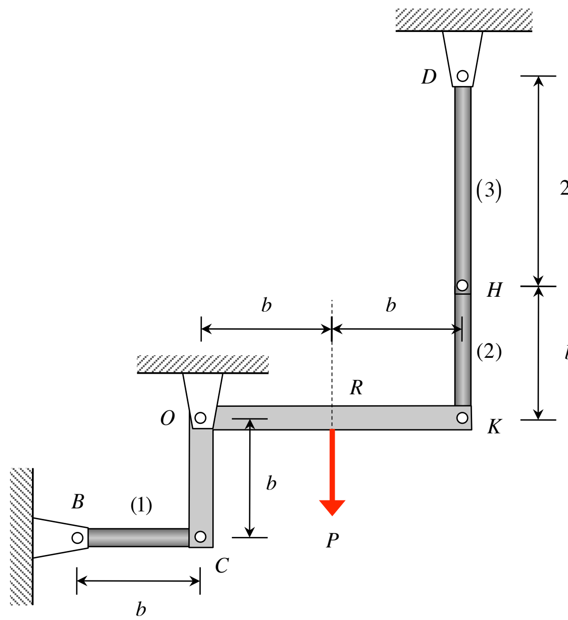
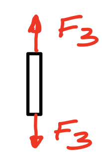
ME 323: Mechanics of Materials — ***Homework Set H09***

Summer 2023 — ***Assigned/Due: June 22/June 26***

A structure is made up of a rigid member CK and three rod elements (1), (2) and (3). The cross-sectional area and the coefficient of thermal expansion for each rod element are A and α, respectively. The Young's moduli for elements (1), (2) and (3) are E, $2E$ and E, respectively. A load P is applied to member CK as shown, with the temperature of elements (1) and (3) increased by ΔT, while the temperature of element (2) remains unchanged. The load P is given by $P = 2\alpha\Delta TEA$.



1) ***Equilibrium***. Draw free body diagrams (FBDs) of member CK and joint H. Write down the appropriate equilibrium equations from your FBDs. Is this system determinate?
2) ***Force/elongation equations***. Write down the force/elongation equations for members (1), (2) and (3).
3) ***Compatibility***. Write down the appropriate compatibility equation(s) relating the elongations of rods (1), (2) and (3).
4) ***Solution***. Solve your equations above for the loads carried by the three members. Also, determine the element elongations for (1), (2) and (3). Write your answers in terms of α, ΔT, A and b.

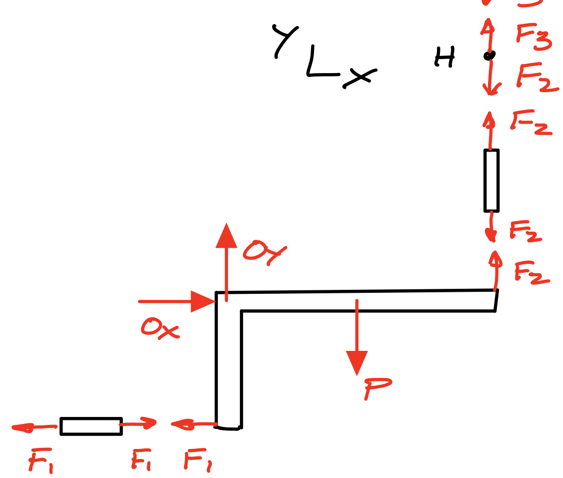


1) Equilibrium

$$\text{(1)}\quad \underline{CK}: \ \Sigma M_O = -F_1(\cancel{b}) - P(\cancel{b}) + F_2(2\cancel{b}) = 0$$
$$\hookrightarrow \ F_1 = 2F_2 - P$$

$$\text{(2)}\quad \underline{H}: \ \Sigma F_y = F_3 - F_2 = 0 \Rightarrow F_2 = F_3$$

Two equations/three unknowns $\Longrightarrow$
INDETERMINATE



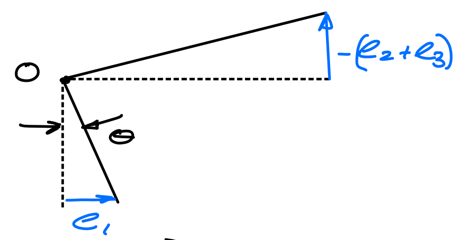
2) Force/elongation

$$\text{(3)}\quad e_1 = \frac{F_1L_1}{E_1A_1} + \alpha_1\Delta T_1 L_1 = \frac{F_1 b}{EA} + \alpha\Delta T b$$

$$\text{(4)}\quad e_2 = \frac{F_2L_2}{E_2A_2} + \alpha_2 \cancel{\Delta T_2}^{\,0} L_2 = \frac{F_2 b}{2EA}$$

$$\text{(5)}\quad e_3 = \frac{F_3L_3}{E_3A_3} + \alpha_3\Delta T_3 L_3 = \frac{2F_3 b}{EA} + \alpha\Delta T(2b)$$

3) Compatibility



$$\theta = \frac{e_1}{b} = \frac{-(e_2+e_3)}{2b}$$

(6) $\hookrightarrow \quad 2e_1 + e_2 + e_3 = 0$

4) Solve: 6 equations / 6 unknowns ($F_1, F_2, F_3, e_1, e_2, e_3$)

(3)-(6): $2\left(\frac{F_1 \cancel{b}}{EA} + \alpha \Delta T \cancel{b}\right) + \frac{F_2 \cancel{b}}{2EA} + \frac{2F_3 \cancel{b}}{EA} + 2\alpha\Delta T \cancel{b}$

(7) $\hookrightarrow \quad 4F_1 + F_2 + 4F_3 = -8\alpha\Delta T EA$

(1), (2): $4(2F_2 - P) + F_2 + 4F_2 = -8\alpha\Delta T EA$

$\hookrightarrow \quad (8+1+4)F_2 = -8\alpha\Delta T EA + 4(\alpha\Delta T EA) = 0$

$\hookrightarrow \quad F_2 = 0 = F_3 \longleftarrow F_2, F_3$

(1) $\Rightarrow \quad F_1 = 2\cancel{F_2}^{\,0} - P = -2\alpha\Delta T EA \longleftarrow F_1$

(3)-(5) $\Rightarrow \quad e_1 = \frac{F_1 b}{EA} + \alpha\Delta T b = -2\alpha\Delta T b + \alpha\Delta T b = -\alpha\Delta T b \longleftarrow e_1$

$$e_2 = \frac{\cancel{F_2}^{\,0} b}{2EA} = 0 \longleftarrow e_2$$

$$e_3 = \frac{2\cancel{F_3}^{\,0} b}{EA} + 2\alpha\Delta T b = 2\alpha\Delta T b \longleftarrow e_3$$