MECHANICS OF MATERIALS

Fall 2023

ME 323- 005

Instructor: Shubhra Bansal

Lecture 2: Normal Stress

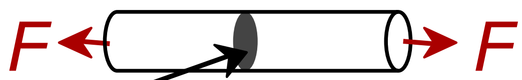
Extensional Strain

Material Properties

PURDUE UNIVERSITY | School of Mechanical Engineering

Common States of Stress

- **Simple tension: cable**

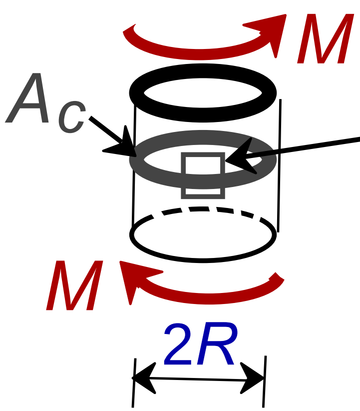


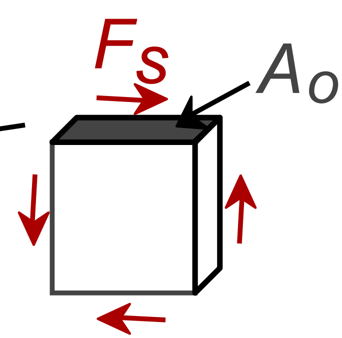
A_o = cross-sectional area (when unloaded)

$$\sigma = \frac{F}{A_o}$$

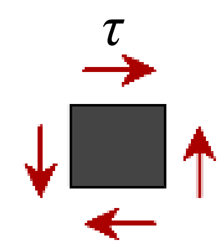
Ski lift (photo courtesy P.M. Anderson)

- **Torsion (a form of shear): drive shaft**





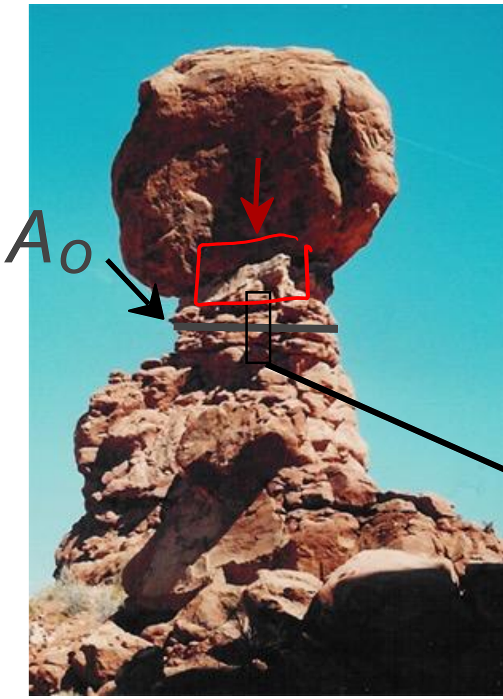
$$\tau = \frac{F_s}{A_o}$$



Note: $\tau = M/A_cR$ here.

Common States of Stress

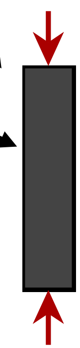
- **Simple compression:**



Balanced Rock, Arches National Park
(photo courtesy P.M. Anderson)

Canyon Bridge, Los Alamos, NM
(photo courtesy P.M. Anderson)

$$\sigma = \frac{F}{A_o}$$



Note: compressive structure member ($\sigma < 0$ here).

N/m^2

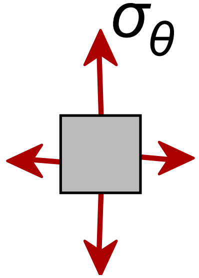
Pa, MPa (10^6), GPa (10^9)

Common States of Stress

- **Bi-axial tension:**

Pressurized tank
(photo courtesy P.M. Anderson)

$\sigma_\theta > 0$

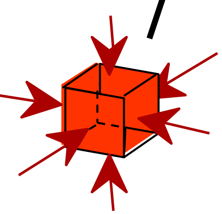


$\sigma_z > 0$

- **Hydrostatic compression:**

Fish under water

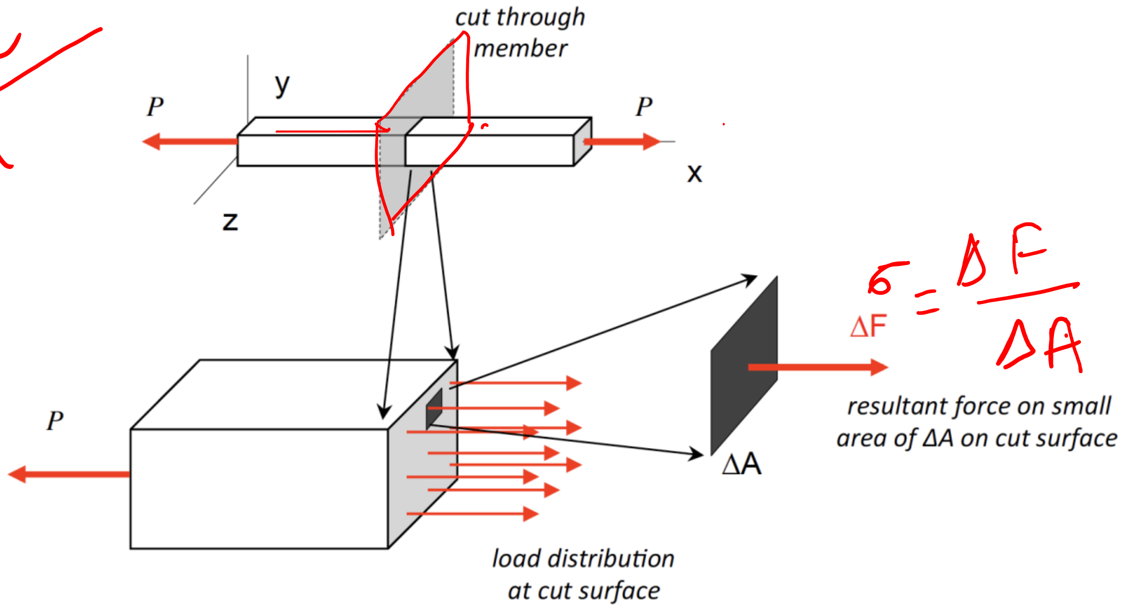
(photo courtesy P.M. Anderson)



