MECHANICS OF MATERIALS

Fall 2023

ME 323- 005

Instructor: Shubhra Bansal

Lecture 1: Introduction

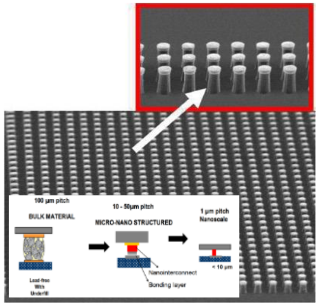
Static Equilibrium

PURDUE UNIVERSITY | School of Mechanical Engineering

About Me

भारतीय प्रौद्योगिकी संस्थान रूड़की
Indian Institute of Technology Roorkee

B. Tech in Metallurgical & Materials Engineering



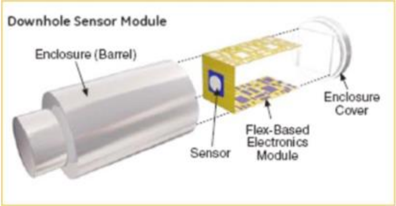
Compliant off-chip Cu pillar interconnects

Georgia Tech

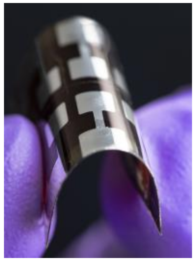
Ph.D. Materials Science and Engineering

GE Research

Lead Materials Scientist



Flexible packaging for sensors > 250°C

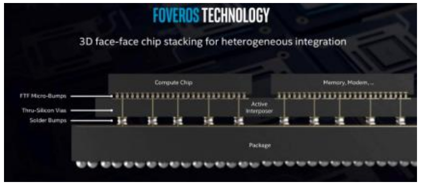


SunShot U.S. Department of Energy

Senior Technical Advisor, Photovoltaics

UNLV UNIVERSITY OF NEVADA LAS VEGAS

Associate Professor, Department of Mechanical Engineering



PURDUE UNIVERSITY

Associate Professor, Joint Appointment in School of Mechanical Engineering and School of Materials Engineering

Course Description

Section	Time (EST)	Instructor	Classroom	Email
005	MWF 7:30-8:20 AM	Prof. Shubra Bansal	PHYS 223	bansal91@purdue.edu
004	MWF 8:30-9:20 AM	Prof. Alex Chortos	PHYS 223	achortos@purdue.edu
001	MWF 9:30-10:20 AM	Prof. Charles Krousgrill	ARMS B061	krousgri@purdue.edu
006	MWF 10:30-11:20 AM	Prof. Klod Kokini	HAMP 1144	kokini@purdue.edu
007	MWF 1:30-2:20 PM	Prof. Charles Krousgrill	PHYS 223	krousgri@purdue.edu
002	MWF 2:30-3:20 PM	Prof. Klod Kokini	HAMP 1144	kokini@purdue.edu
003	MWF 3:30-4:20 PM	Prof. Ryan Wagner	PHYS 223	rbwagner@purdue.edu

Shubhra Bansal
Associate Professor
School of Mechanical Engineering, School of Materials Engineering
Office: FLEX 3081D

- Email: bansal91@purdue.edu

- Office Hours: M-W-F 8:30-9:30am in ME 2187

Teaching Assistants

TEACHING ASSISTANTS

Harshith Adepu *Email:adepu@purdue.edu*	*Khashayar Ghorbani* *Email: kghorban@purdue.edu*	*Ayishe Jin* *Email: jin315@purdue.edu*
Mohit Kumar *Email: kumar589@purdue.edu*	*Amir Mollaali* *Email: mollaali@purdue.edu*	*Ben Obsitnik* *Email bobsitni@purdue.edu*
Tanya Purwar *Email: tpurwar@purdue.edu*	*Michael Verduzco* *Email: verduzm@purdue.edu*	*Tianhao Yu* *Email: yuth@purdue.edu*

T.A. office hours: *Teaching assistants will hold office hours in room TBD.*

Tuesday: Michael (8-10am); Khashayar (10-12); Mohit (2:30-4:30pm); Ben (4:30-7:00pm); Khashayar (7-8pm)

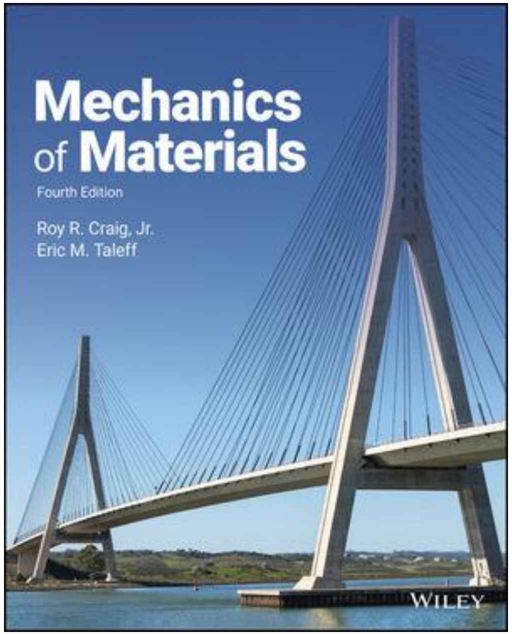
Wednesday: Michael (8-11:30am); Mohit (11:30-12:30; 1:30-3:30pm); Ben (3:30-6pm); Ayishe (6-8:30pm)

Thursday: Michael (8-10am); Khashayar (10-11:50am; 1:30-2pm); Mohit (2-4:30pm); Ben (4:30-7pm)

Friday: Michael (8-10:30am); Ben (11:30am-1pm); Ayishe (1-3:30pm); Mohit (3:30-6pm); Khashayar (6-8:30pm)

Reference Text

- **<u>Required:</u>** Course lecture book to be purchased at University Bookstore.
- Optional: Mechanics of Materials, R.R. Craig, Wiley, 3rd edition, 2011.
- Reference texts: Copies of three different mechanics of materials textbooks (by authors Craig, Hibbeler and Philpot) will be available at the reserve desk for the Engineering/Science library on the second floor of WALC. These books will be available for two-hour checkout starting during the second week of classes.
- Solutions to examples in pdf or Youtube videos
- Discussion forum for students



COURSE BLOG

https://www.purdue.edu/freeform/me323/

ME 323 Website

ME 323
Mechanics of Materials
Purdue University

This blog will be used to deliver all online material for the course (see links in the sidebar) along with providing a forum for the discussion of homework problems and other course issues. Check back often for new material and to see what your colleagues and friends are saying in the course.

PURDUE UNIVERSITY
MECHANICAL ENGINEERING

ME 323: Mechanics of Materials

COURSE INFORMATION | DAILY SCHEDULE - FA 2023 | EXAMS | GENERAL DISCUSSION | HOMEWORK/DISCUSSION - FA23 | INSTRUCTOR-SUPPLIED MATERIAL | LECTUREBOOK EXAMPLES | ME 323 - HOME PAGE | QUIZZES | ANIMATIONS AND DEMOS

Purdue University

FREEform

gradescope

Log in

LINKS

- Animations and demos
- Course information
- Daily Schedule - Fa 2023
- Exams

ME 323 - HOME PAGE

WELCOME!

Welcome to the ME 323 course website for the Fall 2023 term. The material on this site is a complement to the lecture book for the course. And, all material here is accessible without the need to log in. Please review the resources that are available to you in the links on the left sidebar of the page. *Logging in is required only for adding comments to the blog posts.* ***Read me*** for instructions in logging in to the website.

Have a good semester!

Course Procedure and Homeworks

- Conceptual content will be covered directly from the lecture book; you are encouraged to write notes in the book.
- Most in-class examples will be taken from the lecture book.
- Some additional examples will be introduced that are focused on real applications.

Homework:

- Weekly homework
- Posted online on Friday by 7 pm and due the next Friday at 11:59 pm
- Submitted online through Gradescope
- Assignments submitted after the deadline but within 24 hours will be penalized by 20% reduction in score. Assignments submitted more than 24 hours after the deadline will not be accepted except with a university approved justification (e.g. Grief Absence Notification).

Course Schedule

Grade Categories	Percentage
Homework + Quizzes	30%
Exam 1 and 2	25% or 45%
Final Exam	45% or 25%

If you score better on the average of Exams 1 and 2 than you do on the Final, the average of these two midterm exams will count as 45% and the Final will count as 25% of your final grade, vice versa.

