f_3|}{|(\sigma_c)_{allow}|} = \frac{\left|-P\sin\theta/\sin(\theta+\phi)\right|}{85\,MPa}$$

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Example 4.3 from Lecturebook

If the pins have an allowable shear stress of $\tau_{allow} = 12.5\ ksi$ and the allowable tensile stress of rod CB is $\sigma_{allow} = 16.2\ ksi$ determine to the nearest 1/16 in. the smallest diameter of pins A and B and diameter of rod CB necessary. Use: $b = 2\ ft$ and $P = 3kip$. The pin connections at A and C are double-sided, whereas the pin connection at B is single-sided.

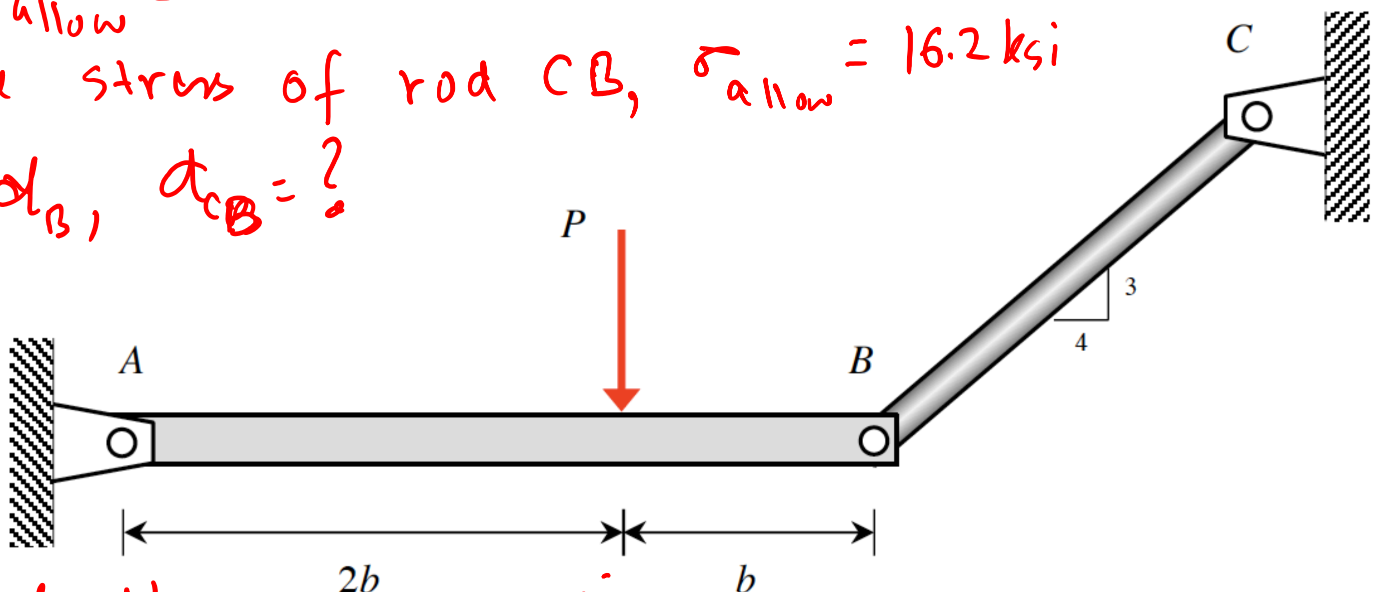
$\tau_{allow} = 12.5$ ksi

allowable stress of rod CB, $\sigma_{allow} = 16.2$ ksi

d_A, d_B, $d_{CB} = ?$

$b = 2$ ft

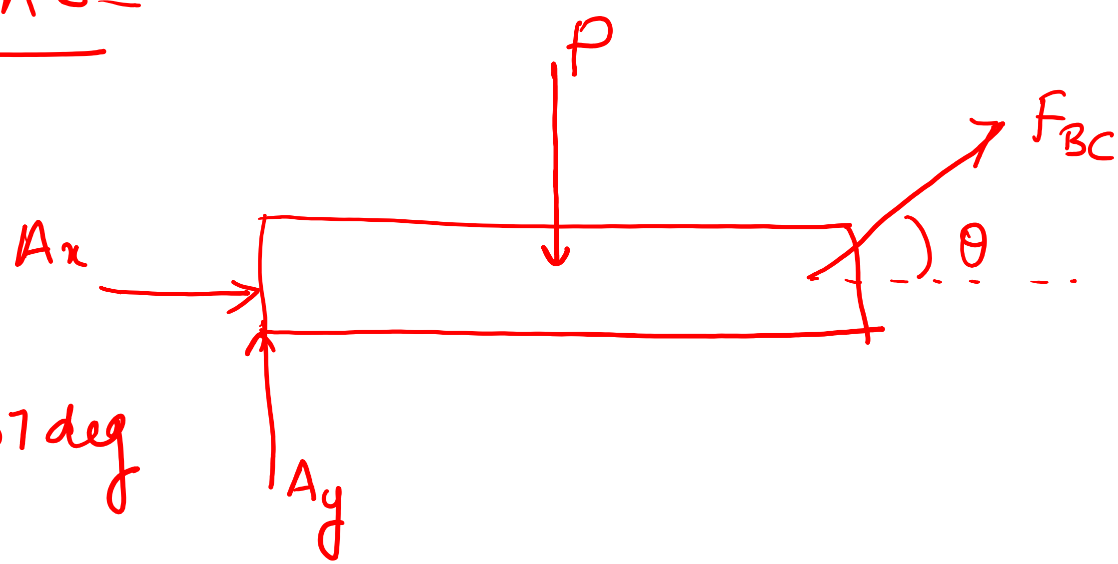
$P = 3$ kips



A, C → double-sided connections

B → single-sided

FBD of AB -



$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87 \text{ deg}$

equilibrium,