$\sigma_h < 0$

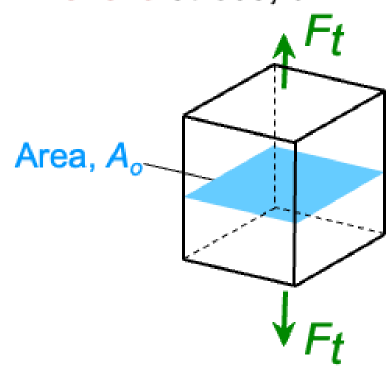
Normal or Axial Stress

Force / area



$\sigma = \frac{\Delta F}{\Delta A}$

- Tensile stress, σ:



$$\sigma = \frac{F_t}{A_o} = \frac{lb_f}{in^2} \text{ or } \frac{N}{m^2}$$

original cross-sectional area before loading

$$\sigma_x = normal\ stress = \lim_{\Delta A \to 0}\left(\frac{\Delta F}{\Delta A}\right) = \frac{dF}{dA} \quad \Rightarrow \quad dF = \sigma_x dA$$

$$\sigma_{ave} = \frac{1}{A}\int_{area} \frac{dF}{dA} \cdot dA$$

Average normal stress $\quad (\sigma_x)_{ave} = \frac{1}{A}\int_{area} \sigma_x\, dA$

$$(\sigma_x)_{ave} = \frac{P}{A}$$

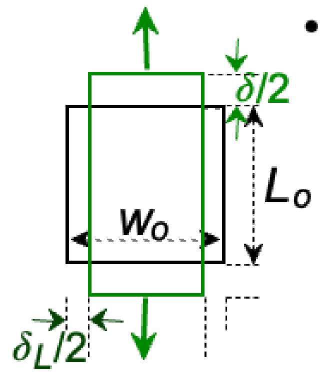
- Assuming axial load is at the centroidal position ✓
- Material is homogeneous and isotropic ✓
- Tensile (+) and compressive (-)

Axial Strain

- **Tensile strain:**

Axial

$$e = \frac{\delta}{L_o}$$

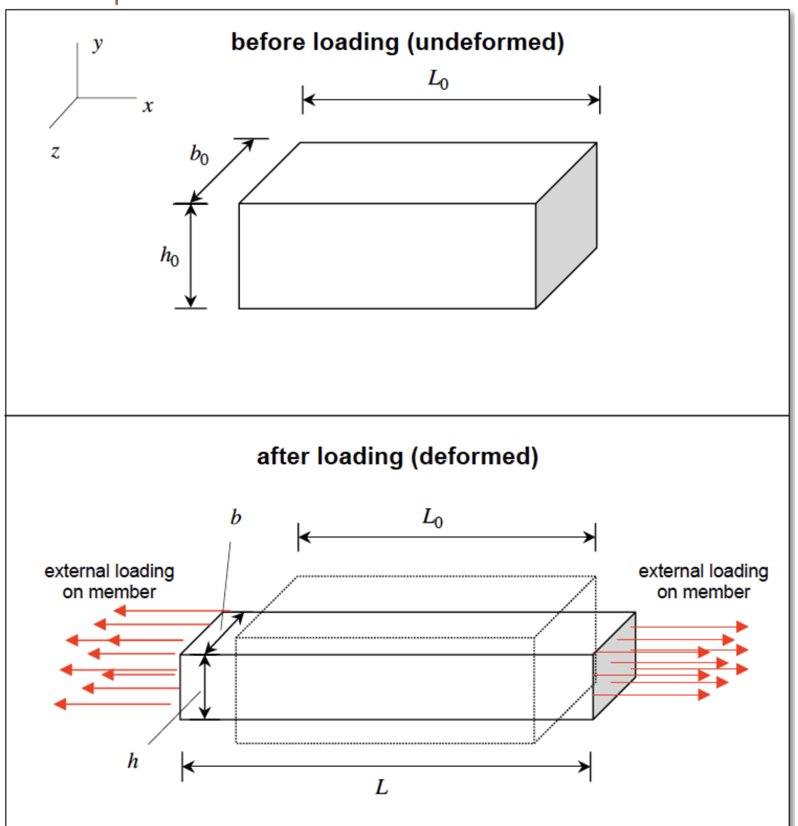


- **Lateral strain:**

$$e_L = \frac{-\delta_L}{w_o}$$

$$\nu = \frac{\text{lateral}}{\text{axial}}$$

$$\text{strain} = \frac{\text{change in dimension}}{\text{original dimension}} = \frac{\Delta L}{L_0} \text{ or } \frac{\Delta d}{d_0}$$

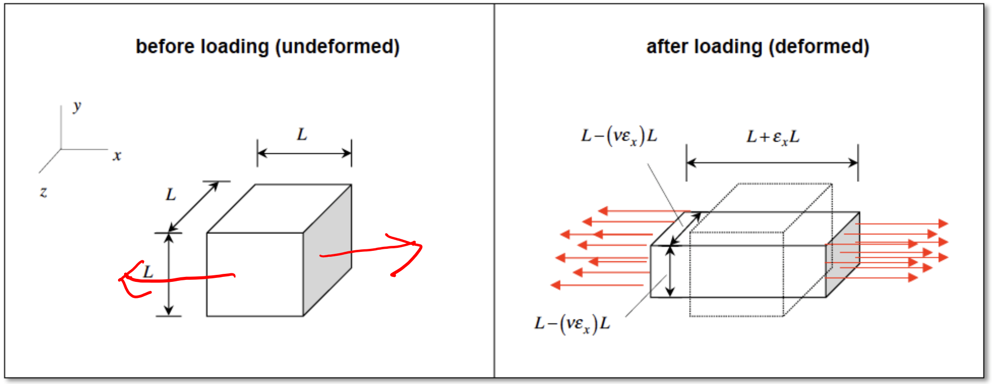


$$\varepsilon_x = \text{strain in } x \text{ (axial) direction} = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0} \quad \text{(elongation in x-direction)}$$

$$\varepsilon_y = \text{strain in } y \text{ direction} = \frac{h - h_0}{h_0} = \frac{\Delta h}{h_0} \quad \text{(contraction in y-direction)}$$

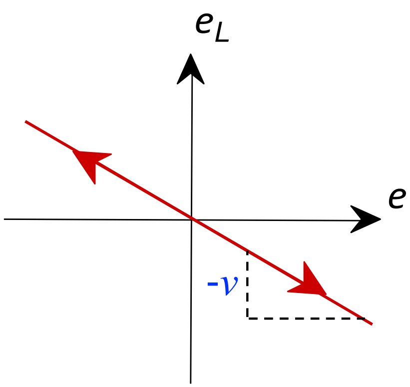
$$\varepsilon_z = \text{strain in } z \text{ direction} = \frac{b - b_0}{b_0} = \frac{\Delta b}{b_0} \quad \text{(contraction in z-direction)}$$

Poisson's Ratio (ν)



$$\nu = -\frac{e_L}{e}$$

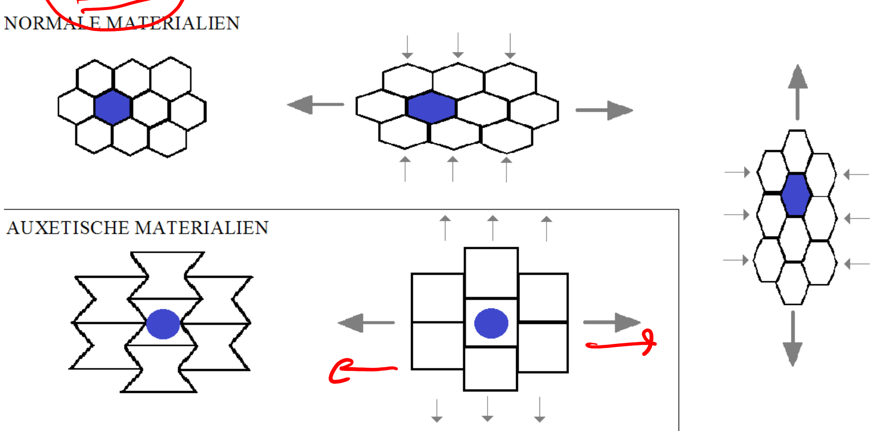
metals: ν ~ 0.33
ceramics: ν ~ 0.25
polymers: ν ~ 0.40