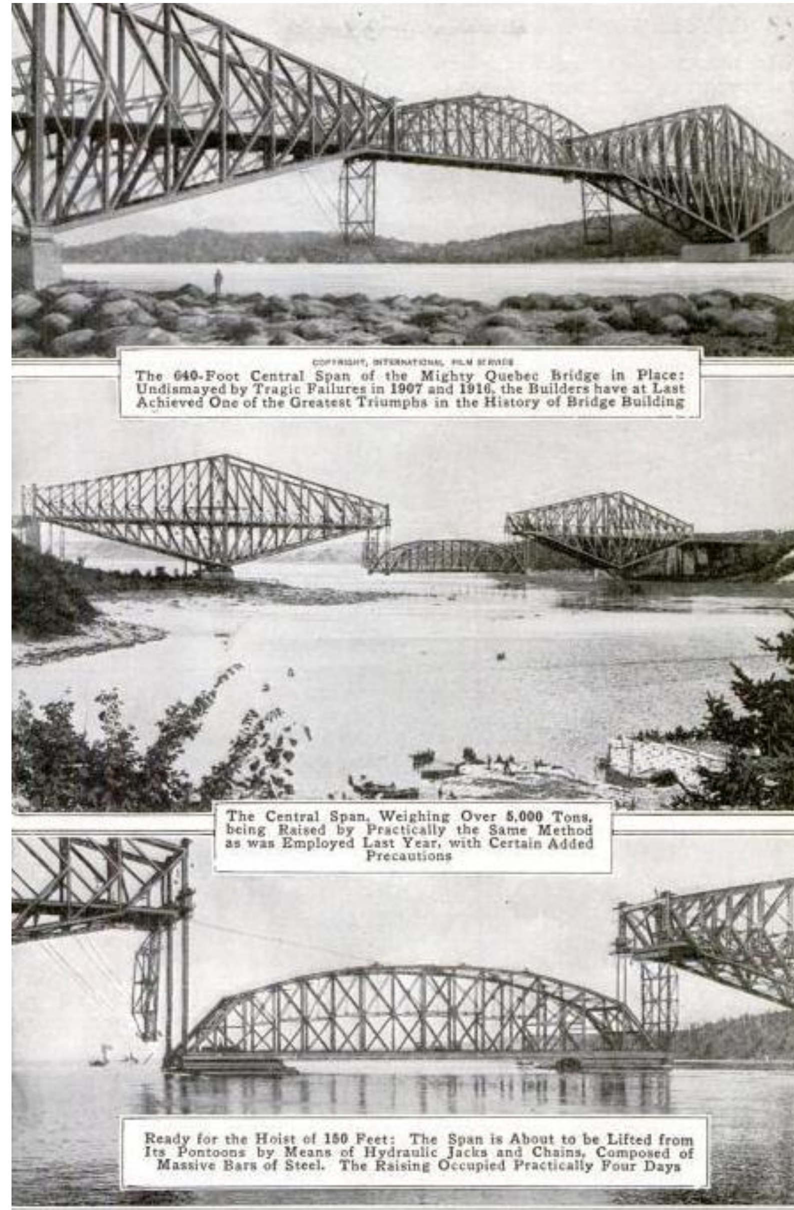
PER	*DATE*	*TOPIC*	*READING**	*HWK DUE*
1 M	21-Aug	Introduction; Static equilibrium	Chap. 1	
2 W	23-Aug	Normal stress and strain; Mechanical properties	Chap. 2	
3 F	25-Aug	Shear stress and strain – direct shear	Chap. 3	
4 M	28-Aug	Stress – introduction to design of deformable bodies	Chap. 4	
5 W	30-Aug	Stress and strain – general definitions	Chap. 5	HW. 1
6 F	1-Sep	Axial members – determinate structures	Chap. 6	
M	**4-Sep**	***Labor Day – no class***		
7 W	6-Sep	Axial members – indeterminate structures	Chap. 6	
8 F	8-Sep	Axial members – planar trusses	Chap. 6	HW. 2
9 M	11-Sep	Axial members – thermal effects	Chap. 7	
10 W	13-Sep	Torsion members – stresses in circular bars	Chap. 8	HW. 3
11 F	15-Sep	Torsion members – statically determinate structures	Chap. 8	
12 M	18-Sep	Torsion members – statically indeterminate structures	Chap. 8	
13 W	20-Sep	Beam stresses – equilibrium and flexural stresses	Chap. 10	HW. 4
14 F	22-Sep	Beam stresses – flexural and shear stresses	Chap. 10	
15 M	25-Sept	Review		
W	***27 Sept***	***Examination 1, 8-10pm (no lecture on Wednesday)***		
16 F	29-Sep	Beam stresses – shear stresses	Chap. 10	
17 M	2-Oct	Shear force/bending moment diagrams - determinate structures	Chap. 9	
18 W	4-Oct	Beam deflections – statically determinate structures	Chap. 11	HW 5
19 F	6 -Oct	Beam deflections – statically indeterminate structures	Chap. 11	
M	**9-Oct**	***October Break - no class***		
20 W	11-Oct	Beam deflections – superposition methods	Chap. 11	
21 F	13-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW. 6
22 M	16-Oct	Energy methods – Castigliano's theorems	Chap. 16	
23 W	18-Oct	Energy methods – Castigliano's theorems	Chap. 16	
24 F	20-Oct	Energy methods – Castigliano's theorems	Chap. 16	HW 7
25 M	23-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
26 W	25-Oct	Shear force/bending moment diagrams – indeterminate structures	Chap. 9	
27 F	27-Oct	Energy methods – introduction to finite element methods	Chap. 17	HW 8
28 M	30-Oct	Review		
W	**1-Nov**	***Examination 2, 8-10p.m. (no lecture on Wednesday)***		
29 F	3-Nov	Energy methods – introduction to finite element methods	Chap. 17	
30 M	6-Nov	Thin-walled pressure vessels – axial and hoop stresses	Chap. 12	
31 W	8-Nov	Stress transformation – principal /maximum shear stresses	Chap. 13	
32 F	10-Nov	Stress transformation – Mohr's circle	Chap. 13	HW 9
33 M	13-Nov	Stress transformation – absolute maximum shear stress	Chap. 13	
34 W	15-Nov	Stresses – combined loading	Chap. 14	
35 F	17-Nov	Stresses – combined loading	Chap. 14	HW 10
36 M	20-Nov	Stresses – combined loading	Chap. 14	
W	**22-Nov**	***Thanksgiving Vacation – no class***		
F	**24-Nov**	***Thanksgiving Vacation – no class***		
37 M	27-Nov	Failure analysis-stress theories	Chap. 15	
38 W	29-Nov	Failure analysis – stress theories	Chap. 15	
39 F	1-Dec	Failure analysis – buckling	Chap. 18	HW. 11
40 M	4-Dec	Practice with combined loadings and failure analysis		
41 W	6-Dec	Practice with combined loadings and failure analysis		
42 F	8-Dec	Review		
	TBA	***Final Examination***		

Homework Schedule

HW #	HW Publish date	HW due date
1	Aug 25	Sept 1
2	Sept 1	Sept 8
3	Sept 8	Sept 15
4	Sept 15	Sept 22
5	Sept 29	Oct 6
6	Oct 6	Oct 13
7	Oct 13	Oct 20
8	Oct 20	Oct 27
9	Nov 3	Nov 10
10	Nov 10	Nov 17
11	Nov 20	Dec 1

Infrastructure Disaster – Quebec Bridge 1907

- Collapsed and killed 86 workers.
- Collapsed again in 1916 and killed 15 workers.



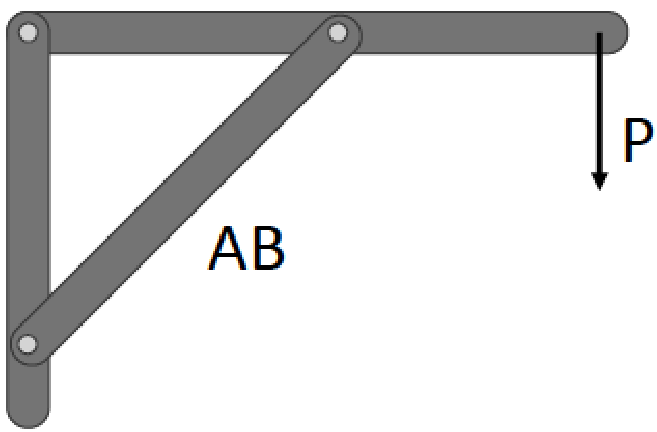
Modern Day Examples..

I-40 bridge, May 2021

Southwest 1380, April 2018

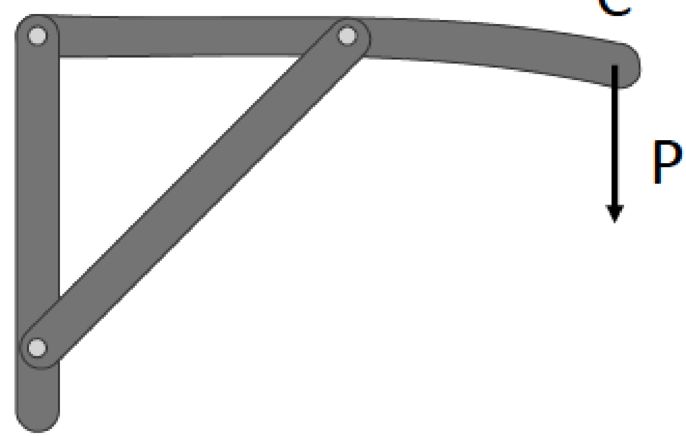
ME323: Statics with Deformable Structures

ME270



When you put a force at C, how much force does AB experience?

ME323



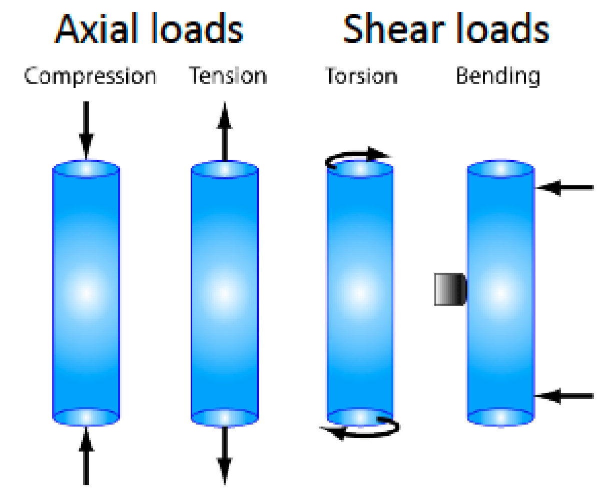
When you put a force at C, how much will point C deflect? At what value of the force will the system fail?

Mechanics of Materials

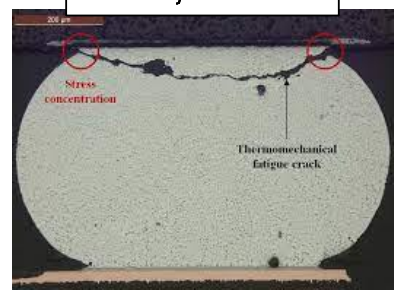
- To study internal effects of stress and strain in a solid body that is subjected to an external loading..

