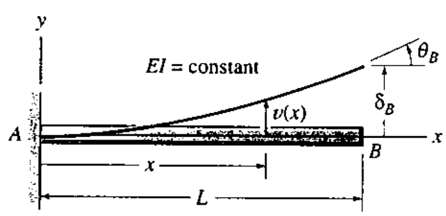
APPENDIX E

DEFLECTIONS AND SLOPES OF BEAMS; FIXED-END ACTIONS

R.R. CRAIG

E.1. Deflections and Slopes of Uniform Cantilever Beams*



Notation

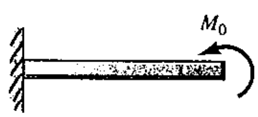
$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\delta_B \equiv v(L)$ = deflection at end B

$\theta_B \equiv v'(L)$ = slope at end B

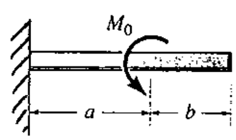
1



$$v = \frac{M_0 x^2}{2EI} \qquad v' = \frac{M_0 x}{EI}$$

$$\delta_B = \frac{M_0 L^2}{2EI} \qquad \theta_B = \frac{M_0 L}{EI}$$

2

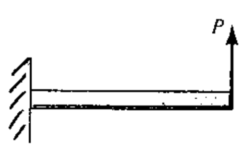


$$v = \frac{M_0 x^2}{2EI} \qquad v' = \frac{M_0 x}{EI} \qquad 0 \le x \le a$$

$$v = \frac{M_0 a}{2EI}(2x - a) \qquad v' = \frac{M_0 a}{EI} \qquad a \le x \le L$$

$$\delta_B = \frac{M_0 a}{2EI}(2L - a) \qquad \theta_B = \frac{M_0 a}{EI}$$

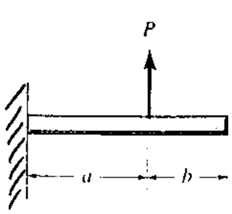
3



$$v = \frac{Px^2}{6EI}(3L - x) \qquad v' = \frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

4

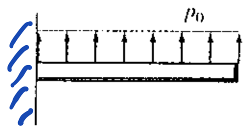


$$v = \frac{Px^2}{6EI}(3a - x) \qquad v' = \frac{Px}{2EI}(2a - x) \qquad 0 \le x \le a$$

$$v = \frac{Pa^2}{6EI}(3x - a) \qquad v' = \frac{Pa^2}{2EI} \qquad a \le x \le L$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

5

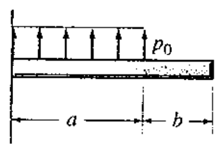


$$v = \frac{p_0 x^2}{24EI}(6L^2 - 4Lx + x^2)$$

$$v' = \frac{p_0 x}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{p_0 L^4}{8EI} \qquad \theta_B = \frac{p_0 L^3}{6EI}$$

6



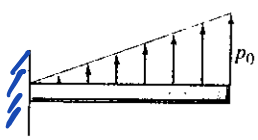
$$v = \frac{p_0 x^2}{24EI}(6a^2 - 4ax + x^2) \qquad 0 \le x \le a$$

$$v' = \frac{p_0 x}{6EI}(3a^2 - 3ax + x^2) \qquad 0 \le x \le a$$

$$v = \frac{p_0 a^3}{24EI}(4x - a) \qquad v' = \frac{p_0 a^3}{6EI} \qquad a \le x \le L$$

$$\delta_B = \frac{p_0 a^3}{24EI}(4L - a) \qquad \theta_B = \frac{p_0 a^3}{6EI}$$

7

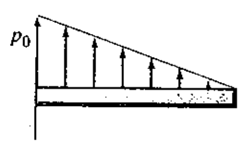


$$v = \frac{p_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$v' = \frac{p_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

$$\delta_B = \frac{11 p_0 L^4}{120EI} \qquad \theta_B = \frac{p_0 L^3}{8EI}$$

8

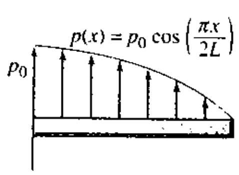


$$v = \frac{p_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$v' = \frac{p_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

$$\delta_B = \frac{p_0 L^4}{30EI} \qquad \theta_B = \frac{p_0 L^3}{24EI}$$

9



$$v = \frac{p_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 Lx^2 - \pi^3 x^3\right)$$

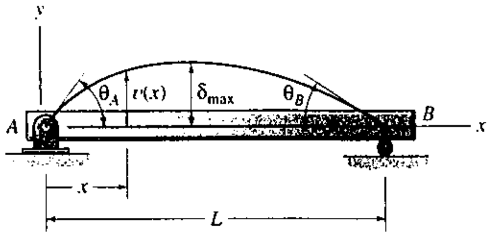
$$v' = \frac{p_0 L}{\pi^3 EI}\left(2\pi^2 Lx - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$

$$\delta_B = \frac{2 p_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \qquad \theta_B = \frac{p_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

*Beam-deflection theory is covered in Chapter 7. The sign convention used here is the same as in Chapter 7.