Material	Poisson's ratio
rubber	0.4999 [5]
gold	0.42–0.44
saturated clay	0.40–0.49
magnesium	0.252-0.289
titanium	0.265-0.34
copper	0.33
aluminium-alloy	0.32
clay	0.30–0.45
stainless steel	0.30–0.31
steel	0.27–0.30
cast iron	0.21–0.26
sand	0.20–0.45
concrete	0.1-0.2
glass	0.18–0.3
foam	0.10–0.50
cork	0.0

Negative Poisson's ratio

Auxetic Materials- negative Poisson's Ratio

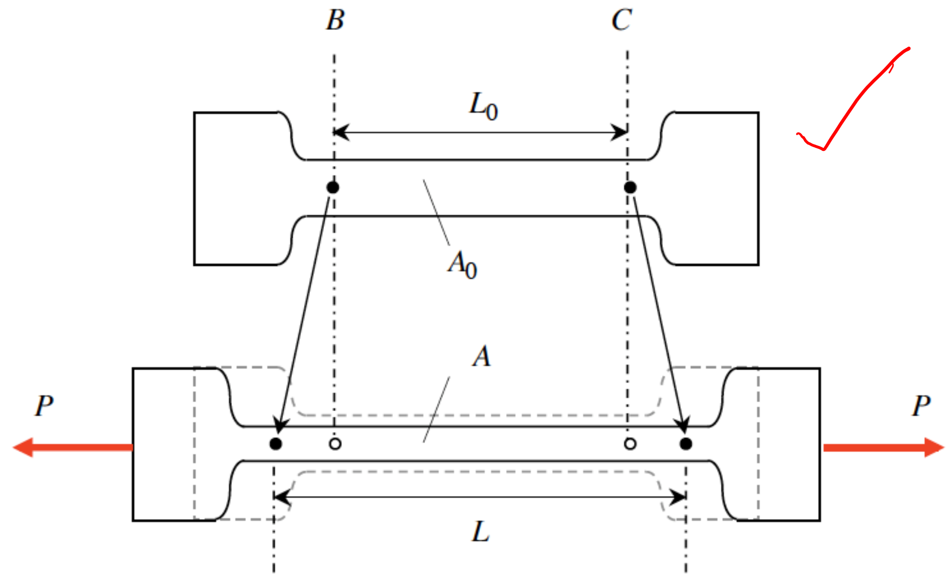


In footwear, auxetic design allows the sole to expand in size while walking or running, thereby increasing flexibility.

Mechanical Properties of Materials

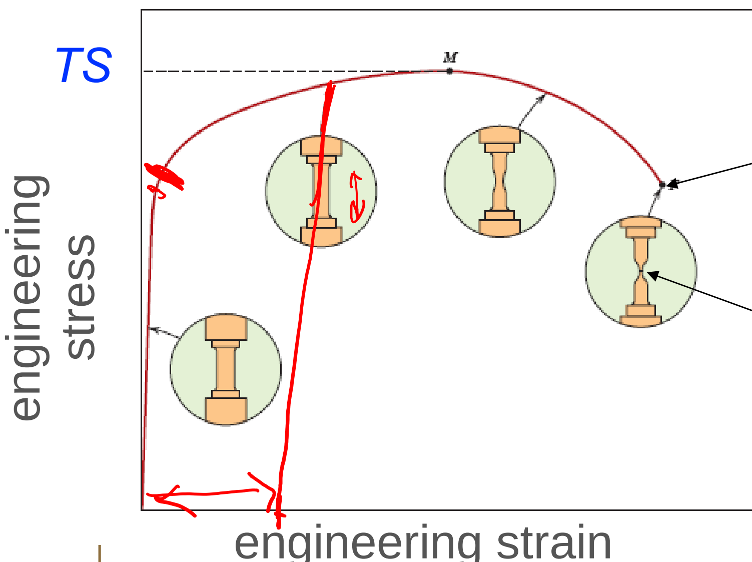
$$\sigma = \frac{P}{A_0}$$

$$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$



Adapted from Fig. 6.11, *Callister & Rethwisch 9e.*

necking → cross-section area decreases



F = fracture or ultimate strength

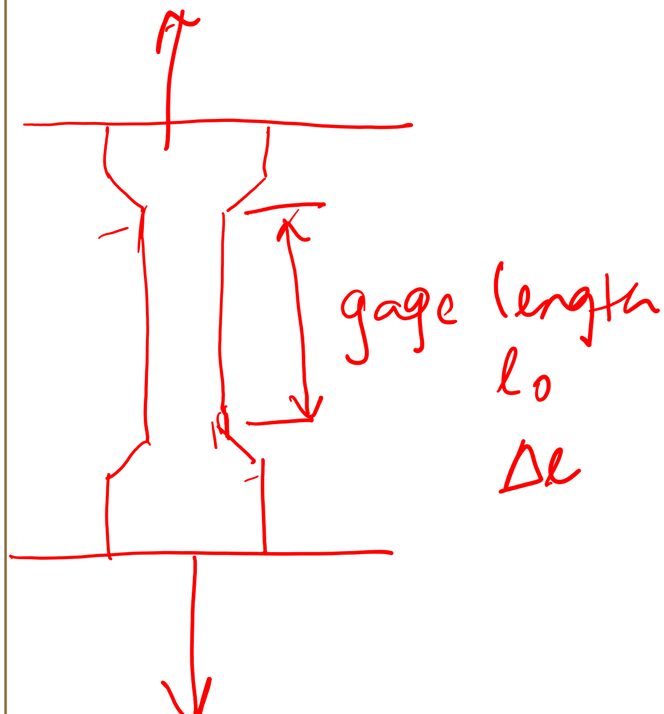
Neck – acts as stress concentrator

- Metals: occurs when noticeable necking starts.
- Polymers: occurs when polymer backbone chains are aligned and about to break.

