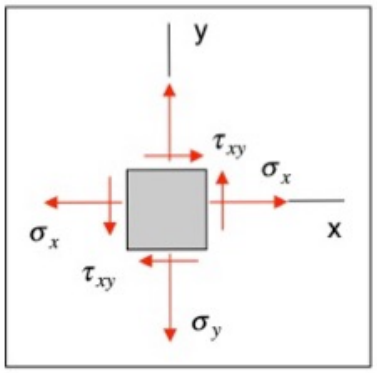
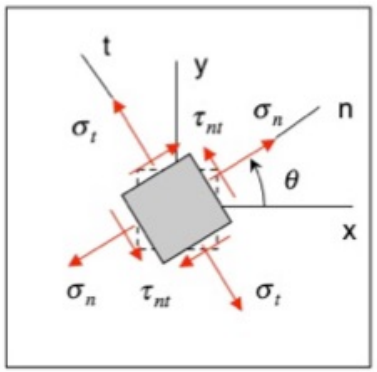
Summary: Stress transformation

PROBLEM: Given a state of plane stress at a point on a body in terms of its *xy*-components: σ_x, σ_y and τ_{xy}. The goal is to find the components of stress if the stress element is rotated counterclockwise through an angle of θ.



xy-components of stress



nt-components of stress

RESULTS:

General stress transformation results:

$$\sigma_n(\theta) = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) cos2\theta + \tau_{xy} sin2\theta$$

$$\sigma_t(\theta) = \sigma_n(\theta + 90°) = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2}\right) cos2\theta - \tau_{xy} sin2\theta$$

$$\tau_{nt}(\theta) = -\left(\frac{\sigma_x - \sigma_y}{2}\right) sin2\theta + \tau_{xy} cos2\theta$$

Max/min values of stress components

$$\sigma_{P2} \le \sigma_n \le \sigma_{P1}$$
$$-R \le \tau_{nt} \le R$$

where

$$\sigma_{P1} = \sigma_{ave} + R$$
$$\sigma_{P2} = \sigma_{ave} - R$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$