

**EQUATIONS**

Small angle ( $\theta \approx 0$ ) approximations

$$\sin(\theta) \approx \theta \quad \cos(\theta) \approx 1 \quad \tan(\theta) \approx \theta$$

Average stress

$$(\sigma)_{ave} = \frac{P}{A} \quad (\tau)_{ave} = \frac{V}{A}$$

Generalized Hooke's Law

$$\begin{aligned}\epsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha\Delta T \\ \epsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] + \alpha\Delta T \\ \epsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha\Delta T \\ \sigma_x &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z) - (1+\nu)\alpha\Delta T] \\ \sigma_y &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_y + \nu(\epsilon_x + \epsilon_z) - (1+\nu)\alpha\Delta T] \\ \sigma_z &= \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_z + \nu(\epsilon_x + \epsilon_y) - (1+\nu)\alpha\Delta T] \\ \gamma_{xy} &= \frac{1}{G}\tau_{xy} \\ \gamma_{yz} &= \frac{1}{G}\tau_{yz} \\ \gamma_{xz} &= \frac{1}{G}\tau_{xz} \\ G &= \frac{E}{2(1+\nu)}\end{aligned}$$

Axial deformations

$$\begin{aligned}e_{AB} &= u_B - u_A \\ e &= \int_0^L \frac{F}{AE} dx + \int_0^L \alpha\Delta T dx \\ e &= \frac{FL}{AE} + \alpha\Delta TL \\ e &= u \cos \theta + v \sin \theta\end{aligned}$$

Torsional deformations

$$\begin{aligned}\phi_{AB} &= \phi_B - \phi_A \\ \phi &= \int_0^L \frac{T(x)}{G(x)I_p(x)} dx \\ \phi &= \frac{TL}{GI_p} \\ \gamma &= \rho \frac{d\phi}{dx} \\ \gamma &= \frac{\rho T}{I_p} \\ \tau &= G\rho \frac{d\phi}{dx} \\ \tau &= \frac{\rho T}{I_p} \\ I_p &= \frac{\pi}{2}r^4 \text{ (solid)} \quad I_p = \frac{\pi}{2}(r_o^4 - r_i^4) \text{ (hollow)}\end{aligned}$$