Material Properties

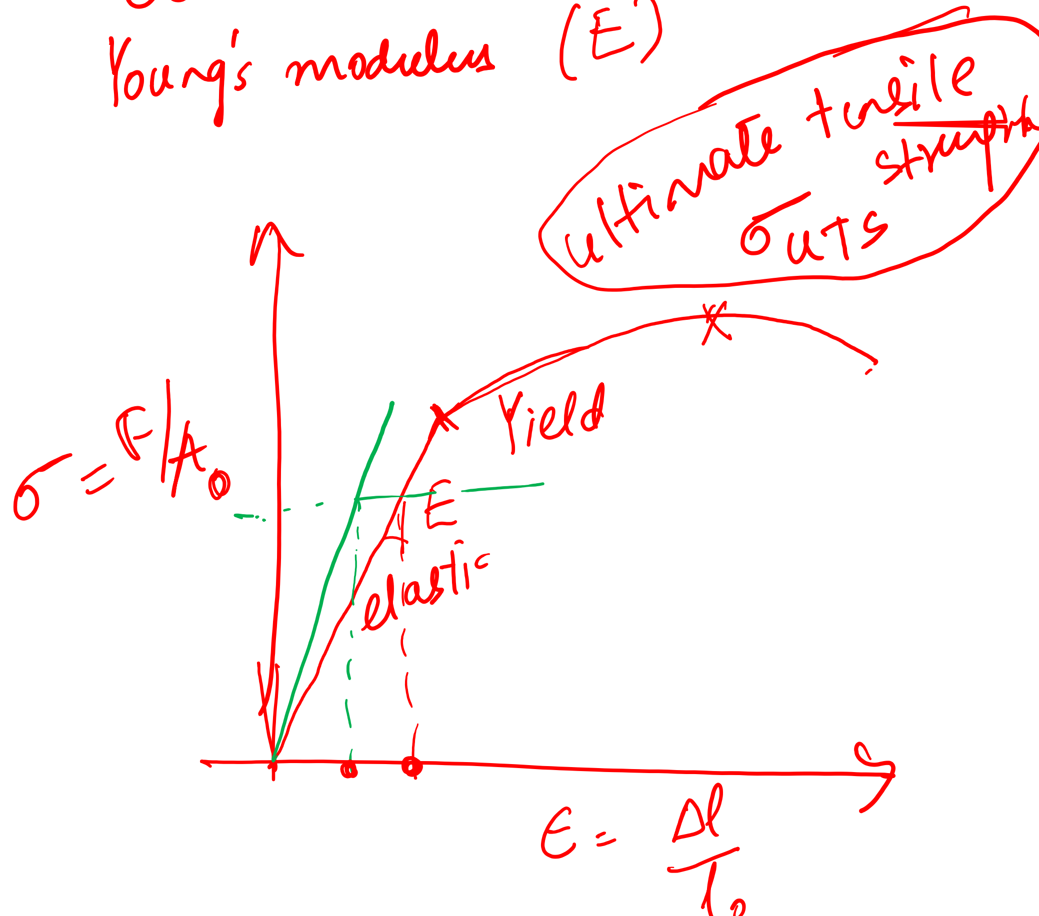
mechanical

- elastic → reversible
- plastic → permanent



elastic modulus → stiffness

Young's modulus (E)