- In this course, we study the statics of deformable bodies upon applying external loads.
  - Entry point into the studies of solid and continuum mechanics
  - Design of structures against failure

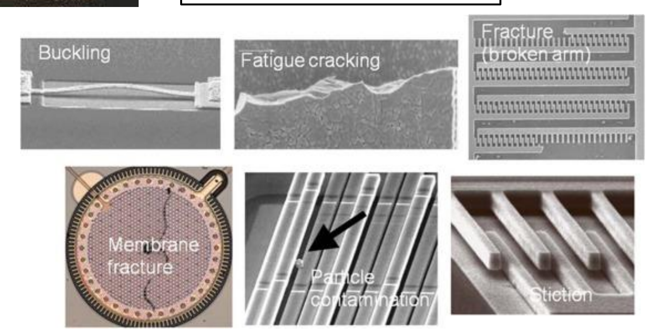
- Diverse applications:
  - automobile components
  - bridges, buildings
  - bone mechanics and tissue engineering
  - data storage systems
  - electronic packaging
  - Microelectromechanical systems (MEMS)
  - packaging of consumer products
  - sports equipment



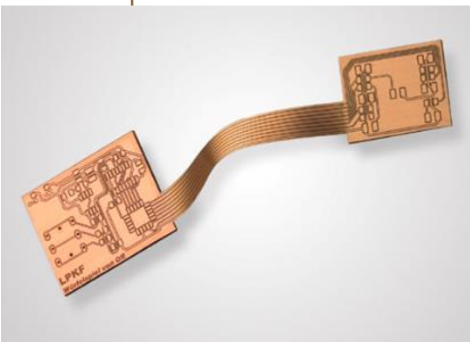
Solder joint failure

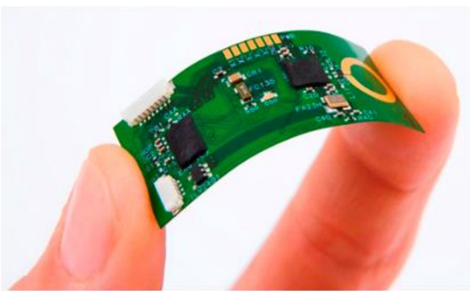


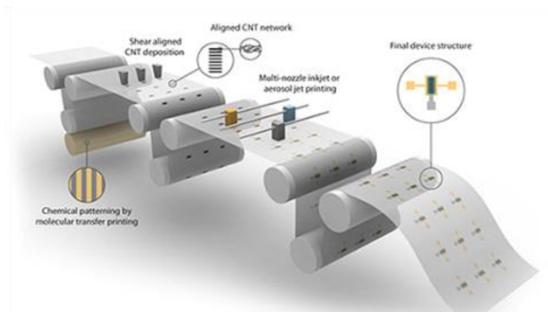
MEMS device failures

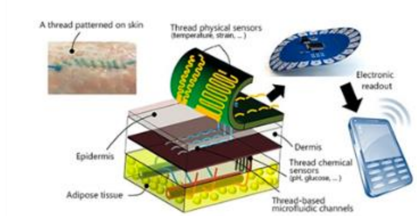


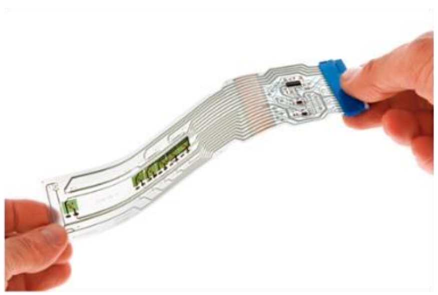
Modern Day Applications

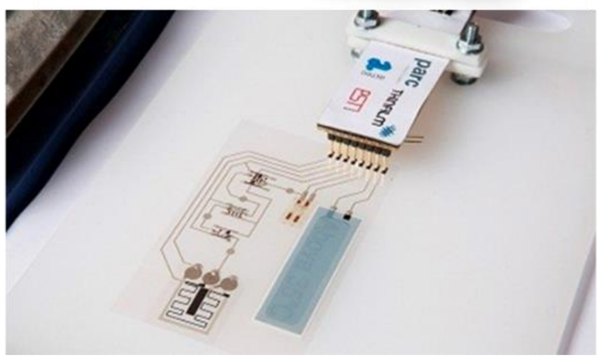


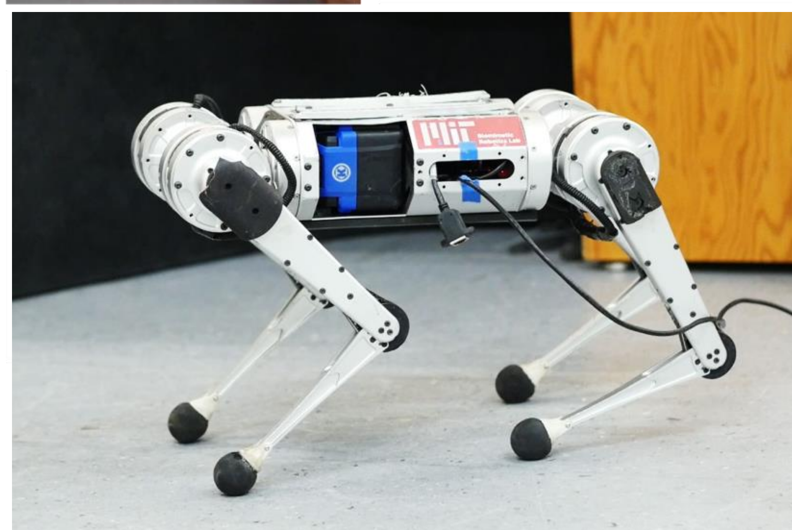












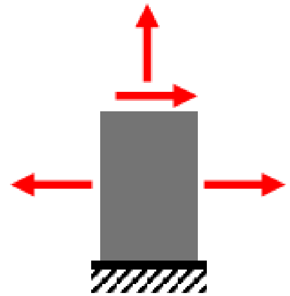
Course Overview

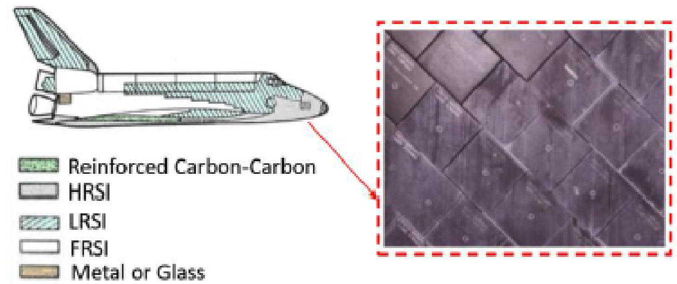
Lecture 2: Uniaxial stress

Lecture 3: Shear stress

Lecture 4: Design

Lecture 5: General stress





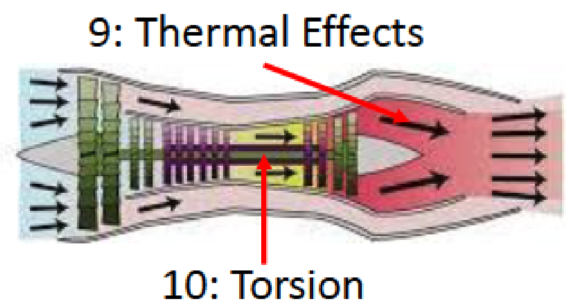
Lecture 6: Axial determinate

Lecture 7: Axial indeterminate

Lecture 8: Planar trusses

Lecture 9: Thermal effects

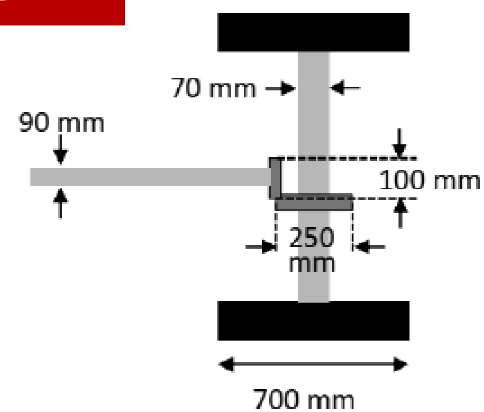
How can we use knowledge of materials properties to solve stress distributions in complex assemblies