$$\left(\Sigma M\right)_A = (-P)(2b) + (F_{BC})(\sin\theta)(3b)$$

$$F_{BC} = \frac{2Pb}{(\sin\theta)(3b)} = \frac{2P}{3\sin\theta}$$

$$\left(\Sigma F\right)_x = A_x + F_{BC}\cos\theta = 0 \Rightarrow A_x = -F_{BC}\cos\theta$$

$$A_x = -\frac{2P\cos\theta}{3\sin\theta} = -\frac{2P}{3}\cot\theta$$

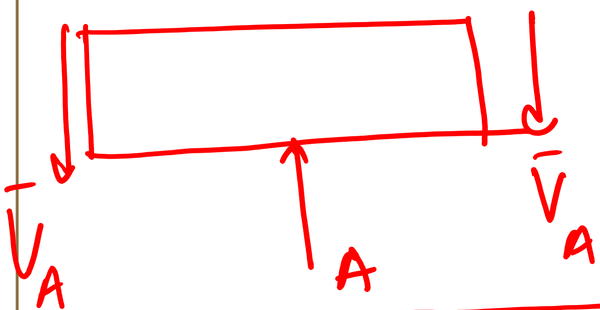
$$\boxed{A_x = -\frac{2}{3}P\cot\theta}$$

$$\Sigma F_y = A_y + F_{BC}\sin\theta - P = 0$$

$$A_y = P - F_{BC}\sin\theta = P - \frac{2}{3}P = P/3$$

$$\boxed{A_y = P/3}$$

analyze pin A $\rightarrow$ double-sided shear



$$= \sqrt{A_x^2 + A_y^2}$$

$$\Sigma F = -2\bar{V}_A + A = 0$$

$$\bar{V}_A = A/2$$

$$= \frac{1}{2}\sqrt{A_x^2 + A_y^2}$$

$$\bar{V}_A = \frac{1}{2}\left[\frac{4}{9}P^2(\cot\theta)^2 + \frac{P^2}{9}\right]^{1/2}$$

$$\boxed{\bar{V}_A = \frac{P}{6}\left[4(\cot\theta)^2 + 1\right]^{1/2}}$$

$$\tau_A = \frac{\bar{V}_A}{A} = \frac{\bar{V}_A}{\pi\left(\frac{d_A}{2}\right)^2} = \frac{4\bar{V}_A}{\pi d_A^2}$$