$$\sigma = E\,\epsilon$$

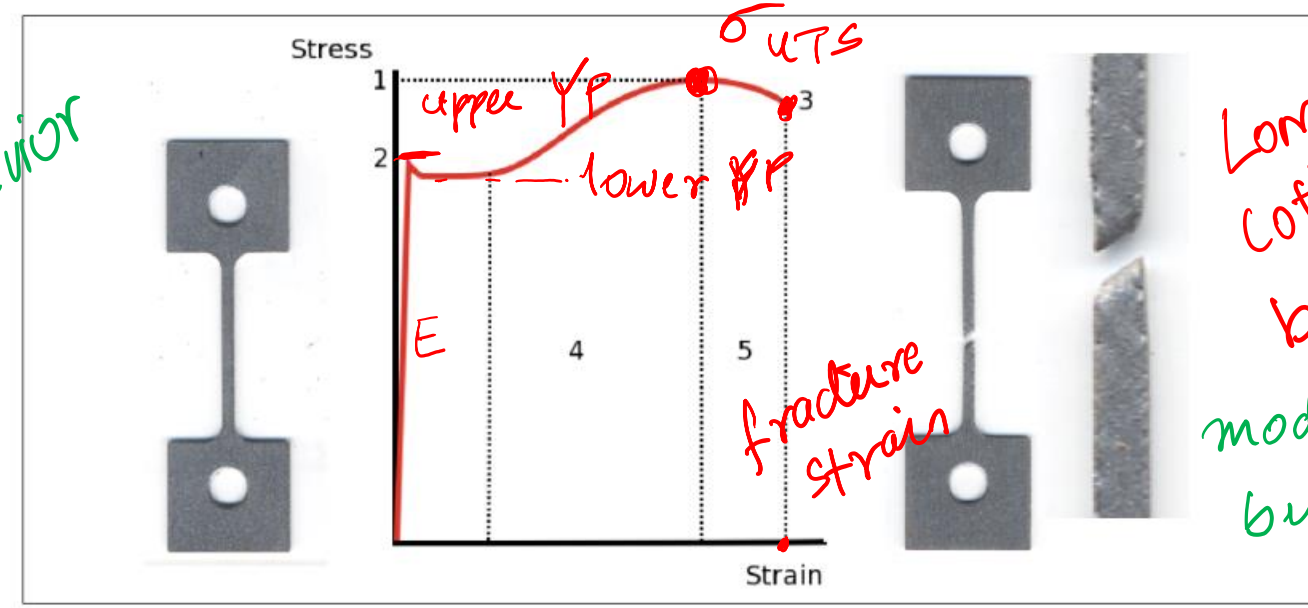
Hooke's Law

Stress-Strain Curves

elastic-plastic behavior

Steel

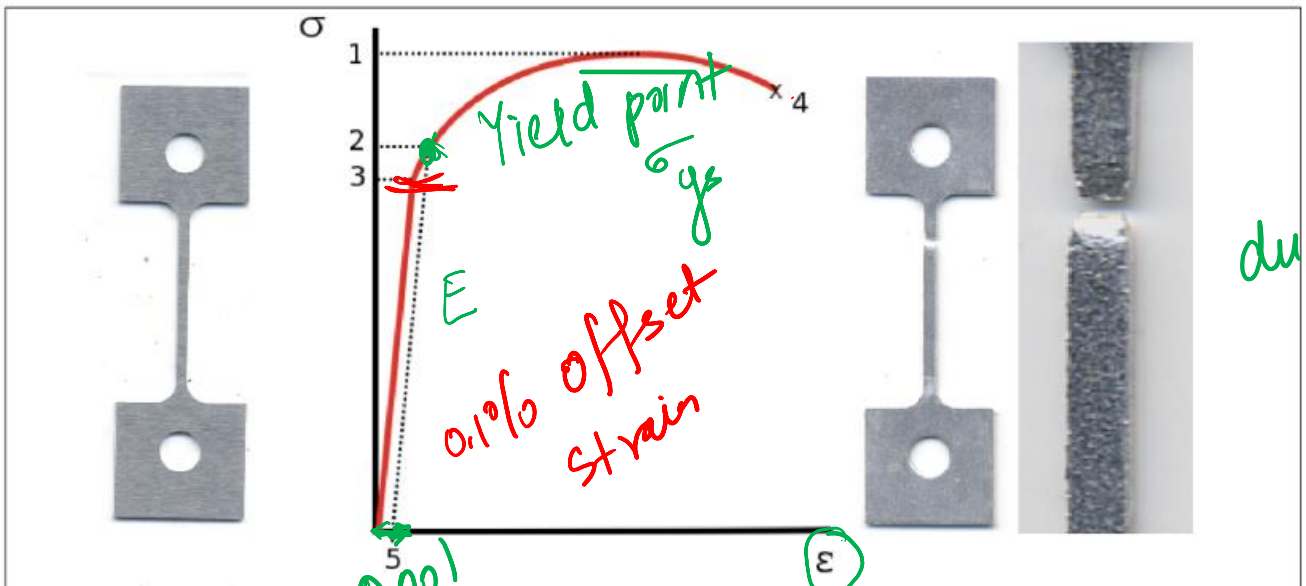
Fe→BCC alloy



Lomer Cottrell behavior

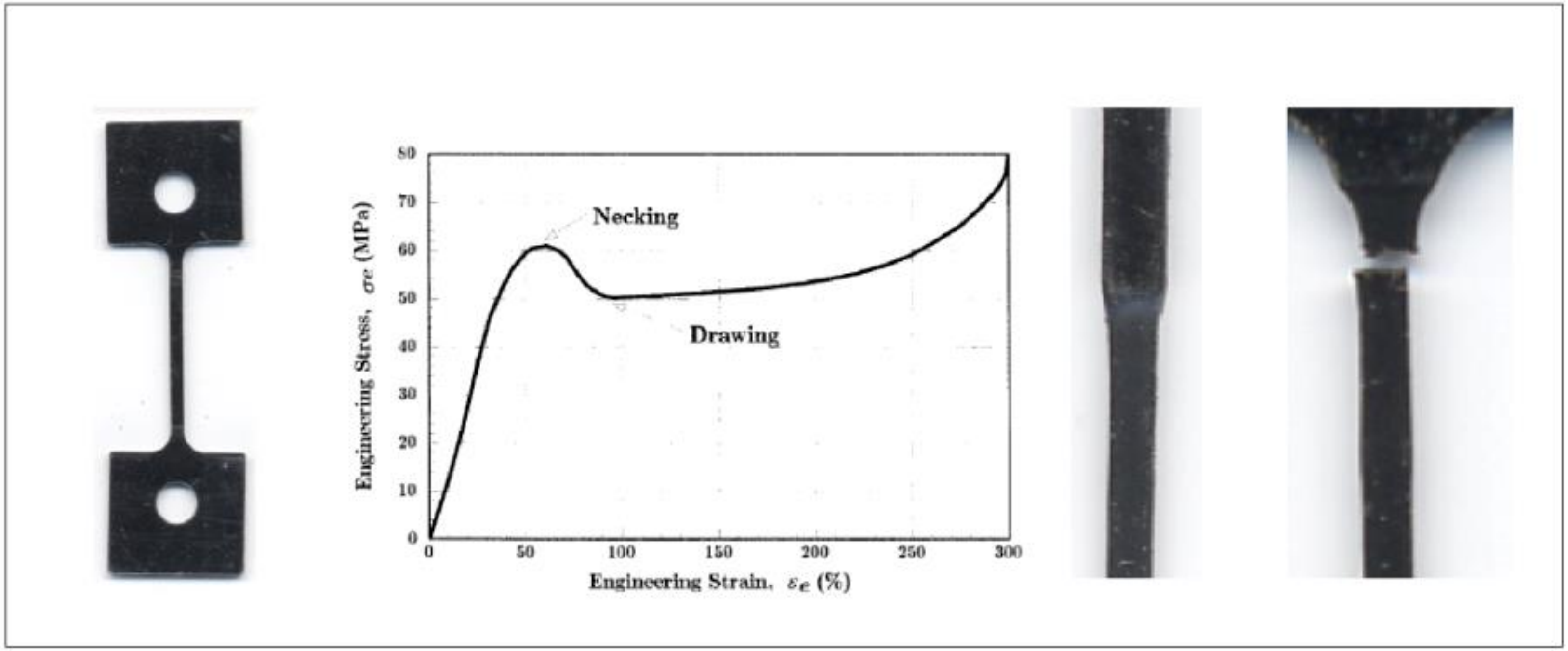
moderately brittle

Aluminum



ductile

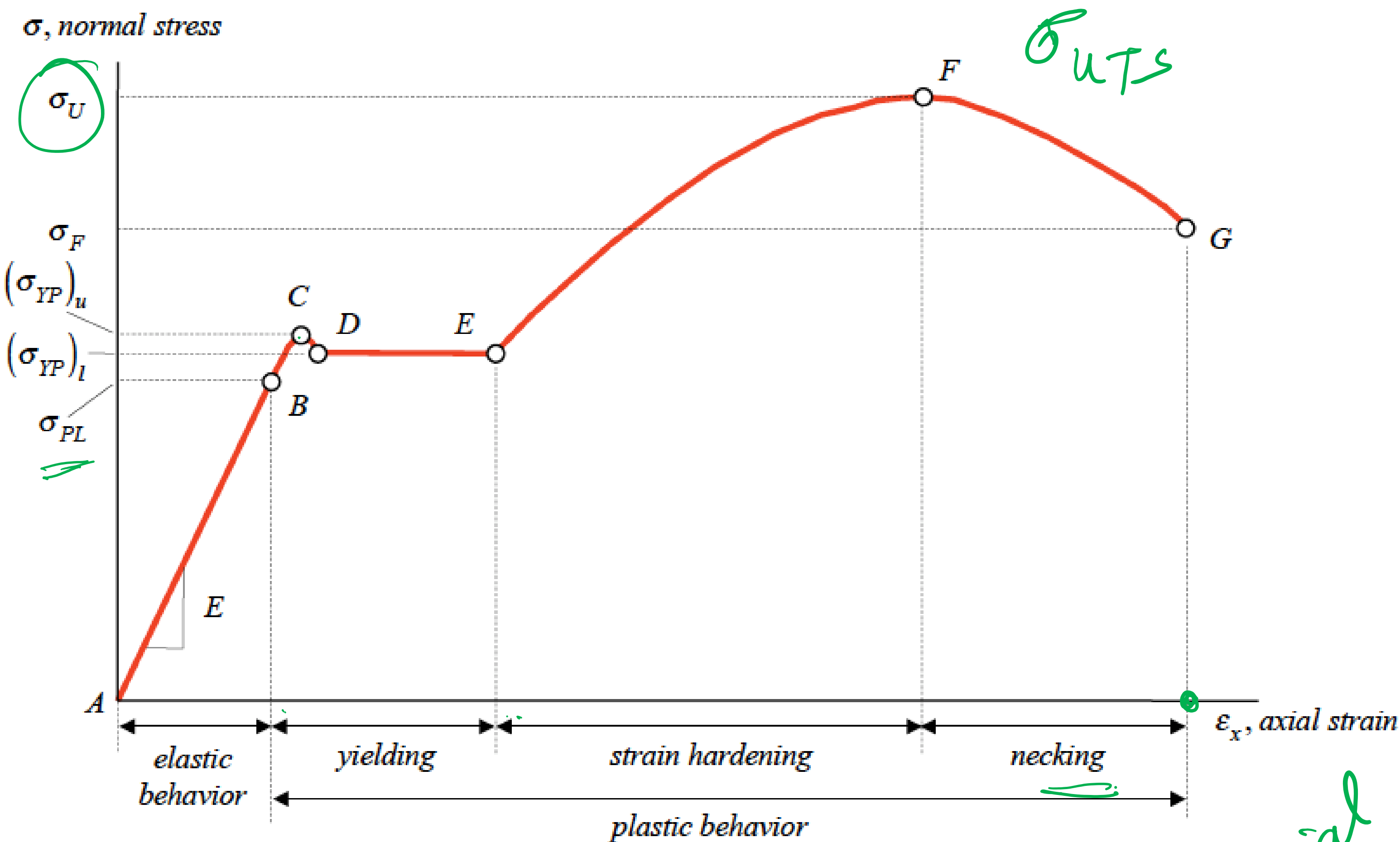
Stress-Strain Curves



Nylon → visco elastic behavior

Figures provided courtesy of Professor Thomas Siegmund, Purdue University

Stress-Strain Curves (Steel)