Lecture 10: Torsion

Lecture 11: Torsion determinate

Lecture 12: Torsion indeterminate

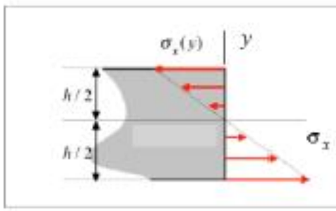


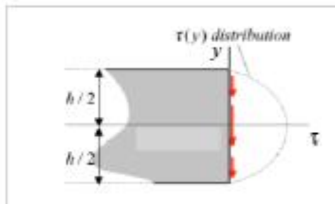
Course Overview

Lecture 13-14: Beams: Flexural stress

Lecture 15: Beams: Shear stress

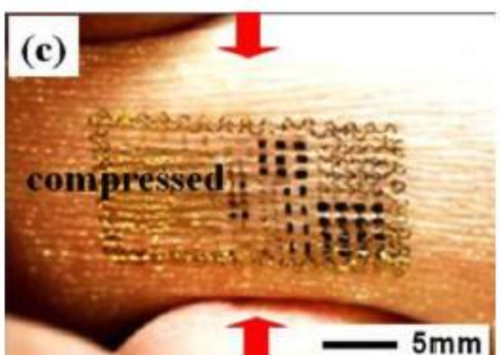
Lecture 16-20: Beam deflections

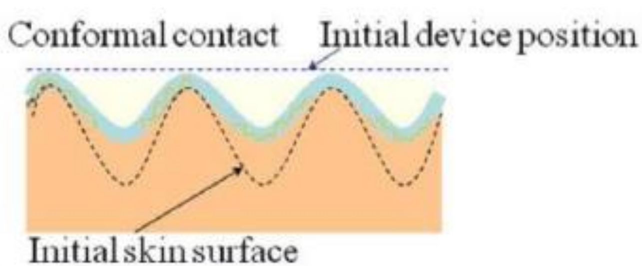




Lecture 21-24: Castigliano's theorems

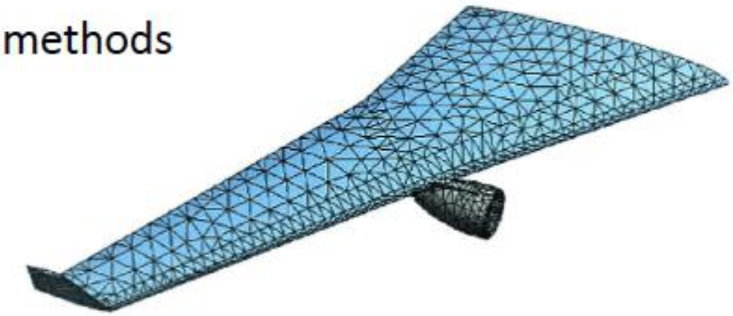
$$U = \frac{1}{2} P e$$



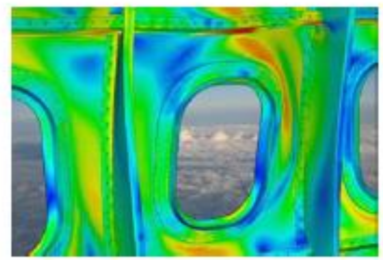


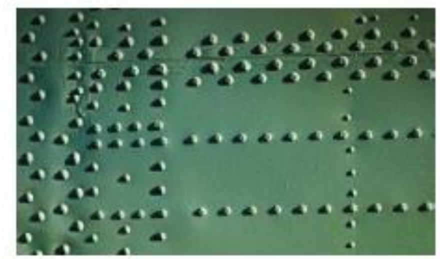
Course Overview

Lecture 27-28: Finite element methods



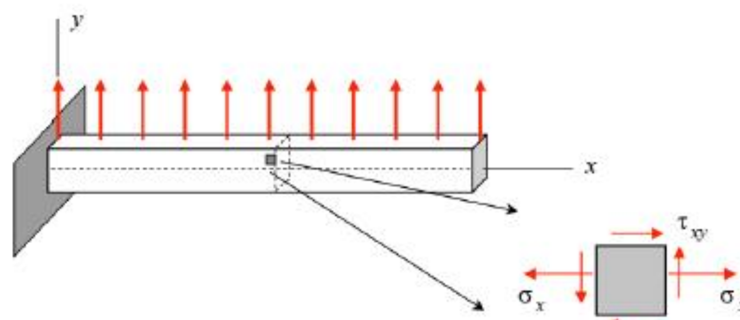
Lecture 30: Pressure vessels





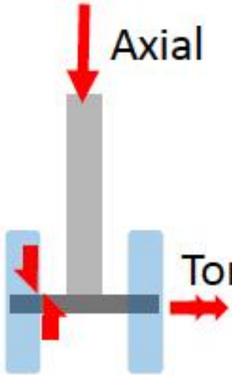
Why is the rivet pattern on an airplane like this?

Lecture 31-33: Stress transformations



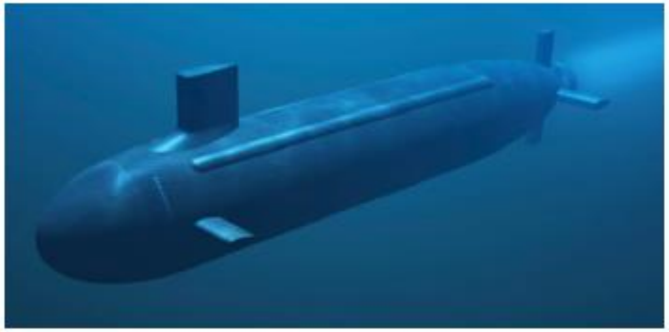
Course Overview

Lecture 34-36: Combined loading



Why does a windmill have 3 blades?

Lecture 37-39: Failure theories



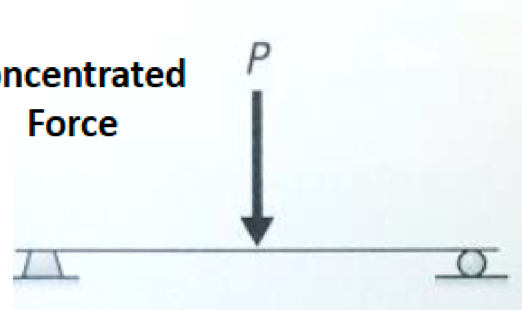
How deep can a submarine dive?

Equilibrium of Deformable Body

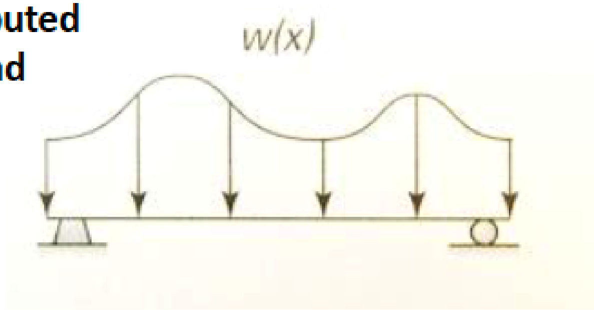
External Loads

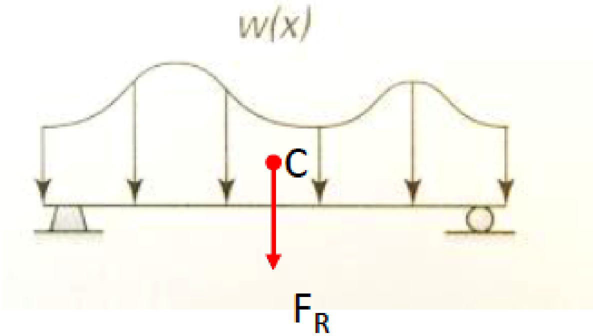
(a) Surface Forces : caused by the direct contact of one body with the surface of another

Concentrated Force



Distributed Load





- The resultant force F_R of w(x) = the area under the distributed loading curve, which acts through the centroid C or geometric center of this area.

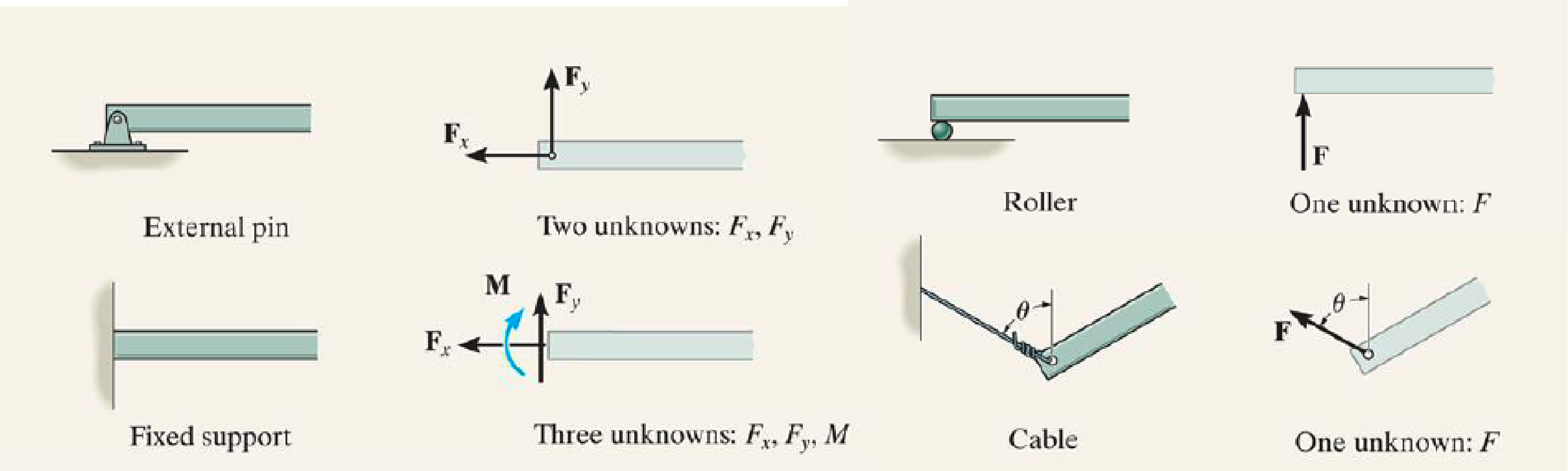
(b) Body Forces : developed when one body applies a force on another body without direct physical contact.

ex) the earth's gravitation/ electromagnetic field

Support Reactions

- Reactions : the surface forces that develop at the points of contact between bodies.

- For two-dimensional problems(coplanar force systems)



- If the support prevents translation in a given direction, then a force must be developed on the member in that direction.
- If rotation is prevented, a moment must be applied on the member.

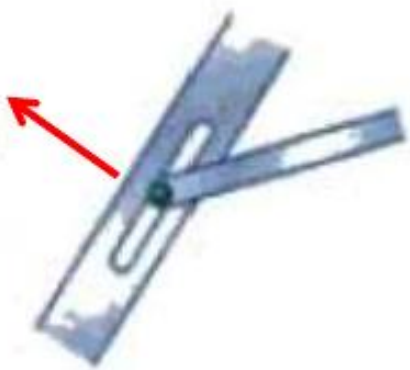
- Does the support prevent translation in x (or y) direction or rotation??

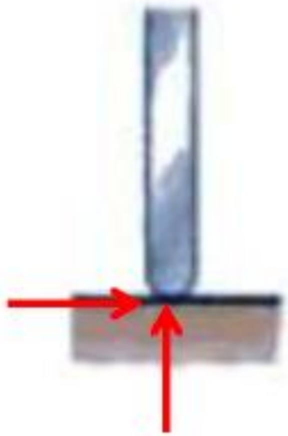
Support Reactions

Frictionless pin in slot

Smooth surface (frictionless surface)

Rough surface





Equations of Equilibrium

- **Equilibrium of a body**

➔ a balance of forces and a balance of moments.

$$\Sigma F=0,\ \Sigma M_O=0$$

ΣF = the sum of all the forces acting on the body
ΣM = the sum of the moments of all the forces about any point O

Necessary conditions for equilibrium of a body (rigid or deformable)

- In x,y,z coordination system,

$$\Sigma F_x=0,\ \Sigma F_y=0,\ \Sigma F_z=0$$
$$\Sigma M_x=0,\ \Sigma M_y=0,\ \Sigma M_z=0$$