$$d_A = \left[\frac{4\bar{V}_A}{\pi \tau_A}\right]^{1/2}$$

$$\tau_A < \tau_{allow}$$

$$d_A > \left[\frac{4\bar{V}_A}{\pi \tau_{allow}}\right]^{1/2}$$

$\tan\theta = 3/4$

$\cot\theta = 4/3$

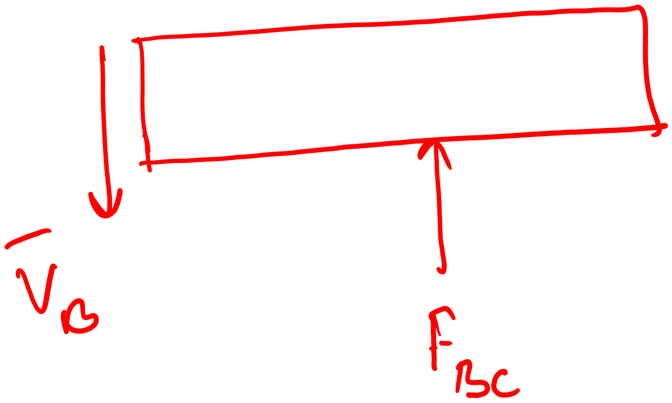
$$\bar{V}_A = \frac{P}{6}\left[4\left(\frac{4}{3}\right)^2 + 1\right]^{1/2} = \frac{P}{6}\left[\frac{64}{9}+1\right] = \frac{P}{18}\sqrt{73}$$

$$\bar{V}_A = 1.424 \text{ kips}$$

$$(d_A)_{min} = \left[\frac{4(1.424 \text{ kips})}{\pi(12.5 \text{ ksi})}\right]^{1/2} = 0.38 \text{ inch}$$

$$\boxed{(d_A)_{min} = 0.38 \text{ inch}}$$

Now lets look at pin B $\rightarrow$ single-sided



$$\sum F = -\bar{V}_B + F_{BC} = 0$$

$$\bar{V}_B = F_{BC}$$

$$\bar{V}_B = \frac{2P}{3\sin\theta}$$

$$\tau_B = \frac{\bar{V}_B}{\pi\left(d_B/2\right)^2} \Rightarrow \tau_B = \frac{4\bar{V}_B}{\pi d_B^2} = \frac{4F_{BC}}{\pi d_B^2}$$

$$\tau_B < \tau_{allow}$$

$$\Rightarrow d_B > \left[\frac{4F_{BC}}{\pi\,\tau_{allow}}\right]^{1/2}$$

$$(d_B)_{min} = \left[\frac{4\left(2P/3\sin\theta\right)}{\pi(12.5\text{ ksi})}\right]^{1/2} = \left[\frac{\frac{8}{3}\left(\frac{3\text{ kips}}{\sin(36.87)}\right)}{\pi(12.5\text{ ksi})}\right]^{1/2}$$

$$\boxed{(d_B)_{min} = 0.457 \text{ inches}}$$

Failure criteria for BC –

$$\sigma_{BC} = \frac{F_{BC}}{\pi\left(\frac{d_{BC}}{2}\right)^2} = \frac{4F_{BC}}{\pi d_{BC}^2} < \sigma_{allow}$$

$$d_{BC} > \left[\frac{4F_{BC}}{\pi \sigma_{allow}}\right]^{1/2}$$

$$(d_{BC})_{min} = \left[\frac{4\left(\frac{2P}{3\sin\theta}\right)}{\pi(16.2\text{ksi})}\right]^{1/2} = \left[\frac{4\left(\frac{(2)(3\text{kips})}{3\sin(36.87)}\right)}{\pi(16.2\text{ksi})}\right]^{1/2}$$