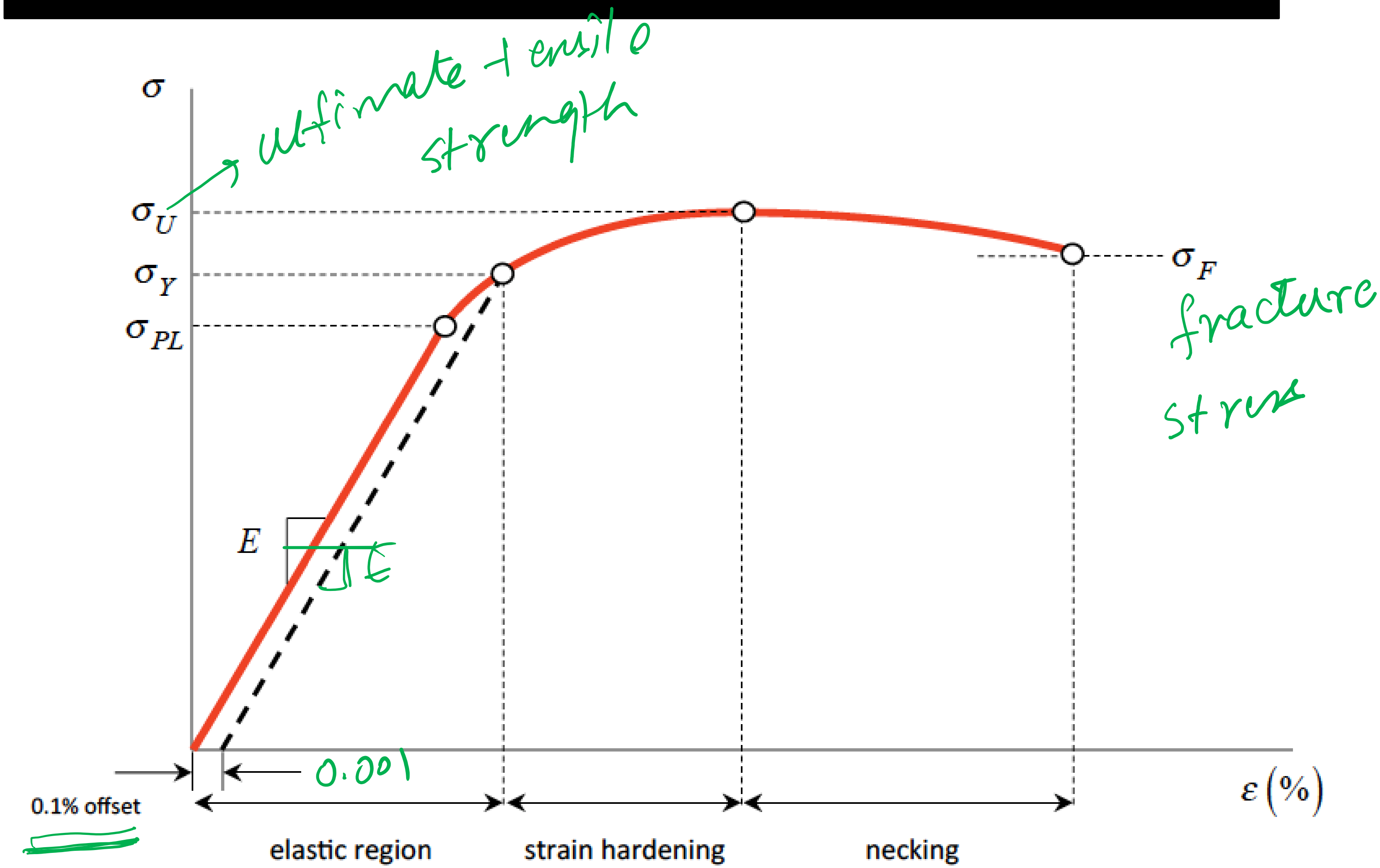
$(\sigma_{YP})_u$ = upper yield point

$(\sigma_{YP})_l$ = lower yield point

(σ_{PL}) = proportionality limit → material starts to soften

Figures provided courtesy of Professor Thomas Siegmund, Purdue University

Stress-Strain Curves (Aluminum)

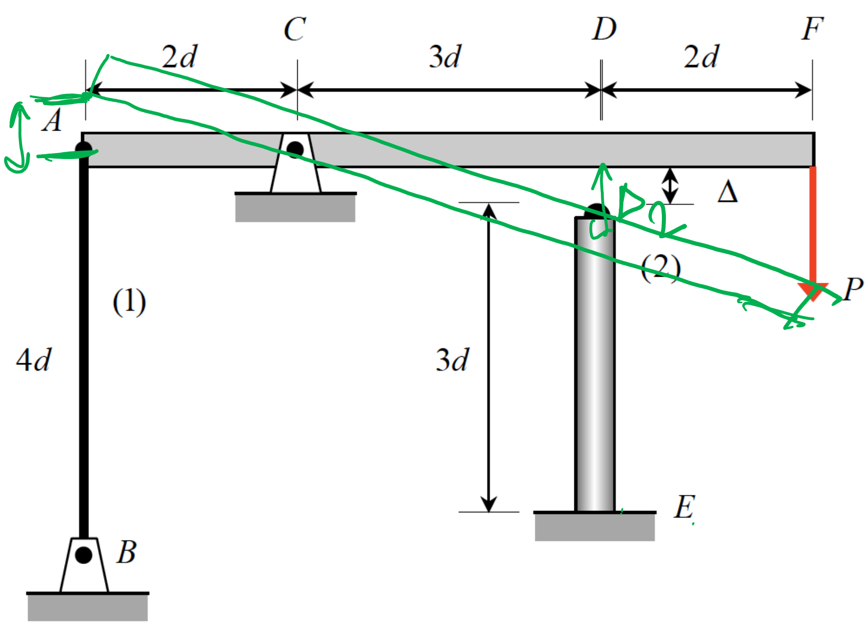


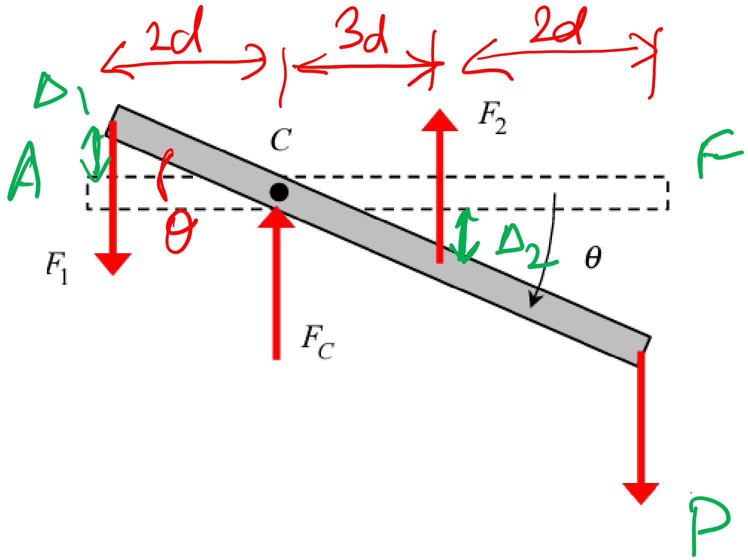
Example 2.2 from Lecture Book

For small loads P, the rotation of the rigid beam AF is controlled by the stretching of rod AB. For larger loads, the beam comes into contact with the top of column DE, and further resistance to rotation is shared by the rod and the column. Assume that the clockwise angle θ through which beam AF rotates is small enough to assume that points on the beam essentially move vertically. The cross-sectional areas of members (1) and (2) are A_1 and A_2, respectively, and the materials of members (1) and (2) have Young's modulus of E_1 and E_2, respectively.