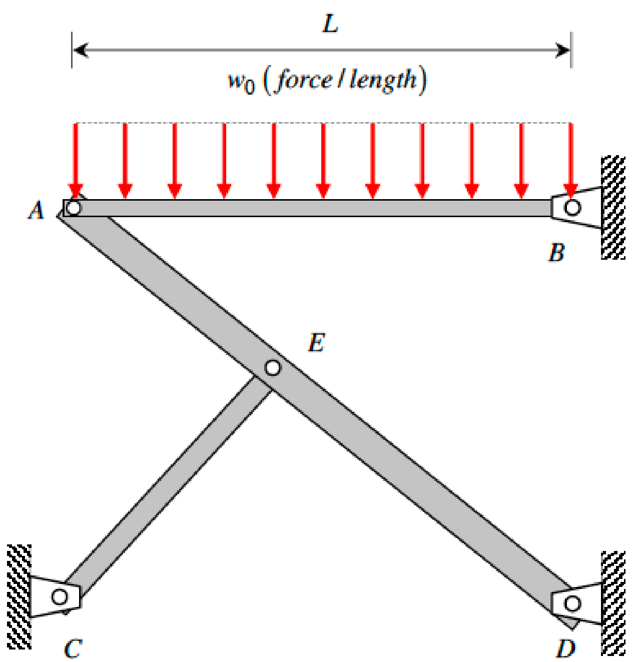
In a system of coplanar forces,

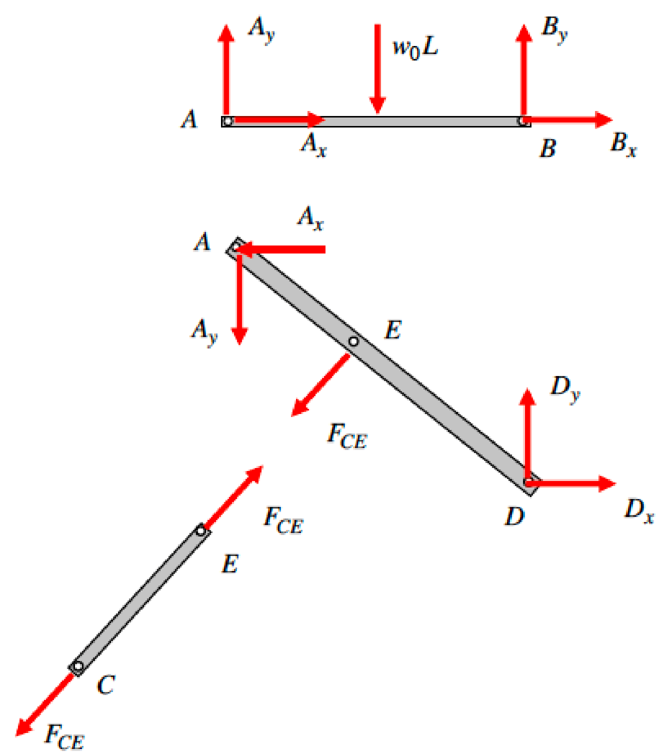
$$\Sigma F_x=0,\ \Sigma F_y=0,\ \Sigma M_O=0$$

* All the moments are summed about point O and so they will be directed along the z axis

Free Body Diagrams

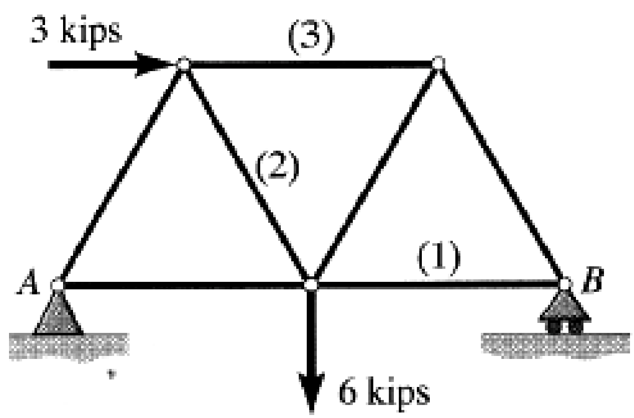
- Determine the extent of the body to be included
- Completely isolate this body from its supports/ other attached bodies
- To determine internal resultants, use sectioning planes
- Positive vs. negative forces (we will consider tension +ve)
- Moments we will follow right hand rule

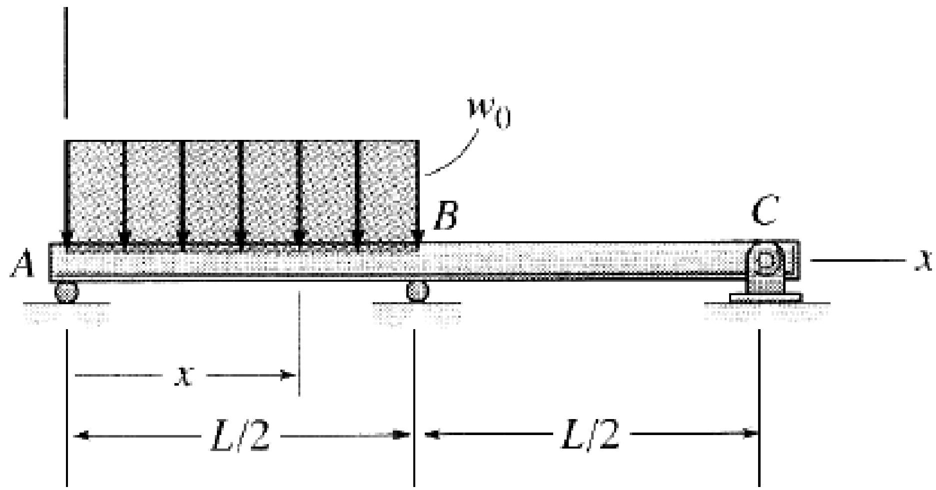




Example 1.1 from Lecturebook

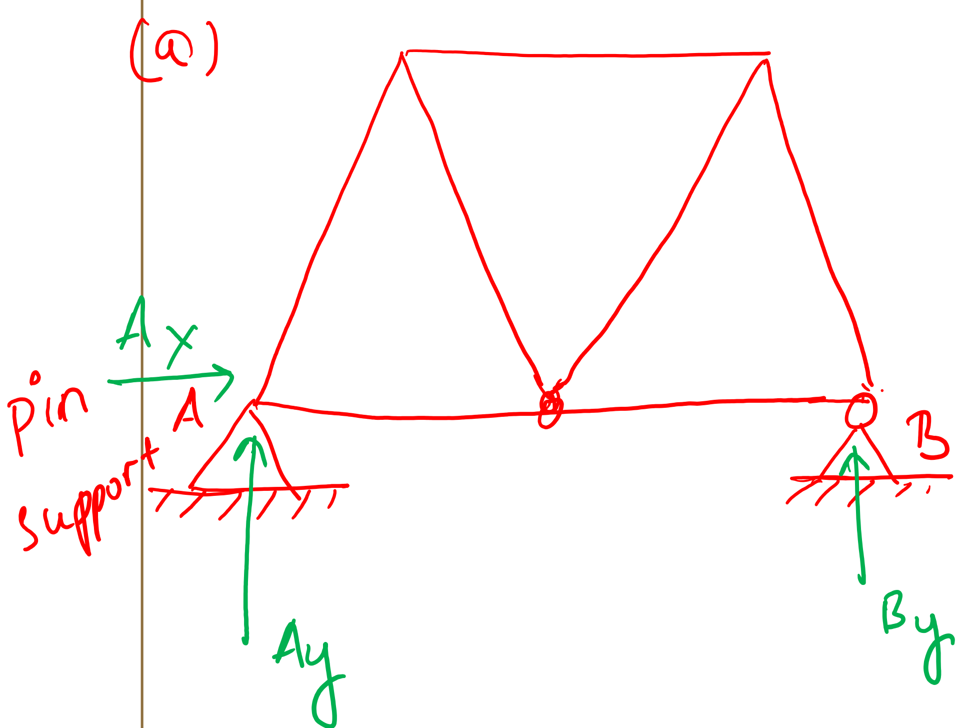
Complete the free body diagrams below. Which of the following systems are statically determinate for the support reactions?





What are statically determinate and indeterminate problems?

