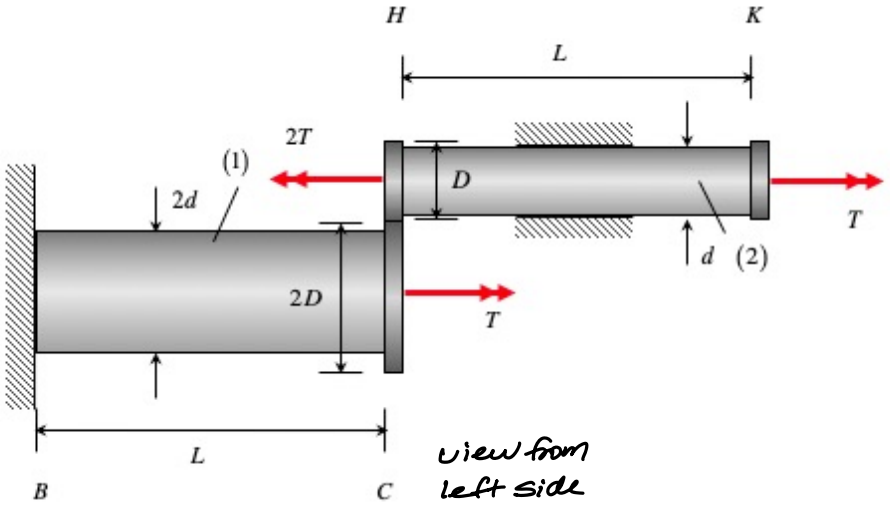
ME 323: Mechanics of Materials ***Homework Set H10 – SOLUTION***

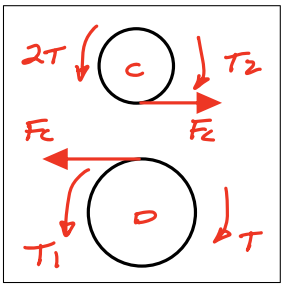
Summer 2023 ***Assigned/Due: June 23/June 27***

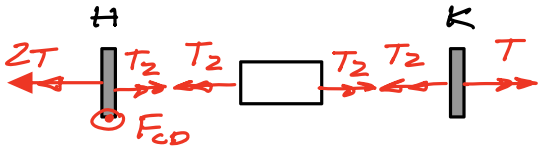
A shaft system is made up of components (1) and (2). Components (1)and (2) have outer diameters $2d$ and d, respectively. Component (1) is attached to a fixed wall at end B and to a rigid gear at C. Component (2) is attached to a rigid gear at H and to a gear at K. Gears C and H, having diameters of $2D$ and D, respectively, mesh without slipping. All shaft components have a shear modulus of G. Torques are applied to the shaft system as shown.

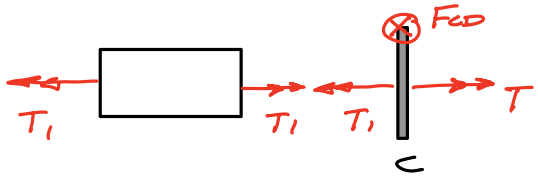
1) ***Equilibrium***. Draw free body diagrams (FBDs) of gears C, H and K. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate? If so, solve for the torques carried by components (1) and (2). Determine the maximum shear stress in the shaft system. At what location(s) does this maximum stress occur?
2) ***Torque/rotation equations***. Write down the torque/rotation equations for components (1) and (2).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the rotation of gears C, H and K to the component rotations of components (1) and (2).
4) ***Solution***. Determine the rotation angles of gears C, H and K.



view from left side







1. Equilibrium

(1) K: $\sum M = T - T_2 = 0$

↳ $T_2 = T$

(2) H: $\sum M = -2T + T_2 - F_{CH}\left(\frac{D}{2}\right) = 0$

↳ $F_{CH} = \frac{T_2 - 2T}{D/2} = -\frac{T}{D/2}$

(3) C: $\sum M = T - T_1 - F_{CH}\left(\frac{2D}{2}\right) = 0$

↳ $T_1 = T - \left(\frac{2D}{2}\right) F_{CH} = 3T$

3 equations / 3 unknowns ⇒ DETERMINATE

$$|\tau_{1,max}| = \frac{|T_1|\rho_{1,max}}{I_{p1}} = \frac{3T\,2d/2}{\frac{\pi}{2}d^4} = \frac{6T}{\pi d^3} \quad ; \quad I_{p1} = \frac{\pi}{2}\left(\frac{2d}{2}\right)^4 = \frac{\pi}{2}d^4$$

$$|\tau_{2,max}| = \frac{|T_2|\rho_{2,max}}{I_{p2}} = \frac{T(d/2)}{\frac{\pi}{32}d^4} = \frac{16T}{\pi d^3} \quad ; \quad I_{p2} = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi}{32}d^4$$

2. Torque/rotation

(4) $\Delta\phi_1 = \frac{T_1L_1}{G_1I_{p1}} = \frac{T_1L}{G(\frac{\pi}{2}d^4)} = \frac{3T_1L}{\pi Gd^4}$

(5) $\Delta\phi_2 = \frac{T_2L_2}{G_2I_{p2}} = \frac{T_2L}{G(\frac{\pi}{32}d^4)} = \frac{32T_2L}{\pi Gd^4}$

3|4. Compatibility & solve

$$\phi_C = \cancel{\phi_B^{\,0}} + \Delta\phi_1 = \Delta\phi_1 = \frac{2(3T)L}{\pi Gd^4} = \frac{6}{\pi}\frac{TL}{Gd^4}$$

$$\phi_H = -2\phi_C = -2\Delta\phi_1 = -\frac{12}{\pi}\frac{TL}{Gd^4}$$

$$\phi_K = \phi_H + \Delta\phi_2 = -2\Delta\phi_1 + \Delta\phi_2 = -2\frac{6}{\pi}\left(\frac{TL}{Gd^4}\right) + \frac{32TL}{\pi Gd^4} = \frac{20TL}{\pi Gd^4}$$