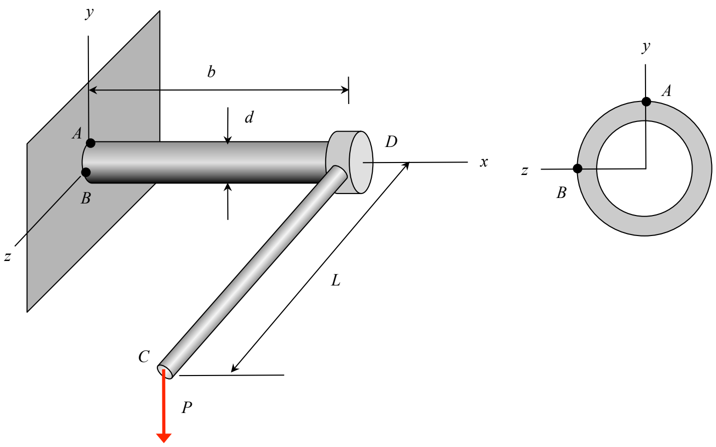
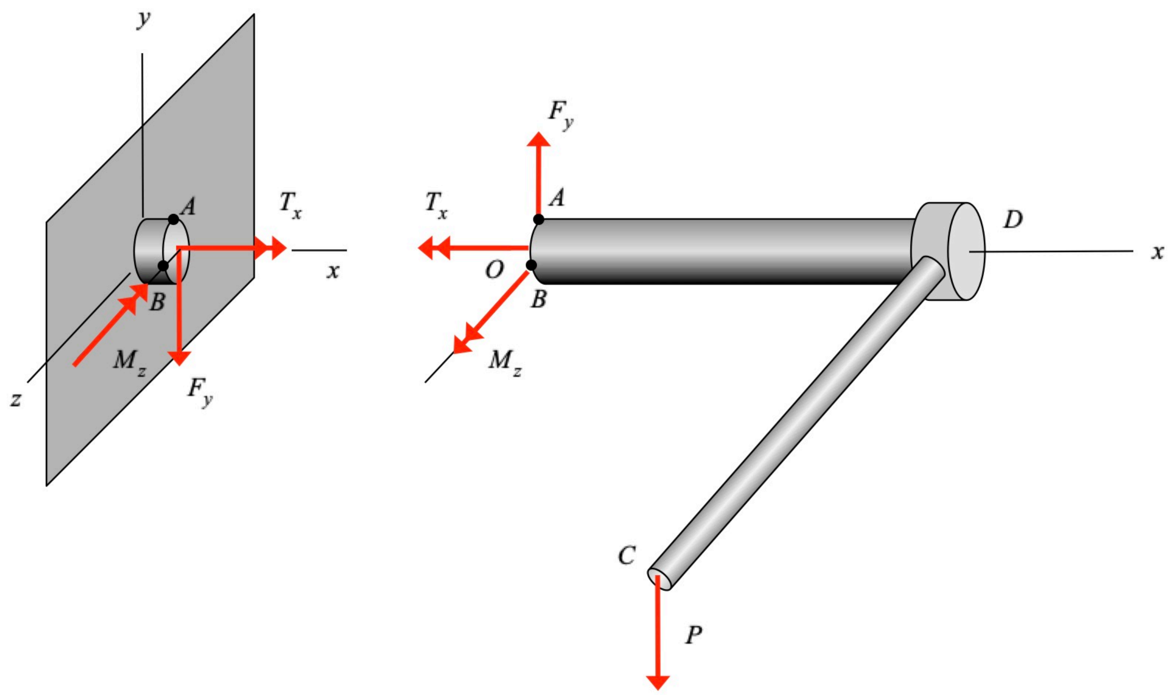
Example 14.4 - SOLUTION

A vertical force of P is applied to the end of a pipe wrench CD, whose handle is parallel to the z-axis. Determine the pipe has an outer diameter of d and wall thickness of t. Determine the principal stresses at points A and B on the cross section of the pipe.



SOLUTION

<u>*Equilibrium*</u>



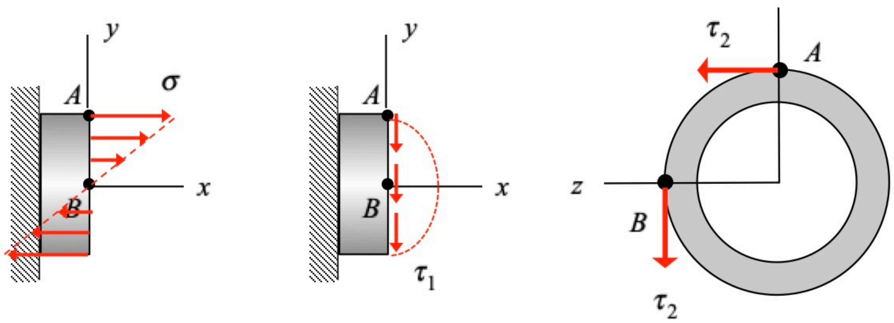
$$\sum F_y = -P + F_y = 0 \quad \Rightarrow \quad F_y = P$$

$$\begin{aligned}\sum M_O &= -T_x\hat{i} + M_z\hat{k} + \vec{r}_{C/O} \times \vec{P} = \vec{0} \\ &= -T_x\hat{i} + M_z\hat{k} + \left(b\hat{i} + L\hat{k}\right) \times \left(-P\hat{j}\right) \\ &= \left(-T_x + PL\right)\hat{i} + \left(M_z - Pb\right)\hat{k} \quad \Rightarrow\end{aligned}$$

$\hat{i}$: $\quad T_x = PL$

$\hat{k}$: $\quad M_z = Pb$

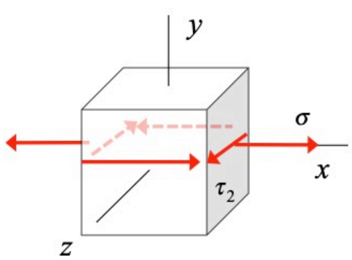
Stress distributions



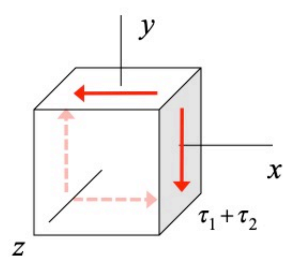
Stresses at A and B

internal resultant	***stress @ A***	***stress @ B***
F_y	0	$\tau_1 = \dfrac{F_y Q}{It} = \dfrac{PQ}{It}$
T_x	$\tau_2 = \dfrac{T_x(d/2)}{2I_P} = \dfrac{PLd}{2I_P}$	$\tau_2 = \dfrac{T_x(d/2)}{2I_P} = \dfrac{PLd}{2I_P}$
M_z	$\sigma = \dfrac{M_z(d/2)}{I} = \dfrac{Pbd}{2I}$	0

Stress elements for A and B



stress element for A



stress element for B

Principal stresses

At A: $\sigma_{ave} = \dfrac{\sigma}{2} = \dfrac{Pbd}{4I}$; $R = \sqrt{\left(\dfrac{\sigma}{2}\right)^2 + \tau_2^2} = \sqrt{\left(\dfrac{Pbd}{4I}\right)^2 + \left(\dfrac{PLd}{2I_P}\right)^2}$; $\sigma_{P1,2} = \sigma_{ave} \pm R$

At B: $\sigma_{ave} = 0$; $R = \sqrt{0 + \tau_2^2} = \dfrac{PLd}{2I_P}$; $\sigma_{P1,2} = \pm R$