a) A load P is applied that is just sufficient to close the Δ gap between the beam and the column. What is the strain ε_1 in rod AB for this value of P?

b) If the load P in a) is doubled, what is the corresponding strain ε_2 in column DE?





$$\tan\theta = \frac{\Delta_1}{2d}$$

$$\tan\theta = \frac{\Delta_2}{3d}$$

$$\frac{\Delta_1}{2d} = \frac{\Delta_2}{3d} \Rightarrow \Delta_1 = \frac{2}{3}(\Delta_2)$$

(a) $$\varepsilon_1 = \frac{\Delta_1}{4d} = \frac{2/3\ \Delta}{4d} = \frac{\Delta}{6d}$$

$P_a = ?$

under equilibrium $(\Sigma M)_c = 0$

$$F_1(2d) + F_2(3d) - P(5d) = 0$$

$$5P = 2F_1 + 3F_2$$

for (a) member (2) is not compressed $\Rightarrow F_2 = 0$

$$5P_a = 2F_1 = 2\sigma_1 A_1 \Rightarrow P_a = \frac{2}{5}\sigma_1 A_1$$

$$P_a = \frac{2}{5} E_1 \varepsilon_1 A_1 = \frac{2}{5} E_1 \left(\frac{\Delta}{6d}\right) A_1$$

$$\boxed{P_a = \frac{1}{15d}\left(E_1 \Delta A_1\right)}$$

$$\boxed{\varepsilon_1 = \frac{\Delta}{6d}}$$

(b) $\varepsilon_2 = ?$

$$P_b = 2P = \frac{2 E_1 \Delta A_1}{15d}$$

$$5P = 2F_1 + 3F_2$$

$$5\left[\frac{2E_1 \Delta A_1}{15d}\right] = 2E_1 A_1 \varepsilon_1 + 3\left(E_2 A_2 \varepsilon_2\right)$$

$$\varepsilon_2 = \Delta_2 - \Delta$$

$$\boxed{P_b = \frac{2 E_1 A_1 \Delta}{15d}}$$

$$\boxed{\varepsilon_2 = \frac{E_1 A_1 \Delta}{18d\, E_2 A_2}}$$

Example 2.4 from Lecture Book

A cylindrical rod having an initial diameter of d_0 and initial length L_0 is made of 6061-T6 aluminum alloy. When a tensile load P is applied to the rod, its diameter is decreased by Δd .

a) Determine the magnitude of the load P.

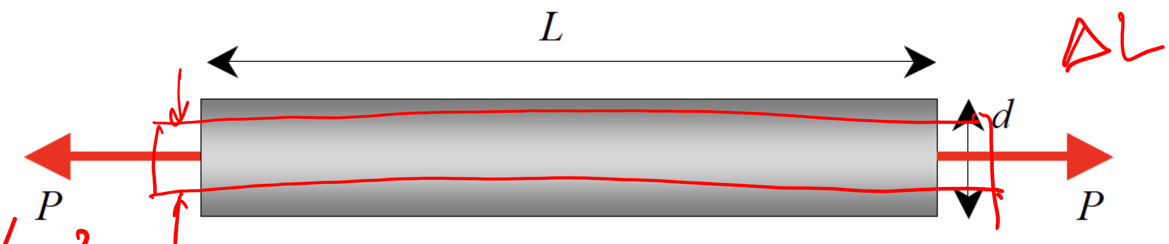
b) Determine the elongation of the rod over the length of the rod.

$d_0 = 8$ mm

$L_0 = 200$ mm

$E = 27 \times 10^9$ N/m^2

$\Delta d = 0.0101$ mm



Material	Young's modulus, E		Poisson's ratio, ν	Yield strength, σ_{YP}		Ultimate strength, σ_U	
	10^3 ksi	GPa		10^3 ksi	GPa	10^3 ksi	GPa
Aluminum alloy 2014-T6	10.6	73	0.33	60	410	70	480
Aluminum alloy 6061-T6	10.0	70	0.33	40	275	45	310
Brass, cold-rolled	15	100	0.34	60	410	75	520
Brass, annealed	15	100	0.34	15	100	40	275
Cast iron, gray	10	70	0.22	-	-	25	170
Steel, ASTM-A36 structural	29	200	0.29	36	250	58	400
Steel, AISI 302 stainless	29	195	0.30	75	520	125	860
Titanium, alloy	16.5	115	0.33	120	830	130	900
Wood, Douglas Fir	1.75	12	-	-	-	7.5	60
Wood, Southern Pine	1.75	12	-	-	-	8.5	60

d_0 = initial diameter

d_1 = final diameter

$\Delta d = d_1 - d_0$

$\nu = 0.33$

$\nu = -\frac{\epsilon_y}{\epsilon_x}$

$$\epsilon_y = \frac{\Delta d}{d_0} = \frac{-0.0101\,mm}{8\,mm} = \frac{-0.0101}{8}$$

$$\epsilon_x = -\epsilon_y/\nu = +\frac{0.0101/8}{0.33} = 0.00382$$

$$\sigma_x = E(\epsilon_x) = (27 \times 10^9 N/m^2)(0.00382)$$

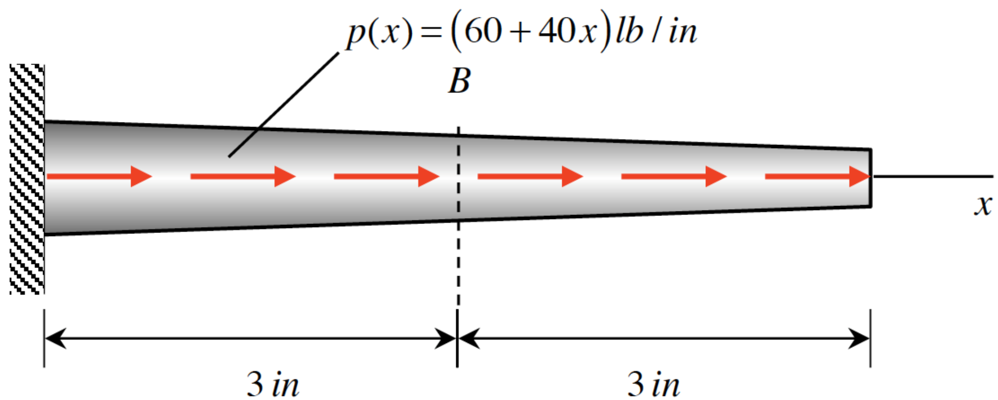
$$\sigma = 0.10314 \times 10^9 N/m^2$$

$$P = \sigma A = \sigma\left[\pi\left(\frac{d_0}{2}\right)^2\right]$$

$$P = 5182\,N$$

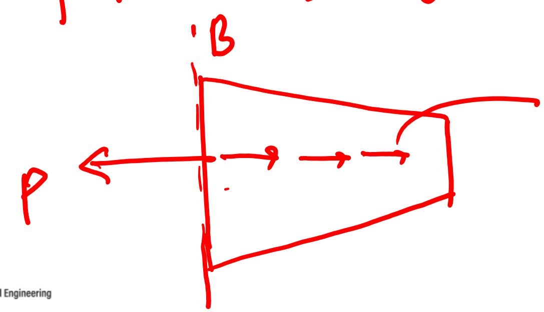
Example 2.6 from Lecture Book

The tapered rod has a radius of r=(2-x/6) in. and is subjected to the distributed loading of p = (60+40x) lb/in. Determine the average normal stress at the center of the rod, B.