$$\boxed{(d_{BC})_{min} = 0.402 \text{ inches}}$$

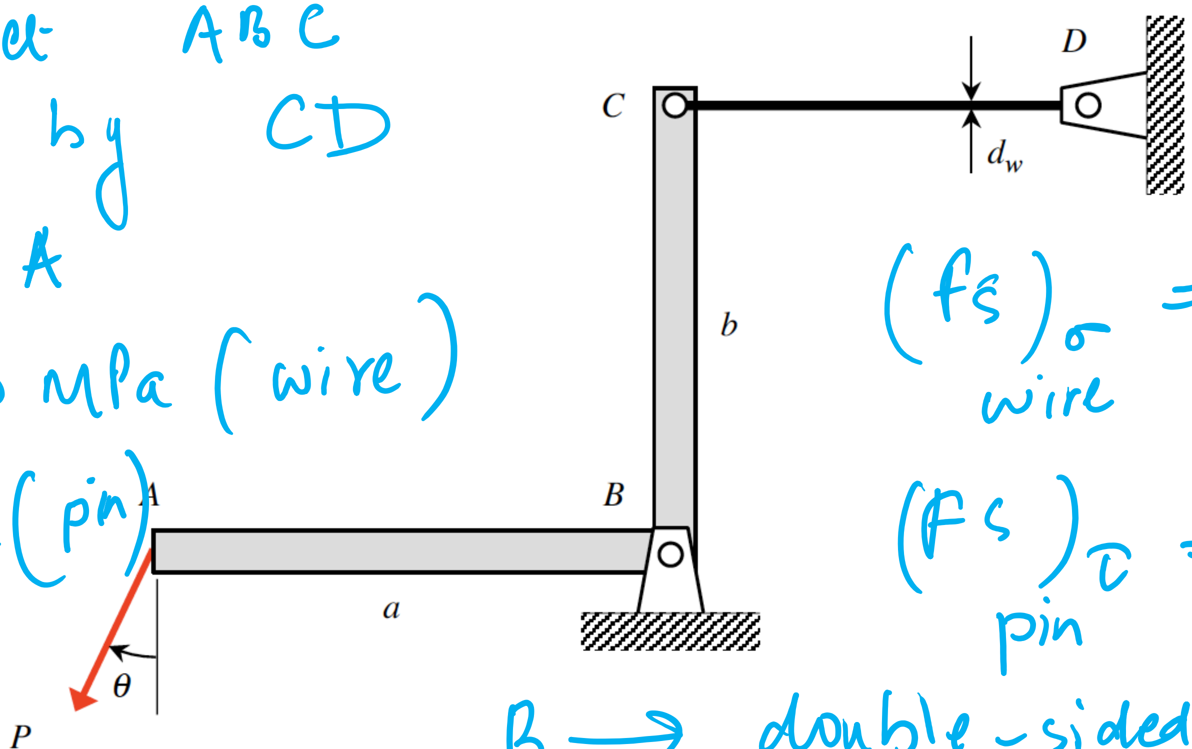
Lecture 4.5 from Lecturebook

An angle bracket ABC is restrained by a high-strength steel wire CD and supports a load P at A, as shown in the figure. The strength properties of the wire and shear pin at B are $\sigma_Y = 350\ MPa$ and $\tau_U = 300\ MPa$, respectively. If the wire and pin are to be sized to provide a factor of safety against yielding of the wire $FS_\sigma = 3.3$ and a factor of safety against ultimate shear failure of the pin of $FS_\tau = 3.5$, what are the required diameters of the wire and of the pin (give each to the nearest mm)? The pin connection at B is double-sided.

- angle bracket ABC
- restrained by CD
- load P @ A

$\sigma_y = 350$ MPa (wire)

$\tau_u = 300$ MPa (pin)