E.2. Deflections and Slopes of Uniform Simply-Supported Beams*

Notation



EI = constant

$v(x)$ = deflection in the y direction

$v'(x)$ = slope of the deflection curve

$\theta_A \equiv v'(0)$ = slope (angle) at end A

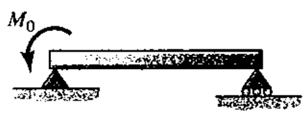
$\theta_B \equiv -v'(L)$ = angle of rotation at end B

x_m = distance from end A to the point of maximum deflection

$\delta_C \equiv |v(L/2)|$ = deflection at the center of the beam

$\delta_{max} \equiv \max |v(x)|$ = maximum deflection

1



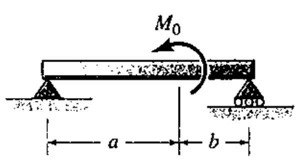
$$v = \frac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$$

$$v' = \frac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$$

$$\theta_A = \frac{M_0 L}{3EI} \qquad \theta_B = \frac{M_0 L}{6EI}$$

$$x_m = L\left(1 - \frac{\sqrt{3}}{3}\right) \text{ and } \delta_{max} = \frac{M_0 L^2}{9\sqrt{3}EI}$$

2

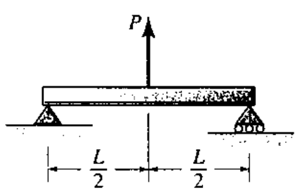


$$v = \frac{-M_0 x}{6LEI}(6aL - 3a^2 - 2L^2 - x^2) \qquad 0 \le x \le a$$

$$v' = \frac{-M_0}{6LEI}(6aL - 3a^2 - 2L^2 - 3x^2) \qquad 0 \le x \le a$$

$$\theta_A = \frac{-M_0}{6LEI}(6aL - 3a^2 - 2L^2) \qquad \theta_B = \frac{-M_0}{6LEI}(3a^2 - L^2)$$

3

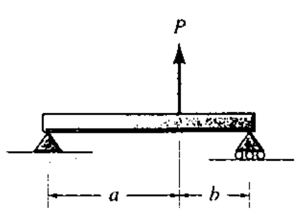


$$v = \frac{Px}{48EI}(3L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$v' = \frac{P}{16EI}(L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$\delta_C = \delta_{max} = \frac{PL^3}{48EI} \qquad \theta_A = \theta_B = \frac{PL^2}{16EI}$$

4



$$v = \frac{Pbx}{6LEI}(L^2 - b^2 - x^2) \qquad 0 \le x \le a$$

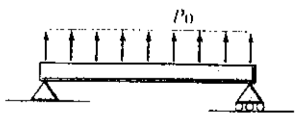
$$v' = \frac{Pb}{6LEI}(L^2 - b^2 - 3x^2) \qquad 0 \le x \le a$$

$$\theta_A = \frac{Pab(L + b)}{6LEI}$$

$$\theta_B = \frac{Pab(L + a)}{6LEI}$$

$$\text{If } a \ge b,\ x_m = \sqrt{\frac{L^2 - b^2}{3}} \text{ and } \delta_{max} = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$$

5

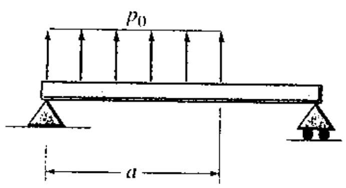


$$v = \frac{p_0 x}{24EI}(L^3 - 2Lx^2 + x^3)$$

$$v' = \frac{p_0}{24EI}(L^3 - 6Lx^2 + 4x^3)$$

$$\delta_C = \delta_{max} = \frac{5p_0L^4}{384EI} \qquad \theta_A = \theta_B = \frac{p_0L^3}{24EI}$$

6



$$v = \frac{p_0 x}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3) \qquad 0 \le x \le a$$

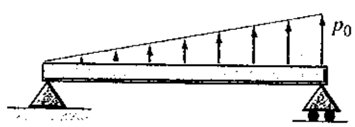
$$v' = \frac{p_0}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3) \qquad 0 \le x \le a$$

$$v = \frac{p_0a^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3) \qquad a \le x \le L$$

$$v' = \frac{p_0a^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2) \qquad a \le x \le L$$

$$\theta_A = \frac{p_0a^2}{24LEI}(2L - a)^2 \qquad \theta_B = \frac{p_0a^2}{24LEI}(2L^2 - a^2)$$

7



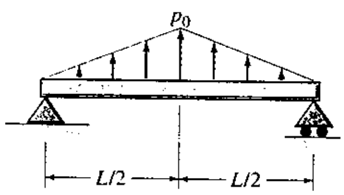
$$v = \frac{p_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$v' = \frac{p_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

$$\theta_A = \frac{7p_0L^3}{360EI} \qquad \theta_B = \frac{p_0L^3}{45EI}$$

$$x_m = 0.5193\ L \qquad \delta_{max} = 0.00652\frac{p_0L^4}{EI}$$

8

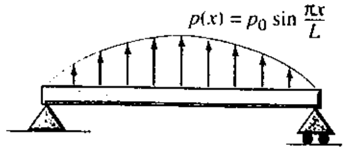


$$v = \frac{p_0 x}{960LEI}(5L^2 - 4x^2)^2 \qquad 0 \le x \le \frac{L}{2}$$

$$v' = \frac{p_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2) \qquad 0 \le x \le \frac{L}{2}$$

$$\delta_C = \delta_{max} = \frac{p_0L^4}{120LEI} \qquad \theta_A = \theta_B = \frac{5p_0L^3}{192EI}$$

9



$$v = \frac{p_0L^4}{\pi^4EI}\sin\left(\frac{\pi x}{L}\right)$$

$$v' = \frac{p_0L^3}{\pi^3EI}\cos\left(\frac{\pi x}{L}\right)$$

$$\delta_C = \delta_{max} = \frac{p_0L^4}{\pi^4EI} \qquad \theta_A = \theta_B = \frac{p_0L^3}{\pi^3EI}$$

*Beam-deflection theory is covered in Chapter 7. The sign convention used here is the same as in Chapter 7.