Summary: work/energy and Castigliano

- *General strain energy and work expressions*:

member loading	strain energy	external work
axial, P	$U_{axial} = \frac{1}{2}\int_0^L \frac{F^2}{EA} dx$	$W_{axial} = \frac{1}{2} F_0 e$
torque, T	$U_{torsion} = \frac{1}{2}\int_0^L \frac{T^2}{GI_P} dx$	$W_{torsion} = \frac{1}{2} T_0 \phi$
bending moment, M	$U_{\sigma} = \frac{1}{2}\int_0^L \frac{M^2}{EI} dx$	$W_{moment} = \frac{1}{2} M_0 \theta$
shear force, V	$U_{\tau} = \frac{1}{2}\int_0^L \frac{f_S V^2}{GA} dx$	$W_{shear} = \frac{1}{2} P_0 y$

- *Work/energy equation:* $U = W$

  A single equation regardless of loading. Not useful for finding deflections when loadings act at multiple points.

- *Castigliano's theorem – determinate structures:* $\Delta_i = \frac{\partial U}{\partial P_i}$
  - *Can be used for finding deflections at a single point even in the presence of loadings at multiple points.*
  - *Useful only when loading is applied at points where deflections are sought. If no loading acts where you want to find deflection, add a DUMMY loading P_d, solve for deflection, and then set $P_d = 0$.*