- Number of independent equilibrium equations
- Number of variables



equilibrium equations

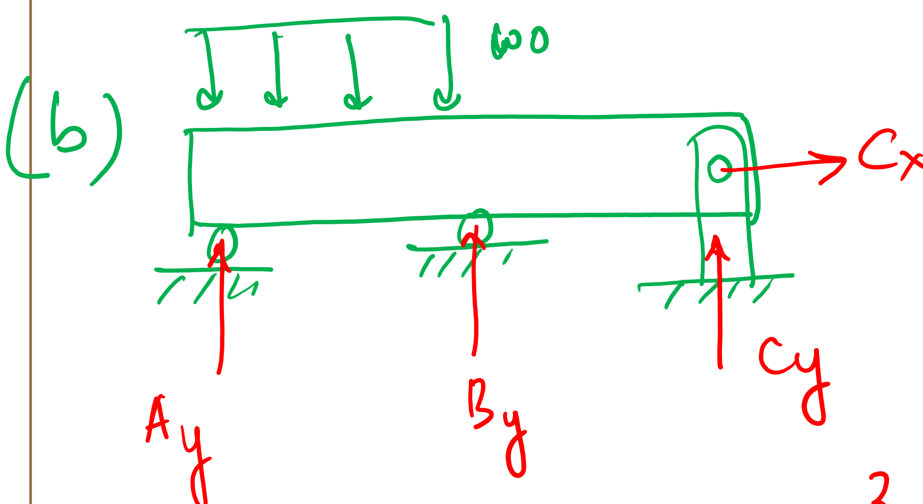
$$\Sigma f_x = 0$$

$$\Sigma f_y = 0$$

$$(\Sigma M) = 0$$

3 equations

3 variables $\Rightarrow$ statically determinate



$$(\Sigma F)_x = 0$$

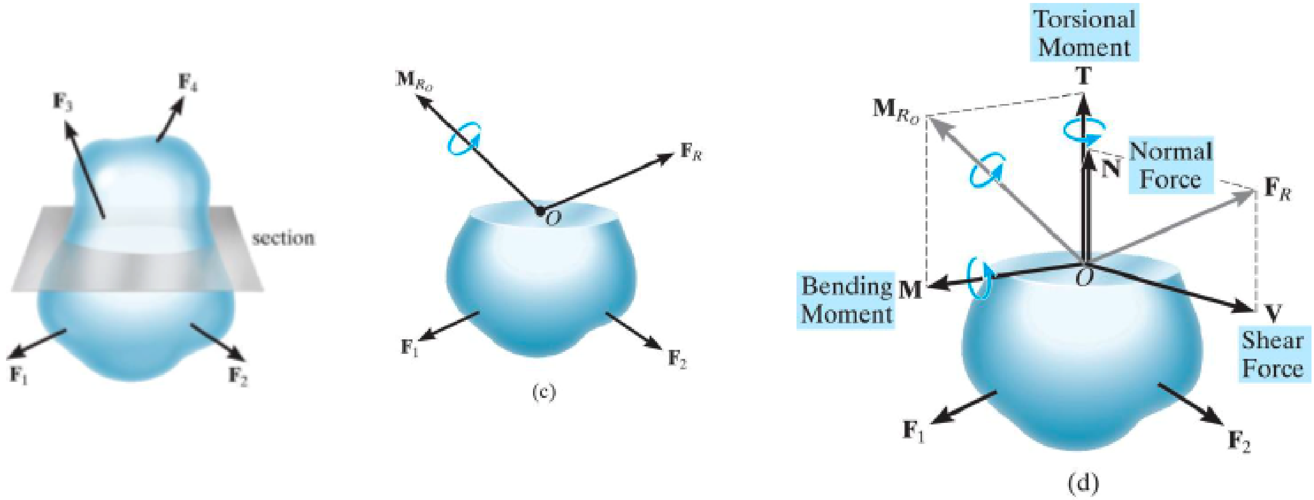
$$(\Sigma F)_y = 0$$

$$(\Sigma M) = 0$$

3 equations, 4 variables

$\Rightarrow$ statically indeterminate

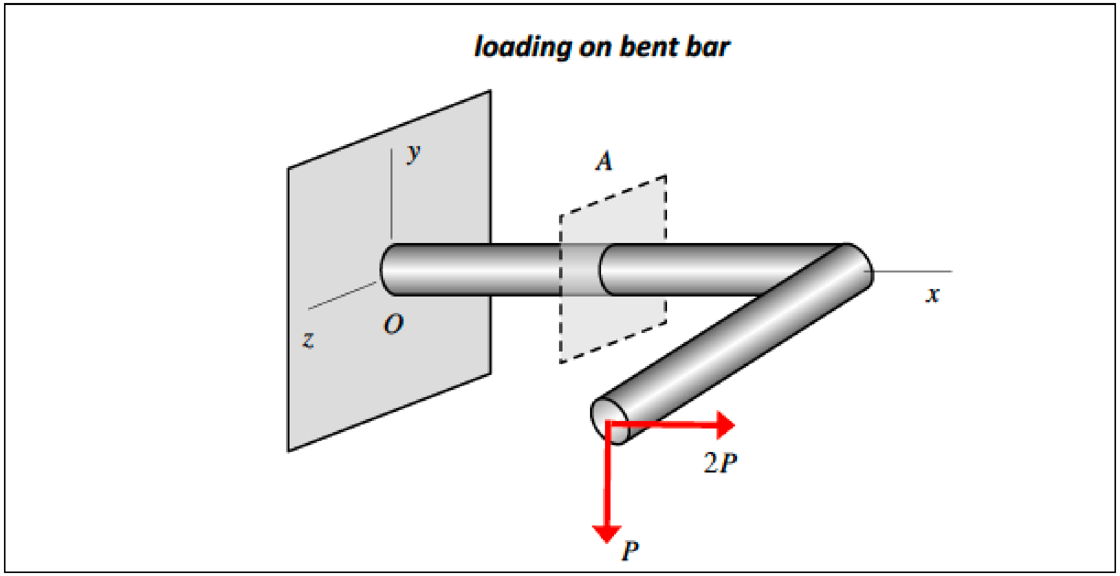
Internal Resultants

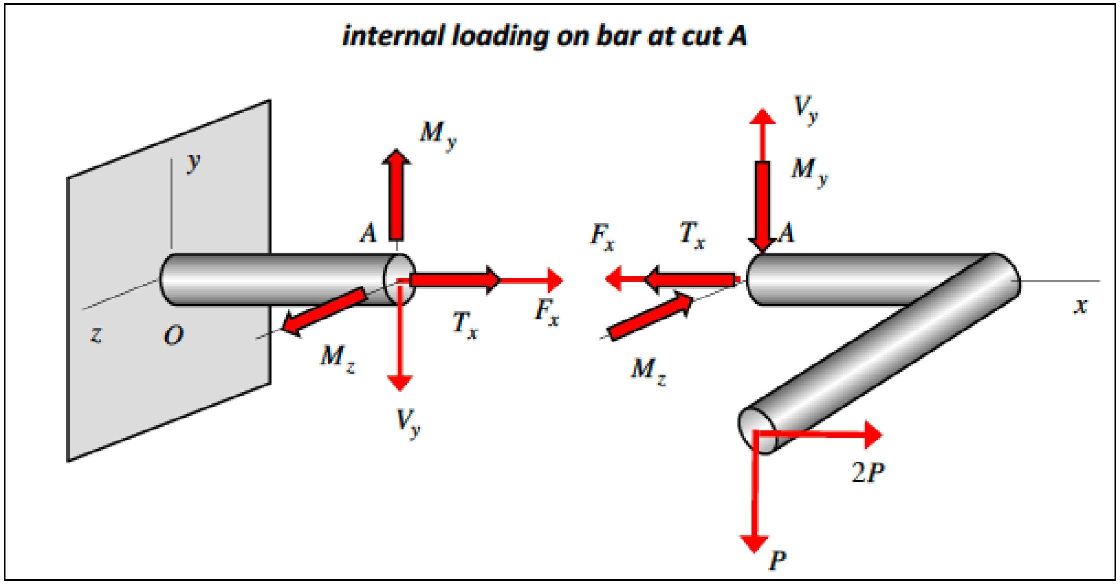


- Four different types of ***resultant loadings***

	Direction (to the sectioned area)	Force	Tendency
F_R	Perpendicular	Normal force, N	pushing or pulling on the body
	Parallel	Shear force, V	sliding over the other body
M_{Ro}	Perpendicular	Torsional moment T (torque)	twisting the body about an axis perpendicular to the area
	Parallel	Bending moment, M	bending the body about an axis lying within the plane of the area

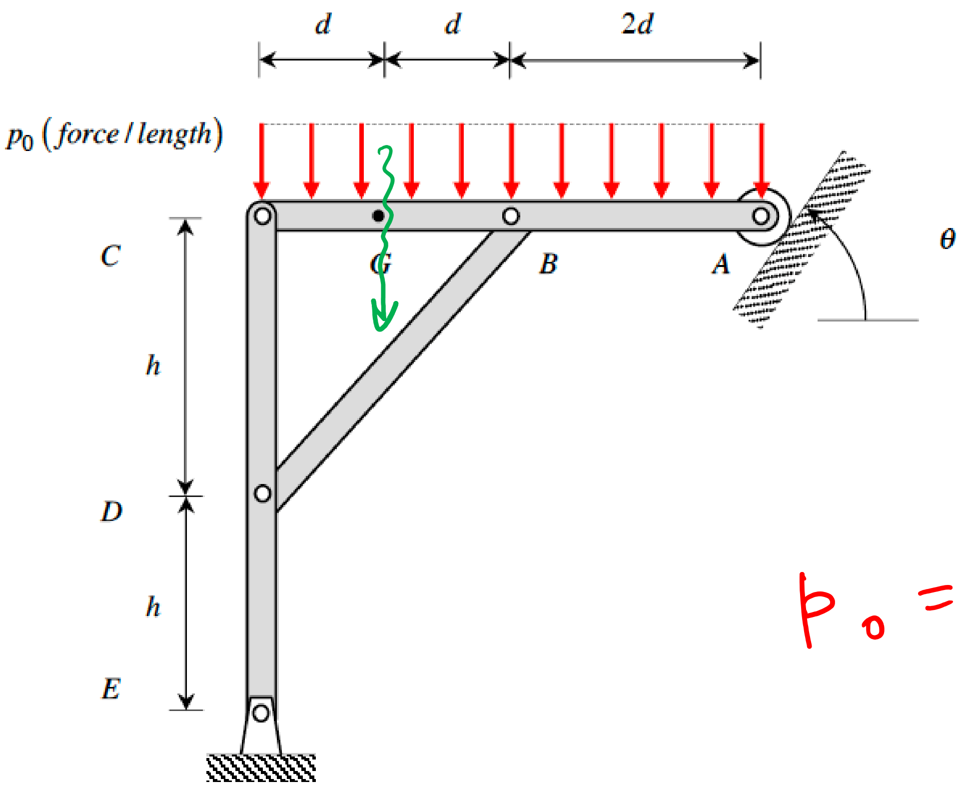
Internal Resultants





Example 1.3 from Lecturebook

The uniform distributed load on member AC has a magnitude of p_0 . Determine the internal axial force, shear force and bending moment acting on the left face of the cross section of member AC at G.



At G –

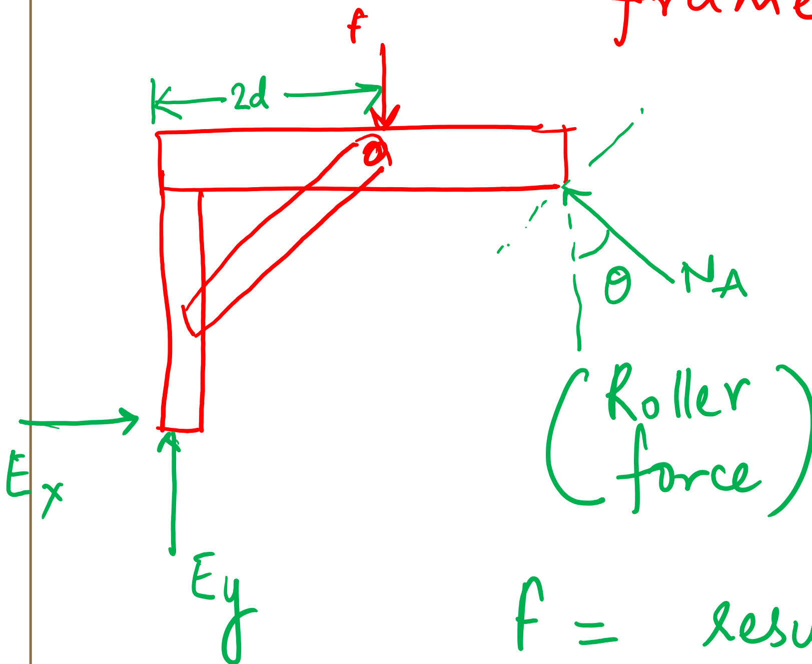
$\theta = 60^\circ$

$h = 2ft$

$d = 1.5ft$

$p_0 = 220 lb/ft$

Step 1 → find external reactions on the frame



N_A = normal force

Resultant ~~force~~ f due to load w_0 at a distance 2d.