$$r(x) = cross - section\ radius = \left(2 - \frac{x}{6}\right) in$$

Let's first section at B →



$p(x) = (60 + 40x)\ lb/in$

at equilibrium $\Sigma F_x = 0$

$$-P + \int_{3\,in}^{6\,in} \left[(60 + 40x)\frac{lb}{in} \right] dx = 0$$

$$P = 60x \Big|_3^6 + \frac{40x^2}{2} \Big|_3^6 = 60(6-3) + 20\left[6^2 - 3^2\right]$$

$$= 180 + (20)(36-9)$$

$$= 180 + 540$$

$$\boxed{P = 720\,lb}$$

$$A = \pi r^2 = \pi\left[2 - \frac{x}{6}\right]^2 = \pi\left[2 - \frac{3}{6}\right]^2 = 7.07\,in^2$$

$$\sigma_{ave} = \frac{P}{A} = \frac{720\,lb}{7.07\,in^2} = 102\,psi$$

$$\boxed{\sigma_{ave} = 102\,psi}$$

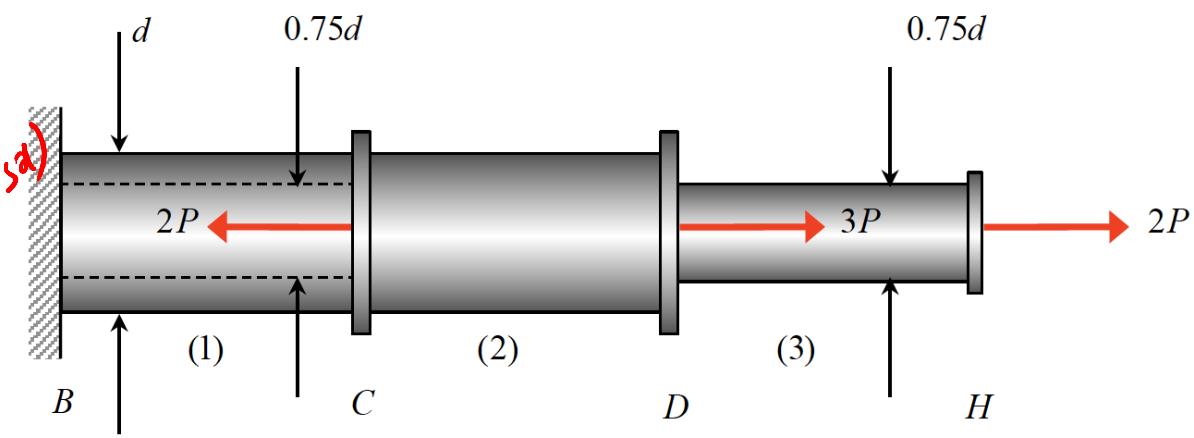
Example 2.7 from Lecture Book

The three-segment axially-loaded member shown below is made up of a tubular segment (1) with an outer diameter of d and inner diameter of 0.75d , a solid segment (2) of outer diameter of d and another solid segment (3) of outer diameter of 0.75d . A set of axial loads are applied at C, D and H. Determine the axial stresses in the three segments.

(1) tubular

(2) solid (d)

(3) Solid (0.75d)



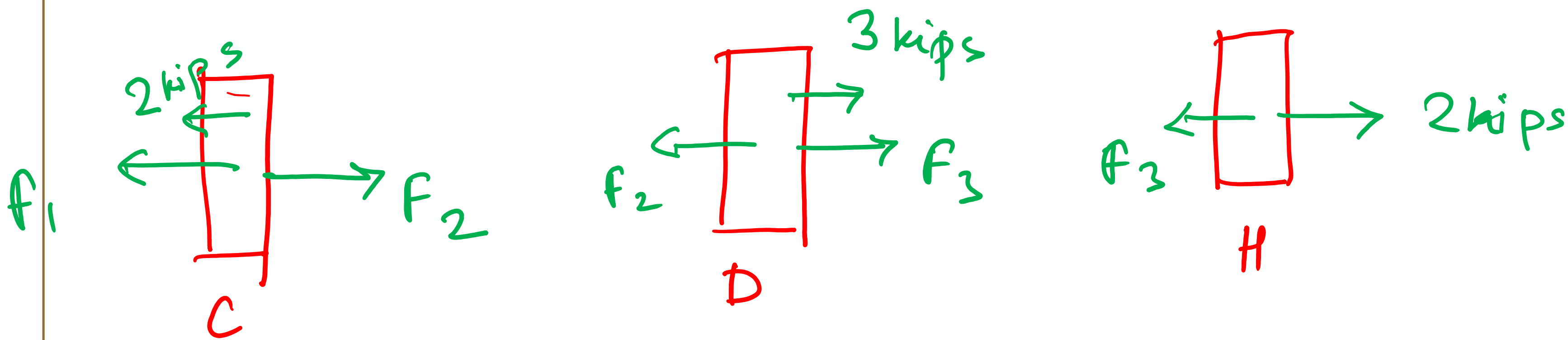
outer diameter = $d = d_o = 1$ inch

$P = 1$ kips

interior dia = $d_{int} = 0.7$ inch

Axial ~~loads~~ at C, B, H are given
find axial stress for each segment
(1), (2), (3)

1. free body diagram of connectors
2. Internal resultants = average load in each segment.



at equilibrium $\quad \sum F_x = 0$

@ H: $\quad -F_3 + 2\text{ kips} = 0 \quad \Rightarrow \quad F_3 = 2\text{ kips}$

@ D $\quad F_2 = 3 + F_3 = 5$ kips

@ C $\quad F_1 + 2 = F_2$

$$F_1 = 3 \text{ kips}$$

now we know loads in each section. And they are all tensile.

$$\sigma_1 = \frac{F_1}{A_1} = \frac{3\text{kips}}{\pi\left(\frac{d_o}{2}\right)^2 - \pi\left(\frac{d_i}{2}\right)^2}$$

$$= \frac{3\text{kips}}{\pi\left(\frac{1}{2}\right)^2 - \pi\left(\frac{0.75}{2}\right)^2}$$

$$\boxed{\sigma_1 = 8.735 \text{ kips/in}^2}$$

$$\sigma_2 = F_2/A_2 = \frac{5\ \text{kips}}{\pi\left(\frac{d_0}{2}\right)^2} = \frac{5\ \text{kips}}{\pi\left(\frac{1}{2}\right)^2}$$

$$\boxed{\sigma_2 = 6.37\ \text{kips/in}^2}$$

$$\sigma_3 = \frac{F_3}{A_3} = \frac{2\ \text{kips}}{\pi\left(\frac{0.75}{2}\right)^2} = 4.53\ \text{kips/in}^2$$

$$\boxed{\sigma_3 = 4.53\ \text{kips/in}^2}$$

THANK YOU