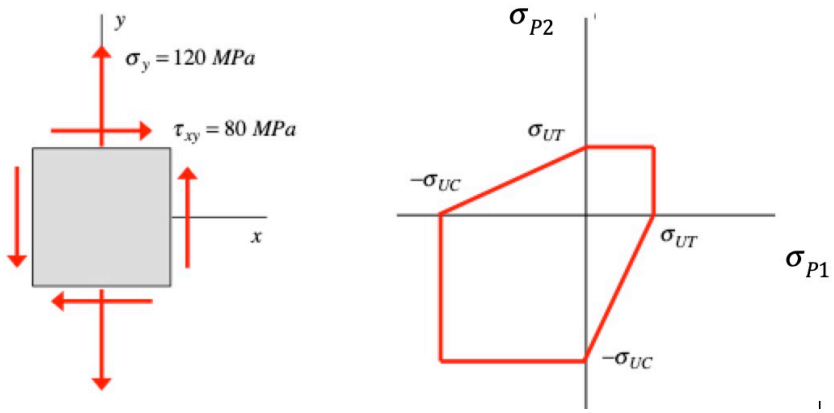
Example 15.7

Consider the state of stress shown below that exists at a location within a component made up of a *brittle* material, where this brittle material has tensile and compressive ultimate strengths of $\sigma_{UT} = 170\ MPa$ and $\sigma_{UC} = 850\ MPa$, respectively. Has the material failed at this location in the component?



SOLUTION
From the above state of stress, we have:

$$\sigma_{ave} = \frac{\sigma_y}{2} = \frac{120}{2} = 60\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100\ MPa$$

Therefore,

$$\sigma_1 = \sigma_{ave} + R = 60 + 100 = 160\ MPa$$

$$\sigma_2 = \sigma_{ave} - R = 60 - 100 = -40\ MPa$$

This state appears in the 4th quadrant of the $\sigma_{P1} - \sigma_{P2}$ plane. Note that the failure boundary in the 4th quadrant for this material is given by:

$$\frac{\sigma_{P1}}{\sigma_{UT}} - \frac{\sigma_{P2}}{\sigma_{UC}} = 1$$

For our state of stress and for our ultimate strengths in compression and tension:

$$\frac{\sigma_{P1}}{\sigma_{UT}} - \frac{\sigma_{P2}}{\sigma_{UC}} = \frac{160}{170} - \frac{(-40)}{850} = 0.988 < 1$$

Therefore, Mohr's theory does NOT predict failure for this material.