$$f = \text{resultant force} = (2d)(h)\,w_0$$

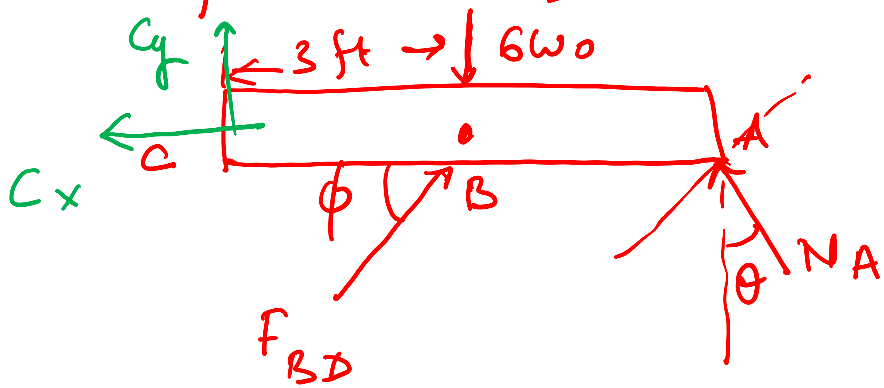
$$F = 6\,w_0$$

$$\left(\sum M\right)_E = -(N_A \cos\theta)(6) - (N_A \sin\theta)(6) + F(2d) = 0$$

$$N_A = \frac{F(2d)}{6(\sin\theta + \cos\theta)} = \frac{(6)(220\ \text{lb/ft})(3\ \text{ft})}{6(\sin 60^\circ + \cos 60^\circ)}$$

$N_A = 661.28$ lbs

Now find F_{BD} then section at G –



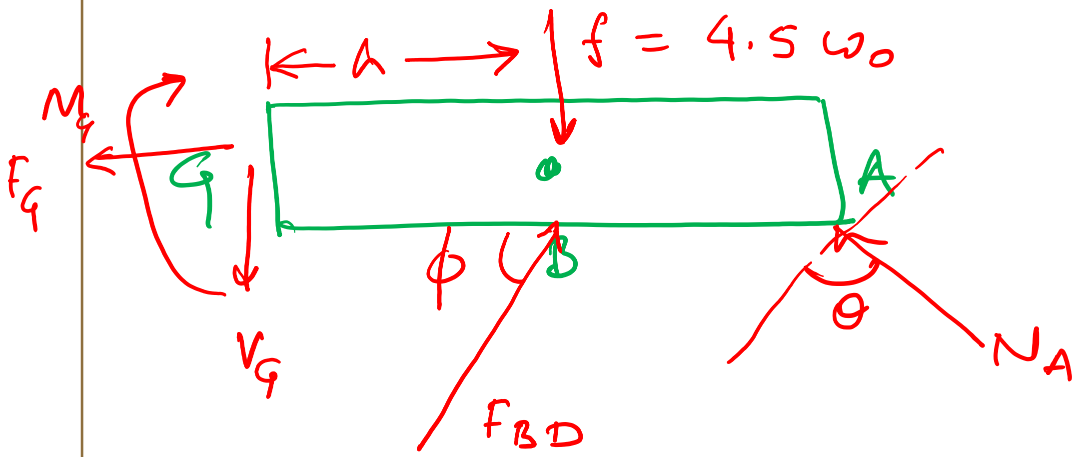
BD → two force member

$\phi = \tan^{-1}\left(\frac{4}{3}\right)$

$$\left(\Sigma M\right)_C = 0 \Rightarrow (6w_0)(3) - \left(F_{BD}\sin\phi\right)(3) - \left(N_A\cos\theta\right)(6) = 0$$

$$\Rightarrow \boxed{F_{BD} = \frac{18w_0 - 6N_A\cos\theta}{3\sin\phi}}$$

final goal is to determine resultant force at G.



$$\Sigma F_x = N_A \sin\theta - F_{BD}\cos\phi + F_G = 0$$

$$F_G = F_{BD}\cos\phi - N_A \sin\theta$$

$$\Sigma F_y = N_A\cos\theta + F_{BD}\sin\phi - f - V_G = 0$$

$$V_G = N_A\cos\theta + F_{BD}\sin\phi - f$$

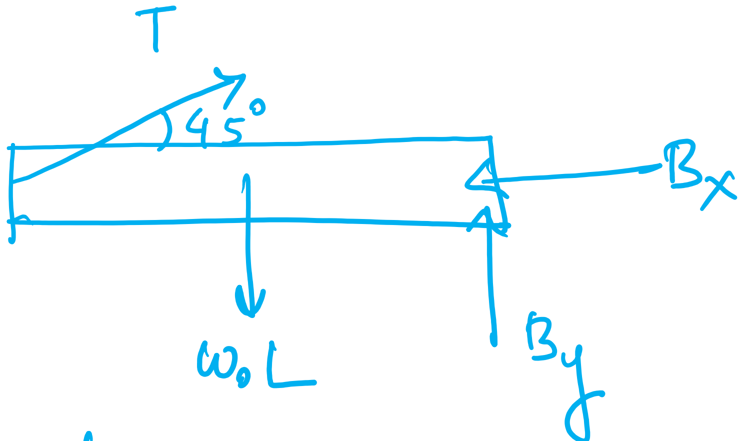
$$(\Sigma M)_G = (F_{BD}\sin\phi)(h) + fh - (N_A\cos\theta)(4.5) + M_G = 0$$

$$M_G = -hF_{BD}\sin\phi - fh + 4.5N_A\cos\theta$$

Example 1.5 from Lecturebook

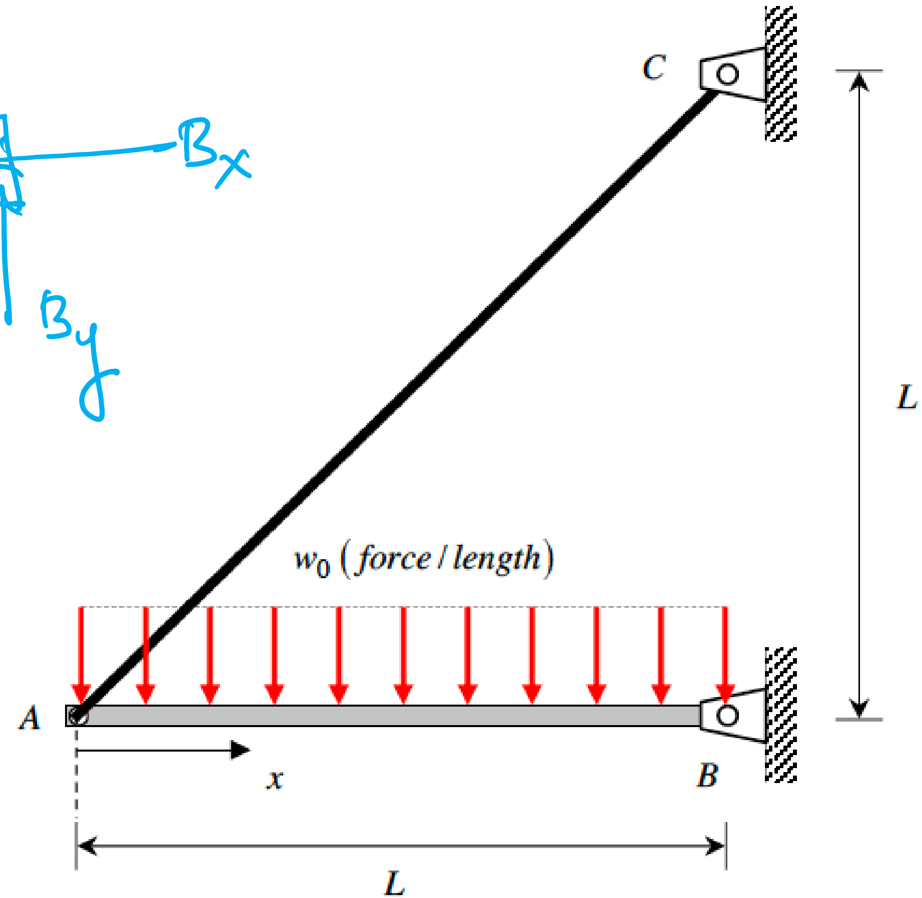
Determine expressions for the internal resultants F(x), V(x), M(x) at an arbitrary point X along AB.

FBD



moments at B. —

$(\Sigma M)_B = 0$



$$(\Sigma M)_B = -(T\sin\theta)(L) + (w_0L)\left(\frac{L}{2}\right) = 0$$

$$T = \frac{w_0 L}{2\sin\theta}$$

$$\Sigma F_x = T\cos\theta - B_x = 0$$

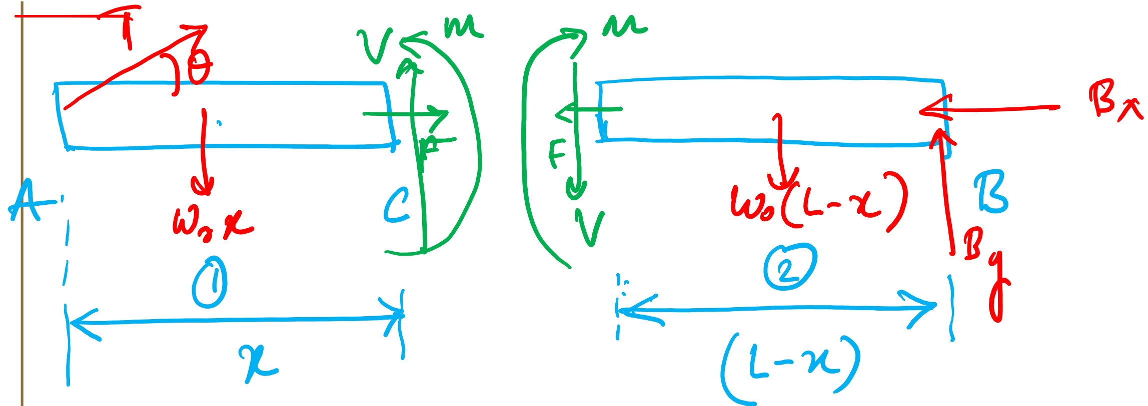
$$B_x = T\cos\theta = \frac{w_0L}{2\sin\theta}(\cos\theta) = \frac{w_0L}{2}(\cot\theta)$$

$$\Sigma F_y = T\sin\theta + B_y - w_0L = 0$$

$$B_y = w_0L - T\sin\theta = w_0L - \frac{w_0L(\sin\theta)}{2\sin\theta}$$

$$B_y = \frac{w_0L}{2}$$

Now we know external reactions. We need to estimate internal resultant at an arbitrary point



"C" section ① and ② at distance x from A.