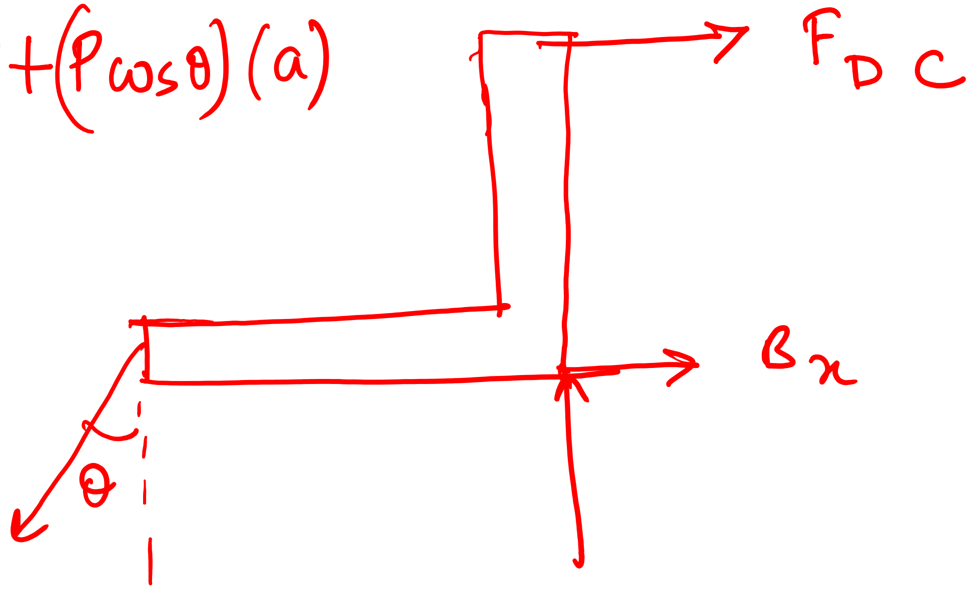
$(FS)_\sigma = 3.3$ wire

$(FS)_\tau = 3.5$ pin

B $\rightarrow$ double-sided shear

free body diagram of bracket ABC

$$(\Sigma M)_B = -(F_{DC})(b) + (P\cos\theta)(a) = 0$$



$$F_{DC} = \frac{a}{b} P \cos\theta$$

$$\Sigma F_x = -P\sin\theta + B_x + F_{DC} = 0$$

$$B_x = P\sin\theta - F_{DC} = P\left[\sin\theta - \frac{a}{b}\cos\theta\right]$$

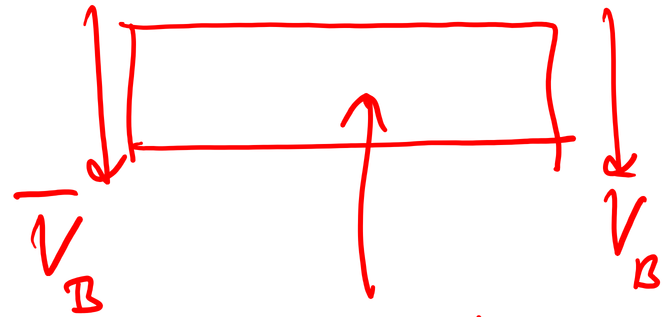
$$\Sigma F_y = -P\cos\theta + B_y = 0$$

$$B_y = P\cos\theta$$

$$B_x = P\left[\sin\theta - a/b\,\cos\theta\right]$$

$$F_{DC} = \frac{a}{b} P\cos\theta$$

for Pin B –



$$B = \sqrt{B_x^2 + B_y^2}$$

$$\sum F = -2\bar{V}_B + B = 0$$

$$\bar{V}_B = B/2$$

$$\bar{V}_B = \frac{1}{2}\left[B_x^2 + B_y^2\right]^{1/2}$$

$$\tau_B = \frac{\bar{V}_B}{A_B} = \frac{\bar{V}_B}{\pi\left(d_B/2\right)^2} = \frac{4\bar{V}_B}{\pi d_B^2} = \frac{\tau_u}{(FS)_{pin}}$$

$$d_B = \left[\frac{(4\bar{V}_B)(3.5)}{\pi(300\,MPa)}\right]^{1/2}$$

$$\bar{V}_B = \frac{1}{2}\left[P^2\left(\sin\theta - \frac{a}{b}\cos\theta\right)^2 + P^2\cos^2\theta\right]^{1/2}$$

$$d_B = \left[\frac{7P\left(1+\frac{a^2}{b^2}\cos^2\theta - \frac{2a}{b}\sin\theta\cdot\cos\theta\right)}{\pi(300\,MPa)} \right]^{1/2}$$

for wire -

$$\sigma_{DC} = \frac{F_{DC}}{\pi\left(d_w/2\right)^2} = \frac{4F_{DC}}{\pi d_w^2}$$

$$(FS_\sigma)_{wire} = \frac{\sigma_{allow}}{\sigma_{DC}} \Rightarrow \sigma_{DC} = \frac{350\,MPa}{3.3}$$

$$\frac{350\,MPa}{3.3} = \frac{4F_{DC}}{\pi d_w^2}$$

$$d_w = \left[\frac{(4F_{DC})(3.3)}{\pi(350\,MPa)} \right]^{1/2}$$

$$\boxed{d_w = \left[\frac{(4a/b)(P\cos\theta)(3.3)}{\pi(350\,MPa)}\right]^{1/2}}$$

leaving d_B and d_w in terms of θ (as it is unknown)

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THANK YOU