①

$$\left(\sum M\right)_C = -(T\sin\theta)(x) + (w_0 x)\left(\frac{x}{2}\right) + m = 0$$

$$m = (T\sin\theta)(x) - w_0\frac{x^2}{2}$$

$$= \left(\frac{w_0 L}{2}\right)(x) - \frac{1}{2}w_0 x^2 = \frac{1}{2}w_0 x(L-x)$$

$$\sum F_y = T\sin\theta - w_0 x + V = 0 \Rightarrow V = -T\sin\theta + w_0 x$$

$$V = -\frac{w_0 L}{2\sin\theta}(\sin\theta) + w_0 x = -\frac{w_0 L}{2} + w_0 x$$

$$\sum F_x = T\cos\theta + F = 0 \Rightarrow F = -T\cos\theta = -\frac{w_0 L}{2\sin\theta}(\cos\theta)$$

$$F = -\frac{w_0 L}{2}(\cot\theta)$$

Section ②

$$\Sigma M_c = 0$$

$$B_y (L - x) - w_0 (L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = \left(\frac{w_0 L}{2}\right)(L - x) - \frac{w_0}{2}\left[L^2 - 2Lx + x^2\right]$$

$$M = \frac{w_0}{2} x (L - x) \checkmark$$

$$\Sigma F_x = -F - B_x = 0 \Rightarrow F = -B_x = \frac{w_0 L}{2} \cot\theta$$

$$F = \left(\frac{w_0 L}{2}\right)(\cot\theta) \checkmark$$

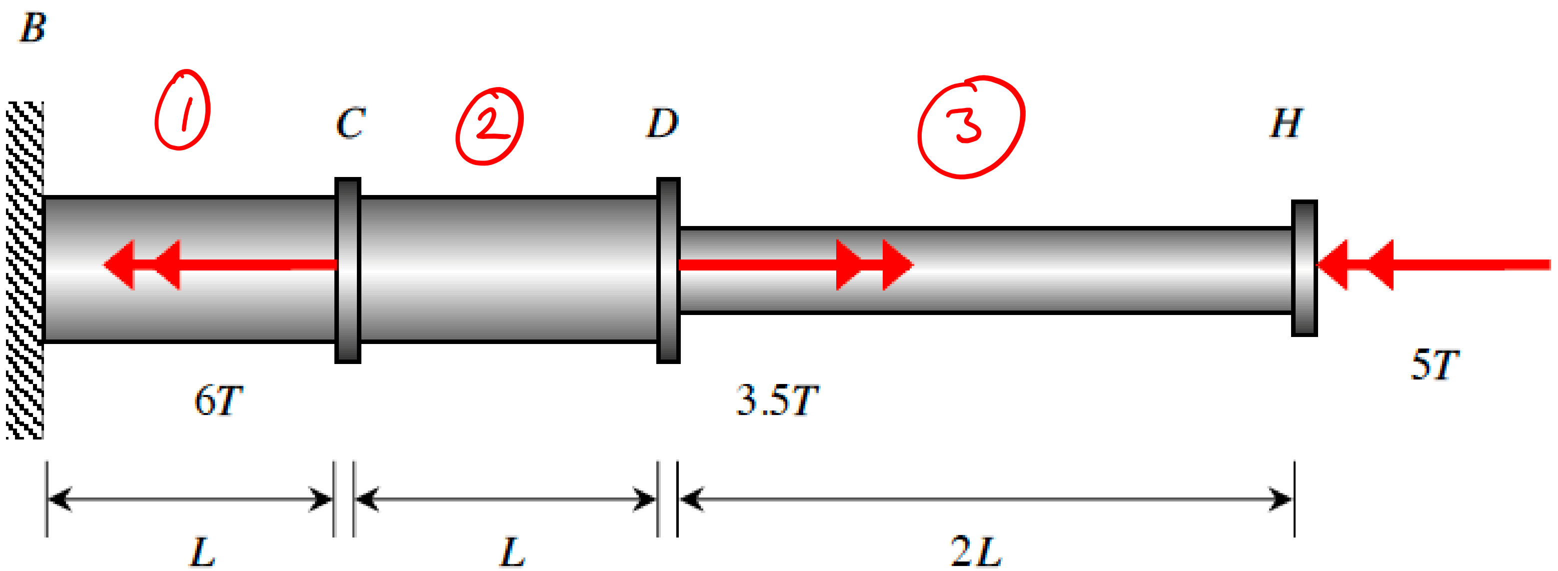
$$\Sigma F_y = B_y - w_0 (L - x) - V = 0 \Rightarrow V = B_y - w_0 (L - x)$$

$$V = \frac{w_0 L}{2} - w_0 L + w_0 x$$

$$V = -\frac{w_0 L}{2} + w_0 x \checkmark$$

Example 1.6 from Lecturebook

Determine expressions for the internal resultant torques in sections CD and DH due to the applied torques at C, D and H.



segments ①, ②, ③

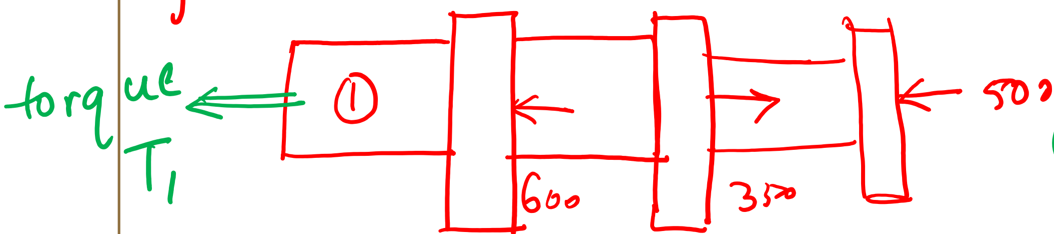
connectors — C, D, H

two approaches

- sectioning
- connectors

Method 1 Sectioning approach

Segment ①



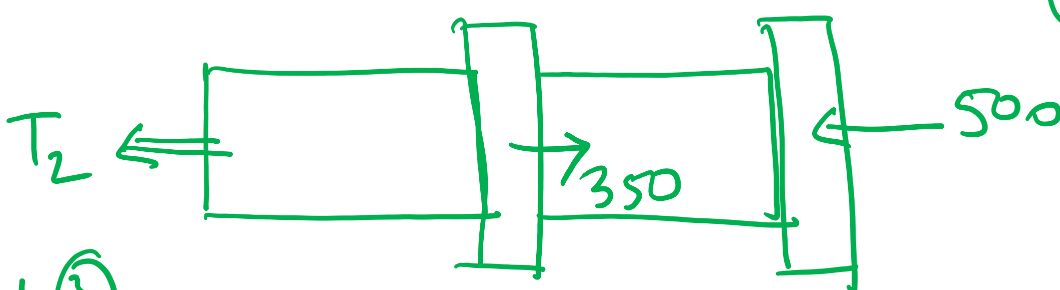
Internal resultant torque T_1 in segment ①

$$(\Sigma T)_1 = -T_1 - 600 + 350 - 500 = 0$$

$$\boxed{T_1 = -750 \text{ ft.lb}}$$

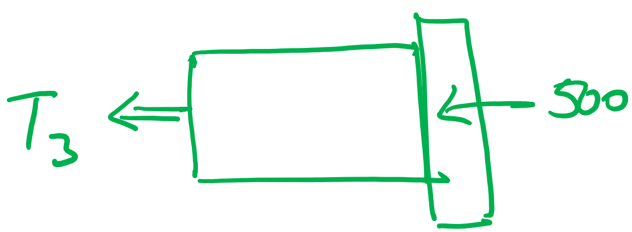
Segment ② –

$$(\Sigma T)_2 = -T_2 + 350 - 500$$



$$\boxed{T_2 = -150 \text{ ft.lb}}$$

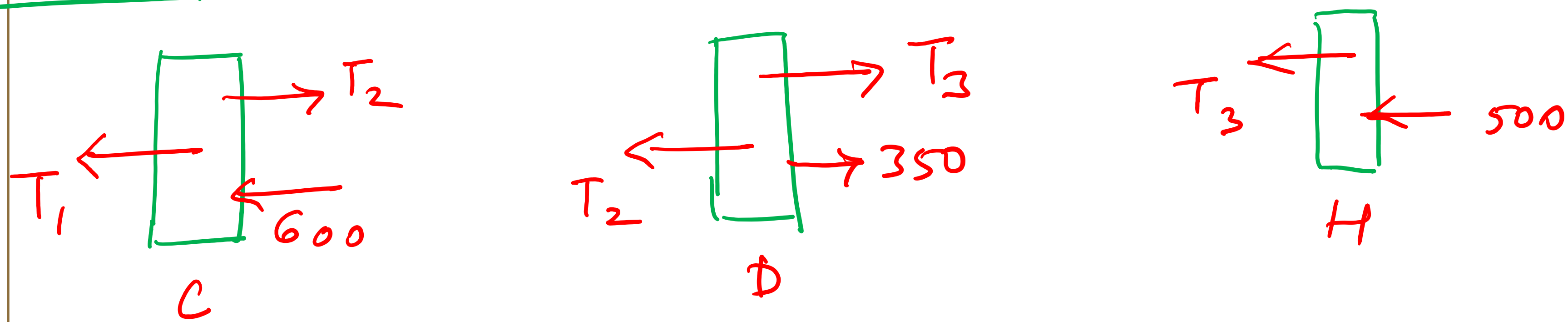
Segment ③ –



$$(\Sigma T)_3 = -T_3 - 500 = 0$$

$$\boxed{T_3 = -500 \text{ ft.lb}}$$

Method 2 FBD of connectors



Sum of torques $(\Sigma T)_C = -T_1 - 600 + T_2 = 0$

$T_2 - T_1 = 600$ —— ①

$(\Sigma T)_D = -T_2 + T_3 + 350 = 0 \Rightarrow T_2 - T_3 = 350$ —— ②

$(\Sigma T)_H = -T_3 - 500 = 0 \Rightarrow$ $\boxed{T_3 = -500 \text{ ft. lb}}$

from eq ② — $\boxed{T_2 = -150 \text{ ft. lb}}$

$T_1 = -150 - 600 = -750 \text{ ft. lb}$

Same answer with two methods. $\boxed{T_1 = -750 \text{ ft. lb}}$

THANK YOU